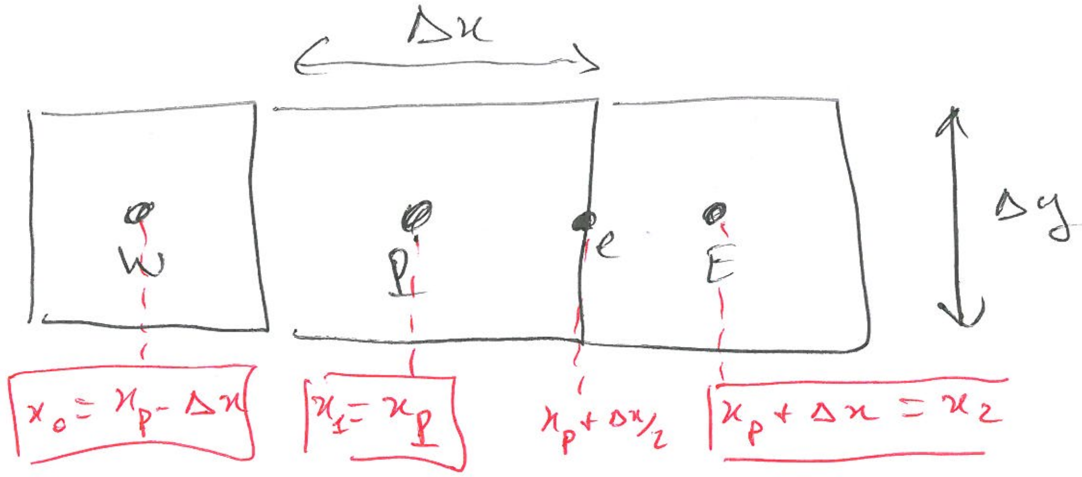
12 Mars 2014

Exo 5
K-scheme



$$\left(F_x^x\right)_e \overset{?}{=} u_e \, \Delta y \, \Phi_e \, .$$

Q1) Quick scheme.

1-c)

$$\phi_e = \frac{\phi_{i+1} + \phi_i}{2} - \frac{\Delta x^2}{2} \, \delta_{xx}^{c} \, \phi_i$$

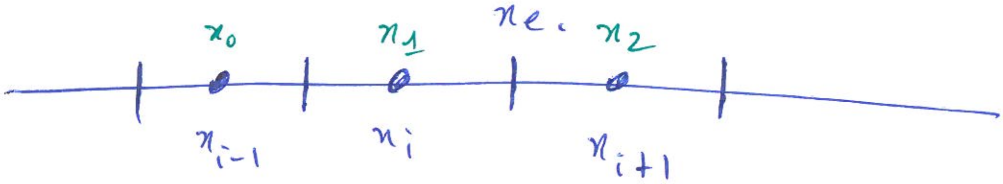
$$= \frac{\phi_{i+1} + \phi_i}{2} - \frac{\Delta x^2}{2} \left(\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2} \right)$$

$$= \phi_{i+1} \underbrace{\left(\frac{1}{2} - \frac{1}{8} \right)}_{3/8} + \phi_i \underbrace{\left(\frac{1}{2} + \frac{1}{4} \right)}_{3/4} - \frac{1}{8} \phi_{i-1}$$

CQFD.

Interpretation: We had a diffusive term to CDS interpolation to smooth the oscillation of central scheme by adding numerical dissipation.

π-Schéma b) | (Quick-interpolation)



$$x_0 = x_{i-1} = x_i - \Delta x.$$

$$x_1 = x_i$$

$$x_2 = x_{i+1} = x_i + \Delta x.$$

$$x_e = x_i + \frac{\Delta x}{2}.$$

$$\Pi_2[\phi(x)] = \sum_{j=0}^{2} f(x_j) L_j(x)$$

$$= L_0(x_e)\,\Phi_{i-1} + L_1(x_e)\,\Phi_i + L_2(x_e)\,\Phi_{i+1}$$

avec $L_0(x) = \dfrac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}.$

$$L_0(x_e) = \frac{(x_i + \Delta x/2 - x_i)(x_i + \Delta x/2 - x_i - \Delta x)}{(x_i - \Delta x - x_i)(x_i - \Delta x - x_i - \Delta x)}$$

$$= \frac{\frac{1}{2}\Delta x \times -\frac{1}{2}\Delta x}{-\Delta x \times -2\Delta x} = -\frac{1}{8} \text{ ok.}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$

$$L_1(x_e) = \frac{(x_i + \Delta x/2 - x_i + \Delta x)(x_i + \Delta x/2 - x_i - \Delta x)}{(x_i - x_i + \Delta x)(x_i - x_i - \Delta x)}$$

$$= \frac{\frac{3}{2}\Delta x \times -\frac{1}{2}\Delta x}{\Delta x \times -\Delta x} = +\frac{3}{4}.$$

h-Schme 7)

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$$L_2(x_e) = \frac{(x_i + \Delta x/2 - x_i + \Delta x)(x_i + \Delta x/2 - x_i)}{(x_i + \Delta x - x_i + \Delta x)(x_i + \Delta x - x_i)}$$

$$= \frac{\frac{3}{2} \times \frac{1}{2}}{2 \times 1} = \frac{3}{8} /_m .$$

$$\Rightarrow \phi_e = \frac{3}{8} \phi_{i+1} + \frac{3}{4} \phi_i - \frac{1}{8} \phi_{i-1} .$$

Q 1 d

$$\phi_e = \frac{3}{8}\phi_{i+1} + \frac{3}{4}\phi_i - \frac{1}{8}\phi_{i-1}$$

DL en $x = x_e$ avec $x_{i+1} = x_e + \Delta x/2$

$x_i = x_e - \Delta x/2$

$x_{i+1} = x_e - \frac{3\Delta x}{2}$

$$\frac{3}{8}\times\phi_{i+1} = \phi_e + \frac{\Delta x}{2}\phi'_e + \frac{\Delta x^2}{8}\phi''_e + \frac{\Delta x^3}{48}\phi'''_e + O(\Delta x^4)$$

$$+\frac{3}{4}\times\phi_i = \phi_e - \frac{\Delta x}{2}\phi'_e + \frac{\Delta x^2}{8}\phi''_e - \frac{\Delta x^3}{48}\phi'''_e + O(\Delta x^4)$$

$$-\frac{1}{8}\times\phi_{i-1} = \phi_e - \frac{3\Delta x}{2}\phi'_e + \frac{9\Delta x^2}{8}\phi''_e - \frac{9}{4}\Delta x^3\phi'''_e + O(\Delta x^4)$$

$\cancel{27}/48$

$$\phi_{\text{recherchée}} = \underbrace{\left(\frac{3}{8}+\frac{3}{4}-\frac{1}{8}\right)}_{1}\phi_e + \Delta x\underbrace{\left(\frac{3}{16}-\frac{3}{8}+\frac{3}{16}\right)}_{0}\phi'_e$$

$$+ \Delta x^2\underbrace{\left(\frac{3}{64}+\frac{3}{32}-\frac{9}{64}\right)}_{=0}\phi''_e$$

$$+ \Delta x^3\underbrace{\left(\frac{3}{8}\times\frac{1}{48}-\frac{3}{4}\times\frac{1}{48}+\cancel{\frac{9}{4\times 8}}\right)}\phi'''_e$$

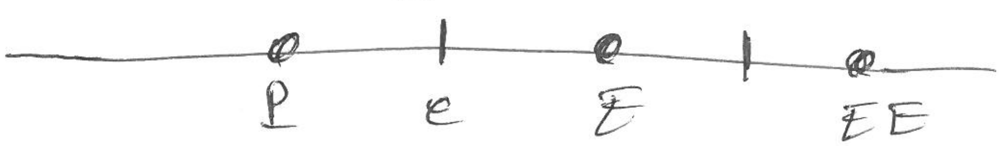
$\frac{35}{128}$ ~~droite~~ ! Faux

$\frac{1}{16}\Delta x^3$

A reprendre

e)

$u_e < 0$



2) $u_e = u_w = u > 0$.

$$\phi_e = \frac{\phi_{i+1} + \phi_i}{2} + \frac{(k-1)}{4}\left[\phi_{i+1} - \cancel{\phi_i} \overset{2\phi_i}{\cancel{-(\phi_i}} + \phi_{i-1})\right]$$

$$\phi_w = \frac{\phi_i + \phi_{i-1}}{2} + \frac{(k-1)}{4}\left[\phi_i - \cancel{\phi_{i-1}} \overset{2\phi_{i-1}}{\cancel{-(\phi_{i-1}}} + \phi_{i-2})\right]$$

$$u D_x^{(u)} \phi_i = \frac{u}{\Delta x}\left[\phi_e - \phi_w\right]$$

$$= \phi_{i+1}\left(\frac{1+k}{4}\right) + \phi_{i}\underbrace{\left(\frac{1}{2} - 2\frac{(k-1)}{4} - \frac{1}{2} - \frac{(k-1)}{4}\right)}_{\frac{3}{4}(1-k)}$$

$$+ \phi_{i-1}\underbrace{\left(-\frac{(1-k)}{4} - \frac{1}{2} + 2\frac{(k-1)}{4}\right)}_{\frac{1}{4}(3k-5)}.$$

$$+ \phi_{i-2}\left(+\frac{1}{4}(1-k)\right).$$

$k = 1$:

$$u D_x^{(u)}\phi = \frac{1}{\Delta x}\left[\frac{1}{2}\phi_{i+1} - \phi_{i-1}\right]$$

$k = -1$:

$$D_x^{(u)}\phi = \frac{a}{4\Delta x}\left[6\phi_i - 8\phi_{i-1} + 2\phi_{i-2}\right]$$

1-Schem 4

$$\left.\frac{du}{du}\right]_h \frac{F^c_{i+\frac{1}{2}} - F^c_{i-\frac{1}{2}}}{\Delta u} = \frac{a}{4\Delta u}\Big[(1+h)\,\phi_{i+1} + 3(1-h)\,\phi_i + (3h-5)\,\phi_{i-1} + (1-h)\,\phi_{i-2} \Big]$$

$$(1+h)\times \phi_{i+1} = \phi_i + \Delta u\,\phi_i' + \frac{\Delta u^2}{2}\phi_i'' + \frac{\Delta u^3}{6}\phi_i''' + \frac{\Delta u^4}{24}\phi_i^{(4)}$$

$$3(1-h)\times \phi_i = \phi_i$$

$$(3h-5)\times \phi_{i-1} = \phi_i - \Delta u\,\phi_i' + \frac{\Delta u^2}{2}\phi_i'' - \frac{\Delta u^3}{6}\phi_i''' + \frac{\Delta u^4}{24}\phi_i^{(4)}$$

$$(1-h)\times \phi_{i-2} = \phi_i - 2\Delta u\,\phi_i' + 2\Delta u^2\phi_i'' - \frac{4\Delta u^3}{3}\phi_i''' + \frac{2\Delta x^4}{3}\phi_i^{(4)}$$

$$\phi_i\Big(\underbrace{1+h+3(1-h)+(3h-5)+(1-h)}_{\text{ok } 0.}\Big)\times\frac{1}{4}$$

$$\phi_{i\,\Delta x}'\Big(\underbrace{(1+h)-(3h-5)-2(1-h)}_{4}\Big)\times\frac{1}{4}$$

$$+\ \phi_i''\Big(\frac{(1+h)}{2} + \frac{(3h-5)}{2} + 2(1-h)\Big)\frac{\Delta x}{4} \longrightarrow \Big(h-\frac{1}{3}\Big)$$

$$+\ \phi_i'''\Big[\underbrace{\frac{(1+h)}{6} - \frac{1}{6}(3h-5) - \frac{4}{3}(1-h)}_{0}\Big]\frac{\Delta u^2}{4}$$

n-schéma-3 |

$$+ \phi_i''' \left[\frac{(1+h)}{24} + \frac{(3h-5)}{24} + \frac{2}{3}(1-h) \right] \frac{\Delta x^3}{4}$$

$$\frac{1}{24} - \frac{5}{24} + \frac{2}{3} = \frac{1\cancel{4}-5+16}{24} = \frac{1}{2}.$$

$$\frac{h}{24} + \frac{3h}{24} - \frac{2}{3}h = \frac{h}{24}\left(1+3-16\right) = h - \frac{1}{2}\times$$

$$\phi_i''' \Delta x^3 \frac{(-h+1)}{8}$$

$$\left(\frac{\partial \phi}{\partial x}\right)\Big|_i^h = \phi_i' + \left(h - \frac{1}{3}\right)\frac{\Delta x^2}{4}\phi_i''' + \frac{(1-h)}{8}\Delta x^3 \phi_i^{(4)}$$

Tous instable

pr EP1 ~~ave~~ pr $\frac{\partial u}{\delta t} + u \frac{\partial u}{\partial x_i}$?