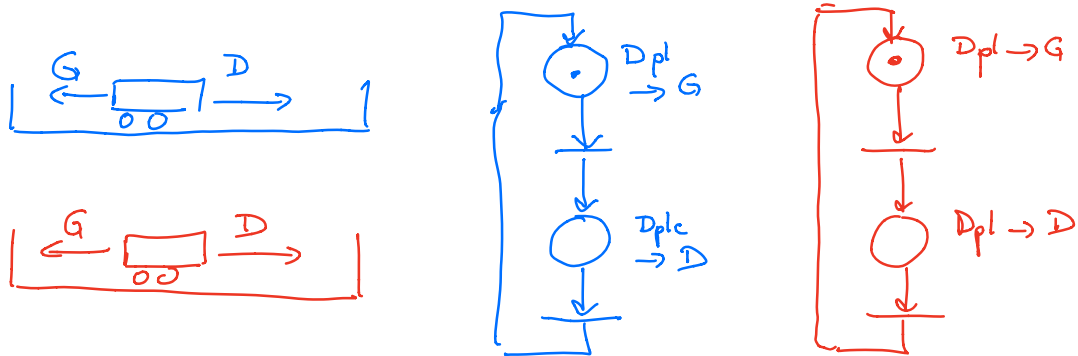


- Rdp G $\rightarrow$ jetons indifférenciés
- Rdp Coloré $\rightarrow$ jeton possède une couleur



$$f(\langle b \rangle) = \langle b \rangle$$

$$f(\langle r \rangle) = \langle r \rangle$$

jeton : 1 ensemble de couleurs
avec 2 couleurs $\langle b \rangle, \langle r \rangle$
positives

jeton : 2 ens de couleurs
 $\{r, b\}$ $\{G, D\}$

$$\langle r, G \rangle$$

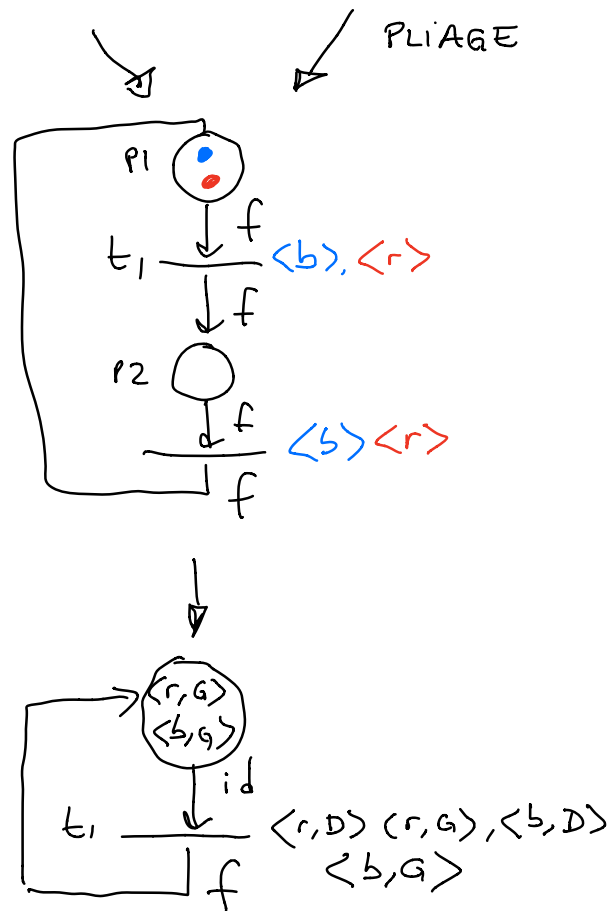
$$\langle b, G \rangle$$

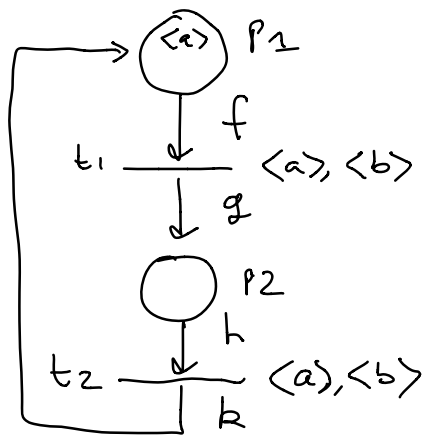
$$f(\langle b, G \rangle) = \langle b, D \rangle$$

$$f(\langle b, D \rangle) = \langle b, G \rangle$$

$$f(\langle r, D \rangle) = \langle r, G \rangle$$

$$f(\langle r, G \rangle) = \langle r, D \rangle$$





$$\Pi_{\text{marquage}} \begin{pmatrix} \langle a \rangle + \langle b \rangle \\ \langle b \rangle \end{pmatrix}$$

Matrice incidence

$$\begin{matrix} & t_1 & t_2 \\ P_1 & \begin{pmatrix} -f & +k \end{pmatrix} \\ P_2 & \begin{pmatrix} +g & -h \end{pmatrix} \end{matrix}$$

Vecteur séquence $\begin{pmatrix} \text{RdPG} \\ \sigma = t_1 t_2 t_1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\hookrightarrow \text{RdPC } G = t_1/a \ t_1/b \ t_2/a$

$$S_\sigma = \begin{pmatrix} a+b \\ a \end{pmatrix}$$

- $\Pi_k = \Pi_0 + W \cdot S$
- Invariants de marquage

RdPG $2m(P1) + \beta m(P2) = \text{cste} \Rightarrow$

$\nwarrow$ entiers $\nearrow$

$$F^T = (\alpha, \beta)$$

$$\boxed{F^T \cdot W = 0}$$

RdP Coloré

$$\alpha m(P1) + \beta m(P2) = \text{cste}$$

$\uparrow$ fonctions $\uparrow$ couleurs

$C = \{ \langle a \rangle, \langle b \rangle \}$ ens de 2 valeurs

jeton associé à C $\langle a \rangle \ \langle b \rangle \ \langle \bullet \rangle$

$\hookrightarrow$ jeton non coloré

transitions associées à C

$t_1 / \langle a \rangle$ franchissement de t_1
 $t_1 / \langle b \rangle$ par rapport à $\langle a \rangle$
 $\underline{\hspace{1cm}} \langle b \rangle$

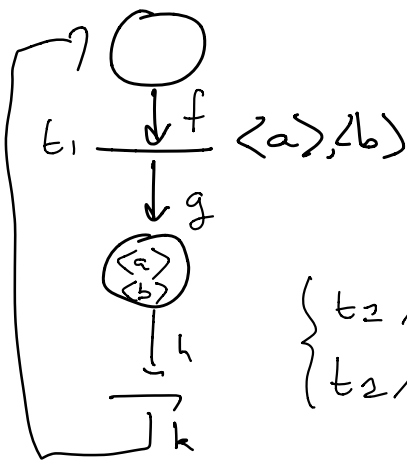
fonctions d'irruence f, g, h, k

$$\left. \begin{array}{l} f(a) = a \\ f(b) = b \end{array} \right\} \text{fonction Identité}$$

$$g(a) = 2a + b$$

$$h(a) = \langle a \rangle$$

Exemple



$$\left\{ \begin{array}{l} f(\langle a \rangle) = \langle a \rangle \\ f(\langle b \rangle) = \langle a \rangle + \langle b \rangle \end{array} \right.$$

$$\left\{ \begin{array}{l} g(a) = a + b \\ g(b) = b \end{array} \right.$$

$$\left\{ \begin{array}{l} t_2 / a ? \\ t_2 / b ? \end{array} \right. \quad \begin{array}{l} \text{oui } f(a) = a \text{ donc il faut} \\ \text{1(a) dans } P_1 \\ \text{non } f(b) = a + b \Rightarrow \langle a \rangle + \langle b \rangle \\ \text{ds } P_1 \end{array}$$

$$t_1 / a \quad \begin{pmatrix} a \\ 0 \end{pmatrix} \xrightarrow{t_1 / a} \begin{pmatrix} 0 \\ a+b \end{pmatrix}$$

$$n_k = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -f & +k \\ +g & -h \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix}$$

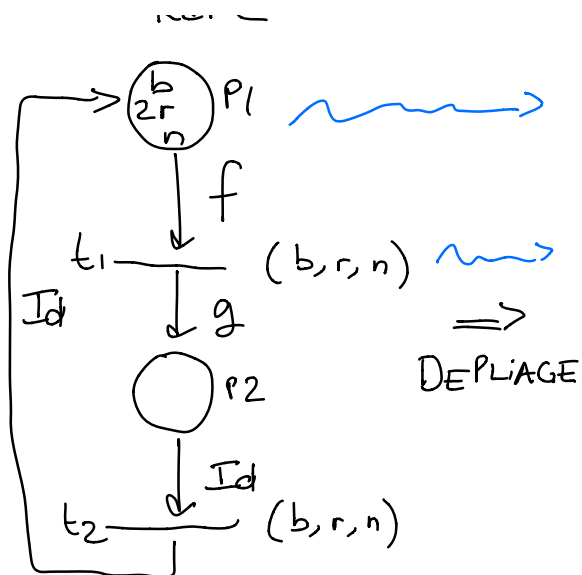
$$S = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

$\hookrightarrow t_1 / a$

marquage initial

$$\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -f(a) \\ +g(a) \end{pmatrix}$$

$$\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -a \\ a+b \end{pmatrix} = \begin{pmatrix} 0 \\ a+b \end{pmatrix}$$



$$\begin{cases} f(b) = r+b \\ f(r) = r \\ f(n) = n+2r \end{cases}$$

1 jeton dans P_{ij} (RdPG)

$\Rightarrow$ 1 jeton de valeur j dans P_i (RdPC)

$$\begin{cases} g(b) = r+n \\ g(r) = b+2n \\ g(n) = 2r \end{cases}$$

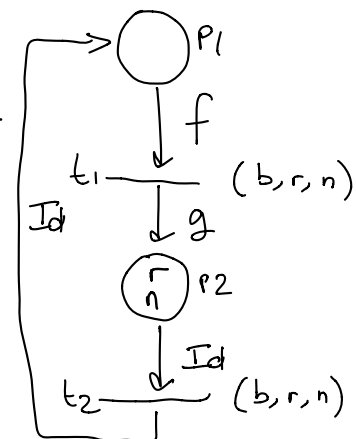
1 jeton ds P_{2n}

$\hookrightarrow$ 1 $\langle n \rangle$ dans P_2

$$W = \begin{pmatrix} t_1 & t_2 \\ -f & Id \\ g & -Id \end{pmatrix}$$

$$\begin{pmatrix} r+b \\ 0 \\ n_0 \end{pmatrix} \xrightarrow{t_1/b} \begin{pmatrix} 0 \\ r+n \\ n_1 \end{pmatrix} \xrightarrow{t_2/r} \begin{pmatrix} r \\ n \\ \dots \end{pmatrix}$$

$$\xrightarrow{t_1/r} \begin{pmatrix} b \\ b+2n \\ \dots \end{pmatrix} \xrightarrow{t_2/n} \begin{pmatrix} n \\ r \\ \dots \end{pmatrix}$$



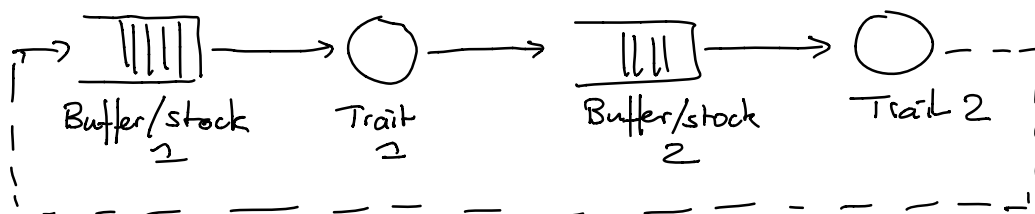
$$\sigma = t_1/b \ t_2/r \rightarrow S_\sigma = t_2 \begin{pmatrix} b \\ r \end{pmatrix}$$

$$\Pi_k = \begin{pmatrix} r+b \\ 0 \end{pmatrix} + \begin{pmatrix} -f & Id \\ +g & -Id \end{pmatrix} \begin{pmatrix} b \\ r \end{pmatrix}$$

$$\begin{pmatrix} r+b \\ 0 \end{pmatrix} + \begin{pmatrix} -f(b) + Id(r) \\ g(b) - Id(r) \end{pmatrix}$$

$$\begin{pmatrix} r+b \\ 0 \end{pmatrix} + \begin{pmatrix} -r-b+r \\ r+n-r \end{pmatrix} = \begin{pmatrix} r \\ n \end{pmatrix}$$

Exemple 2

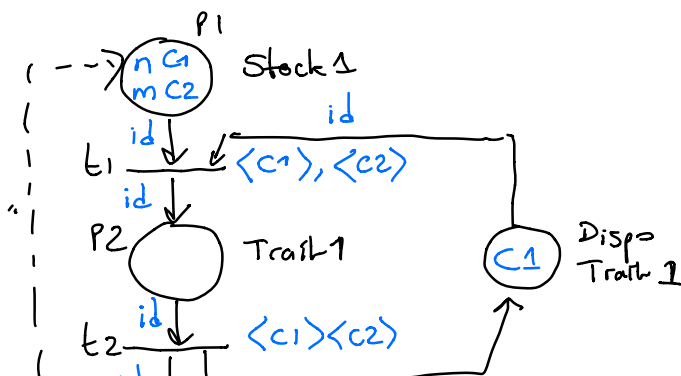


2 types d'objets/données qui circulent type 1 type 2

↳ 2 couleurs C1 C2

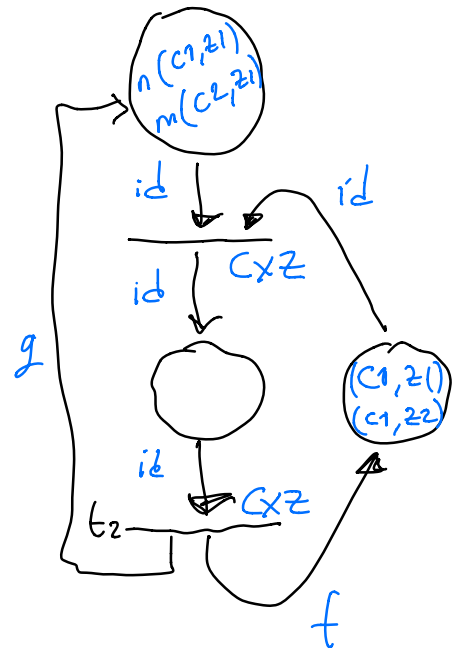
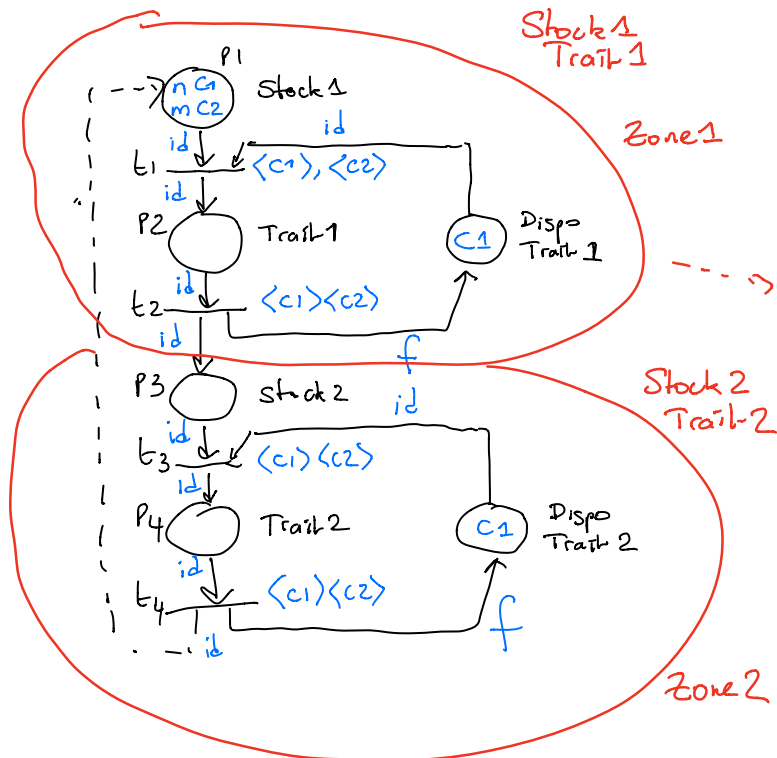
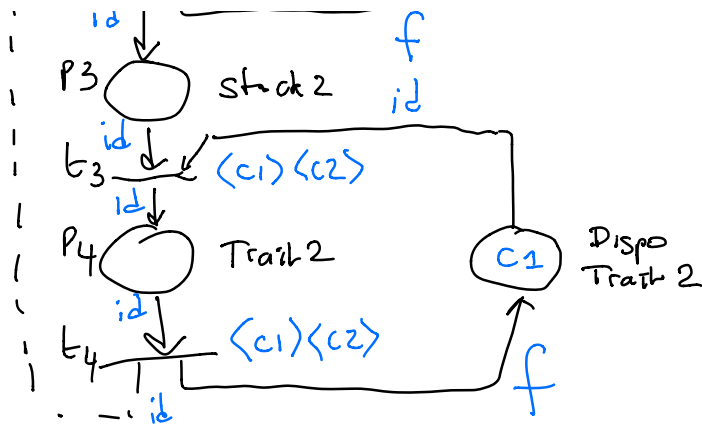
Traitement : alternance entre C1 et C2

Etat initial $\begin{cases} n \text{ objets de type 1} \\ m \text{ ————— 2} \end{cases}$



Trait 1 C1
C2
C1
C2
⋮

$$\begin{cases} f(C1) = C2 \\ f(C2) = C1 \end{cases}$$



↓
Couleurs Zone 1, Zone 2 -
↳ $\{z1, z2\} = z$

$C = \{c1, c2\}$

jeton $\rightarrow$ $C \times z$
 $(c1, z1)$
 $(c1, z2)$
...

Alte nance c1 puis c2
 $f(c1, z1) = c2, z1$
 $f(c1, z2) = c2, z2$
 $f(c2, z1) = (c1, z1)$
 $f(c2, z2) = (c1, z2)$

Alte nance m1 $\rightarrow$ Zone 2
 $g(c1, z1) = c1, z2$
 $g(c1, z2) = c1, z1$

2c