

Exo 1

Spl

$$1) \quad W = \begin{pmatrix} -1 & +1 & 0 & 0 \\ 1 & 0 & -1 & +1 \\ 0 & -1 & +1 & -1 \end{pmatrix}$$

0,5

$$2) \quad a) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & +1 & 0 & 0 \\ 1 & 0 & -1 & +1 \\ 0 & -1 & +1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (0,5)$$

b) $(x \ y \ z) \begin{pmatrix} -1 & +1 & 0 & 0 \\ 1 & 0 & -1 & +1 \\ 0 & -1 & +1 & -1 \end{pmatrix} = 0 \quad \begin{cases} -x+y=0 \\ x-z=0 \\ -y+z=0 \\ y-z=0 \end{cases} \Rightarrow x=y=z.$

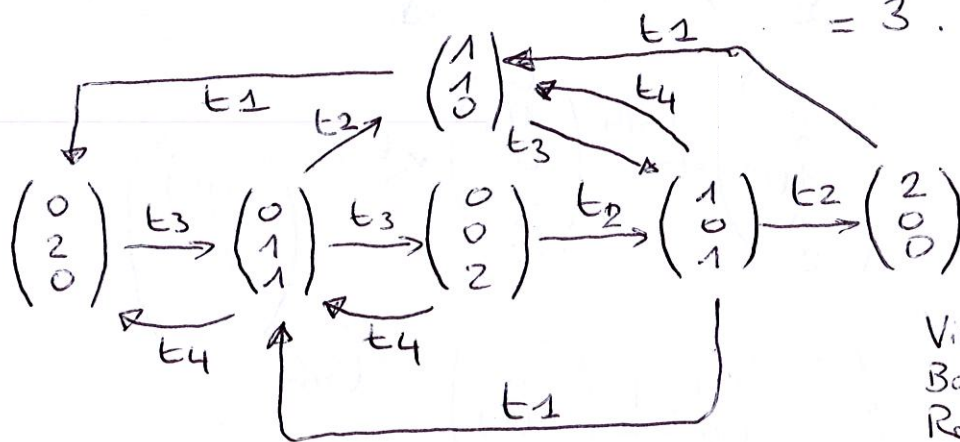
①

181 $F^T = (1 \ 1 \ 1)$

Invariant $m(P1) + m(P2) + m(P3) = F^T \cdot \eta_0 = (1 \ 1 \ 1) \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

$$= 3.$$

3) a)



②

Vivant
Borné (2)
Reint
Sans blocage

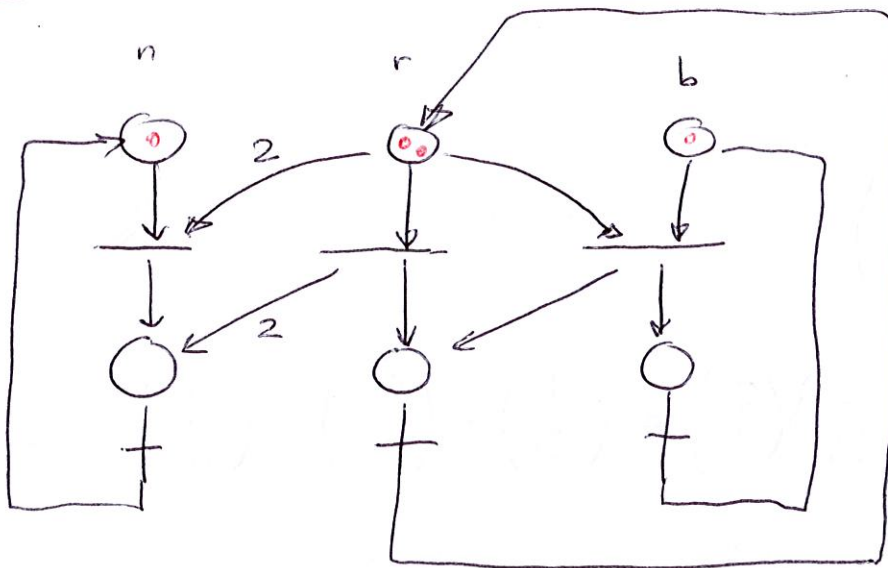
0,5

b) Changement de valeur de la constante

$$m(P_1) + m(P_2) + m(P_3) = 2.$$

0,5

Exo3



Exo4

a)

$$\begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix} \xrightarrow{t_1 t_2 / 0} \begin{pmatrix} 0 \\ 1(4) \\ 1(2) \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix} \xrightarrow{t_4 / 2} \begin{pmatrix} 0 \\ 1(2) \\ 0 \\ 0 \\ 1(3) \\ 0 \\ 1 \end{pmatrix} \xrightarrow{t_3 / 2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(4) \\ 1(1) \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(4) \\ 0 \\ 0 \\ 2 \end{pmatrix} \xleftarrow{t_3 / 4} \begin{pmatrix} 0 \\ 1(4) \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \xleftarrow{t_1 / 6} \begin{pmatrix} 1(6) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix} \xleftarrow{t_5 / 4} \begin{pmatrix} 1(6) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1(3) \\ 2 \end{pmatrix} \xleftarrow{t_4 / 2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1(2) \\ 0 \\ 0 \\ 1 \end{pmatrix} \xleftarrow{t_2 / 6} \begin{pmatrix} 1(6) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

b) non pas de régime stationnaire en blocage.

Exo2

3pts

$$1) \pi_{\min} \geq \text{Pre}(-, t_2)$$

$$\text{PRE} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 5 & 5 \end{pmatrix}$$

$$\pi_{\min} \geq \text{Pre}(-, t_2) \geq \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\begin{cases} x \geq 1 \\ y \geq 5 \end{cases}$$

$$W = \begin{pmatrix} +3 & -1 & +5 \\ +3 & -2 & -5 \end{pmatrix}$$

$$\pi_{\min} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} \geq \text{Pre}(-, t_3)$$

$$\pi_{\min} \geq \begin{pmatrix} 0 \\ +5 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\pi_{\min} \geq \begin{pmatrix} 1 \\ +7 \end{pmatrix}$$

$$\begin{cases} x \geq 6 \\ y \geq -3 \end{cases}$$

~~$$\pi_{\min} \geq \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$~~

$$\boxed{\pi_{\min} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}}$$

①

$$2) \pi_{\min} \geq \text{Pre}(-, t_3)$$

$$\pi_{\min} \geq \begin{pmatrix} 0 \\ 5 \end{pmatrix} \Rightarrow \begin{cases} x \geq 0 \\ y \geq 5 \end{cases}$$

$$\pi_{\min} + C(-, t_3) \geq \text{Pre}(-, t_2)$$

$$+ \begin{pmatrix} +5 \\ -5 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\pi_{\min} \geq \begin{pmatrix} 1 & -5 \\ 5 & 5 \end{pmatrix} \geq \begin{pmatrix} -4 \\ 10 \end{pmatrix}$$

$$\begin{cases} x \geq -4 \\ y \geq 10 \end{cases}$$

$$\boxed{\pi_{\min} = \begin{pmatrix} 0 \\ 10 \end{pmatrix}}$$

①

$$3) \boxed{\pi_{\min} = \sup = \begin{pmatrix} 1 \\ 10 \end{pmatrix}}$$

①

Exo 5

a) $X_5 = X_7.d + X_8.\bar{c} + X_5.\bar{b}$

$$X_6 = X_5.b + X_6(\overline{a+c}) = X_5.b + X_6.\bar{a}.\bar{c}$$

$$X_7 = X_6.a + X_7.\bar{d}$$

$$X_8 = X_6.c + X_8.c$$

b)

$$A_5 = X_7.d + X_8.\bar{c}$$

$$A_6 = X_5.b$$

$$A_7 = X_6.a$$

$$A_8 = X_6.c$$

$$X_5 = A_5 + X_5.\bar{b}$$

$$X_6 = A_6 + X_6.\bar{a}.\bar{c}$$

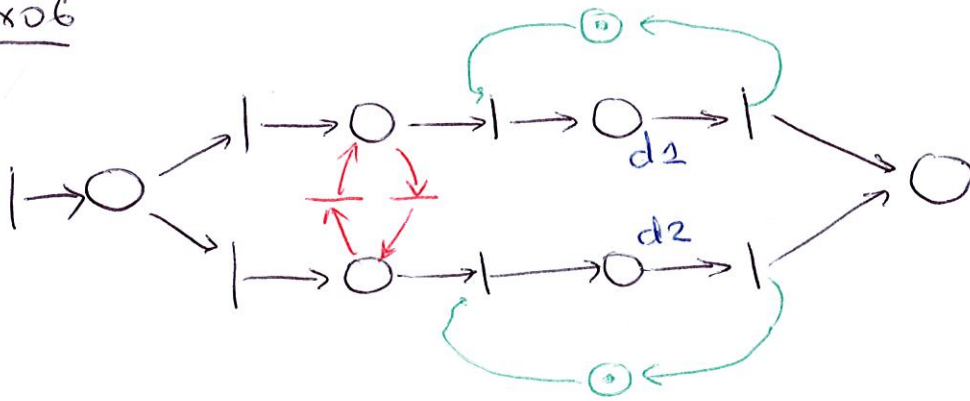
$$X_7 = A_7 + X_7.\bar{d}$$

$$X_8 = A_8 + X_8.c$$

$$S_1 = X_8$$

$$S_2 = X_7$$

Exo6

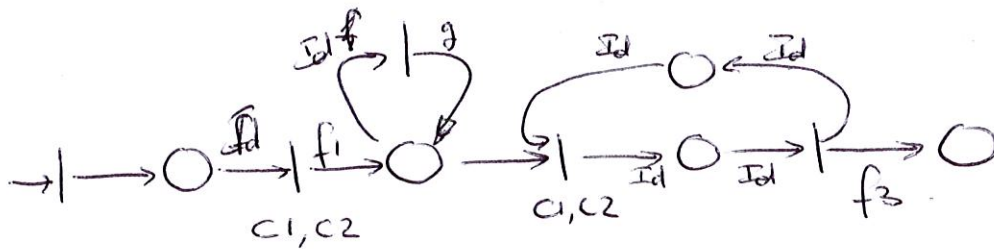


a)

b)

c)

d)



f_3 débolation
 f_1 rotation
 $\begin{cases} g(c_1) = c_2 \\ g(c_2) = c_1 \end{cases}$

e)

