Systèmes triphasés

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2A Energie - ENRS8AB

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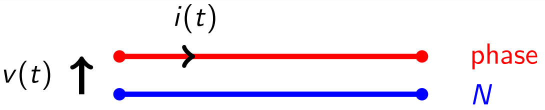
Systèmes monophasés



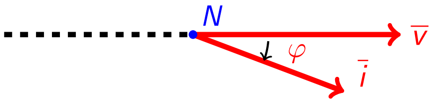
pulsation $\omega = 2\pi f = \dfrac{2\pi}{T}$ $\quad f = 50\text{Hz}$ fréquence. $T = 20ms$ période

tension de phase : $v(t) = \sqrt{2}\, V \cos(\omega t) = \sqrt{2}\, \Re e \left[\overline{V}\, e^{j\omega t} \right]$

$$\overline{V} = V \qquad V = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} \;\; [V]$$

courant de phase : $i(t) = \sqrt{2}\, I \cos(\omega t - \varphi) = \sqrt{2}\, \Re e \left[\overline{I}\, e^{j\omega t} \right]$

$$\overline{I} = I\, e^{-j\varphi} \qquad I = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt} \;\; [A]$$

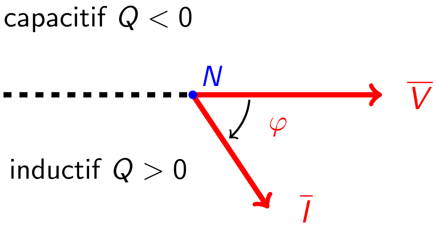


puissance instantanée : $p(t) = v(t)i(t) = VI\cos\varphi + VI\cos(2\omega t - \varphi)$

$$\text{Puissance active} \qquad P = \Re e\left[\overline{V}\,\overline{I}^{*}\right] = V\,I\cos\varphi = <\mathrm{p(t)}> \qquad [\mathrm{W}]$$

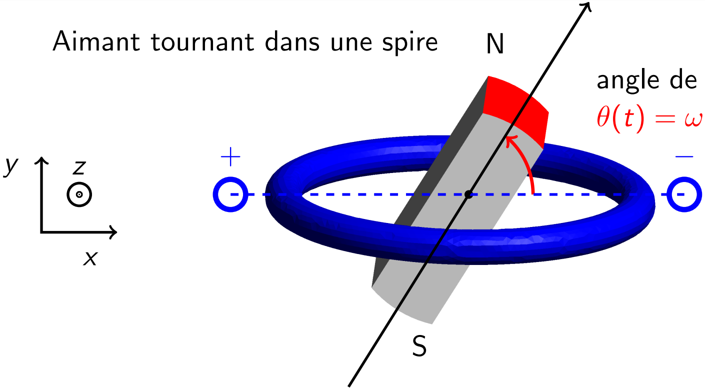
$$\text{Puissance réactive} \qquad Q = \Im m\left[\overline{V}\,\overline{I}^{*}\right] = V\,I\sin\varphi \qquad [\mathrm{Var}]$$

$$\text{Puissance apparente} \qquad \overline{S} = \overline{V}\,\overline{I}^{*} = V\,I\,e^{j\varphi} \qquad [\mathrm{VA}]$$



Alternateur élémentaire monophasé

Aimant tournant dans une spire



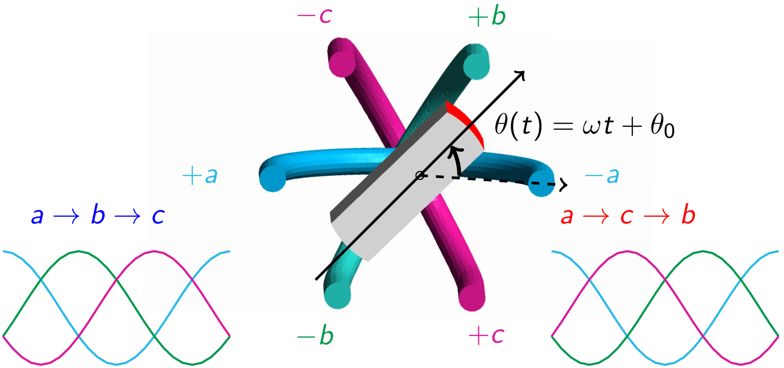
Flux dans la spire

$$\Phi(t) = \sum_{k=0}^{\infty} \Phi_{2k+1} \sqrt{2}\, \sin((2k+1)\,\theta(t)) \simeq \sqrt{2}\Phi \sin\theta(t)$$

FEM (force electromotrice)

$$e(t) = \frac{d\Phi}{dt} = \sqrt{2}\,\Phi\; \cos\theta(t)\, \frac{d\theta}{dt} = \sqrt{2}\,\Phi\,\omega\cos(\omega\, t + \theta_0)$$

Alternateur élémentaire triphasé



Système triphasé direct

$$e_a(t) = E\sqrt{2}\cos(\omega t + \theta_0)$$

$$e_b(t) = E\sqrt{2}\cos(\omega t + \theta_0 - 2\pi/3)$$

$$e_c(t) = E\sqrt{2}\cos(\omega t + \theta_0 + 2\pi/3)$$

Système triphasé inverse

$$e_a(t) = E\sqrt{2}\cos(\omega t + \theta_0)$$

$$e_b(t) = E\sqrt{2}\cos(\omega t + \theta_0 + 2\pi/3)$$

$$e_c(t) = E\sqrt{2}\cos(\omega t + \theta_0 - 2\pi/3)$$

a : racine cubique de l'unité $\quad z^3 = 1 = e^{j2\pi k} \leftrightarrow z = e^{j2\pi k/3} \quad k \in \mathbb{N}$

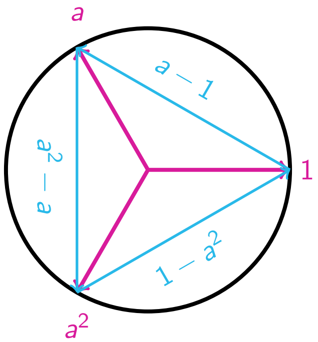
$$a = e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}} = a^* \quad \textit{(conjugué de a)}$$

$$1 + a + a^2 = 0$$

$$1 - a^2 = \sqrt{3}\, e^{j\frac{\pi}{6}} = (1-a)^*$$

$$a - a^2 = j\sqrt{3}$$



Système triphasé direct (Positive sequence)

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} = E\sqrt{2}\begin{bmatrix} \cos(\omega t + \theta_0) \\ \cos(\omega t + \theta_0 - 2\pi/3) \\ \cos(\omega t + \theta_0 + 2\pi/3) \end{bmatrix} = \sqrt{2}\,\Re e\left(\overline{E}_d\, e^{j\omega t}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}\right)$$

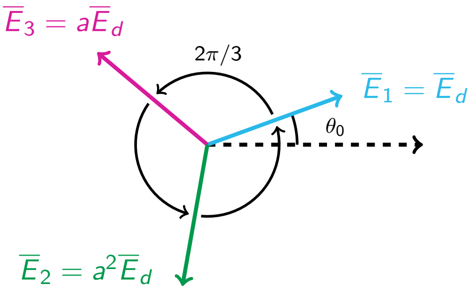
$$\overline{E}_d = E\, e^{j\theta_0}$$

$$\textit{tensions de même module :}\ E = |\overline{E}_1| = |\overline{E}_2| = |\overline{E}_3|$$

$$\textit{déphasage :}\ \arg\left(\frac{\overline{E}_1}{\overline{E}_2}\right) = \arg\left(\frac{\overline{E}_2}{\overline{E}_3}\right) = \arg\left(\frac{\overline{E}_3}{\overline{E}_1}\right) = +\frac{2\pi}{3}$$

$$e_1(t) + e_2(t) + e_3(t) = 0$$

$$\overline{E}_1 + \overline{E}_2 + \overline{E}_3 = 0$$



Système triphasé inverse (Negative sequence)

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} = E\sqrt{2} \begin{bmatrix} \cos(\omega t + \theta_0) \\ \cos(\omega t + \theta_0 + 2\pi/3) \\ \cos(\omega t + \theta_0 - 2\pi/3) \end{bmatrix} = \sqrt{2}\,\Re e\left(\overline{E}_i\, e^{j\omega t} \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} \right)$$

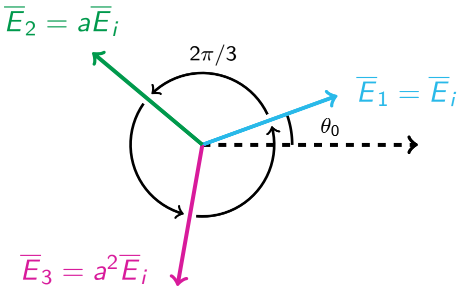
$$\overline{E}_i = E\, e^{j\theta_0}$$

$$\textit{tensions de même module : } E = |\overline{E}_1| = |\overline{E}_2| = |\overline{E}_3|$$

$$\textit{déphasage : } \arg\left(\frac{\overline{E}_1}{\overline{E}_2}\right) = \arg\left(\frac{\overline{E}_2}{\overline{E}_3}\right) = \arg\left(\frac{\overline{E}_3}{\overline{E}_1}\right) = -\frac{2\pi}{3}$$

$$e_1(t) + e_2(t) + e_3(t) = 0$$

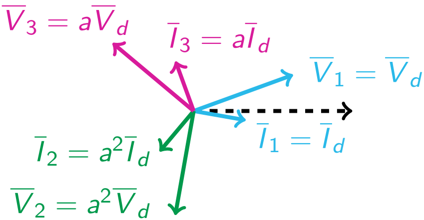
$$\overline{E}_1 + \overline{E}_2 + \overline{E}_3 = 0$$



Puissance apparente en triphasé : système direct

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \sqrt{2}\, \Re e \left(\overline{V}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e^{j\omega t} \right)$$

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \sqrt{2}\, \Re e \left(\overline{I}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e^{j\omega t} \right)$$



$$\begin{aligned} \overline{S} &= \overline{V}_1\,\overline{I}_1^* + \overline{V}_2\,\overline{I}_2^* + \overline{V}_3\,\overline{I}_3^* \\ &= \overline{V}_d\,\overline{I}_d^* + a^2\overline{V}_d\,(a^2\overline{I}_d)^* + a\overline{V}_d\,(a\overline{I}_d)^* \\ P + jQ &= 3\,\overline{V}_d\,\overline{I}_d^* \end{aligned}$$

Puissance instantanée en triphasé : système direct

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \sqrt{2}\ \Re e \left(\overline{V}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e^{j\omega t} \right)$$

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \sqrt{2}\ \Re e \left(\overline{I}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e^{j\omega t} \right)$$

$$\begin{aligned} v_1(t)\, i_1(t) &= \Re e \left[\overline{V}_d\, \overline{I}_d^* \right] + \Re e \left[\overline{V}_d\, \overline{I}_d e^{2j\omega t} \right] \\ v_2(t)\, i_2(t) & \quad + \Re e \left[\overline{V}_d\, \overline{I}_d^* \right] + \Re e \left[\overline{V}_d\, \overline{I}_d\, a^4\, e^{2j\omega t} \right] \\ v_3(t)\, i_3(t) & \quad + \Re e \left[\overline{V}_d\, \overline{I}_d^* \right] + \Re e \left[\overline{V}_d\, \overline{I}_d\, a^2\, e^{2j\omega t} \right] \end{aligned}$$

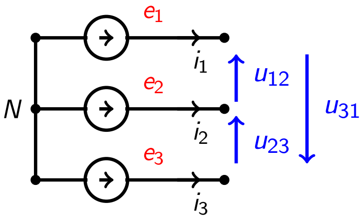
$$p(t) = 3\, \Re e \left[\overline{V}_d\, \overline{I}_d^* \right] = P$$

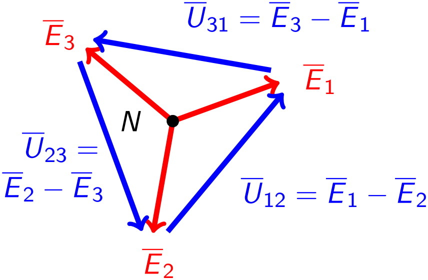
suppression de la puissance fluctuante

Connections en triphasé : couplage étoile

$$i_1 + i_2 + i_3 = 0$$

⋆ couplage des sources



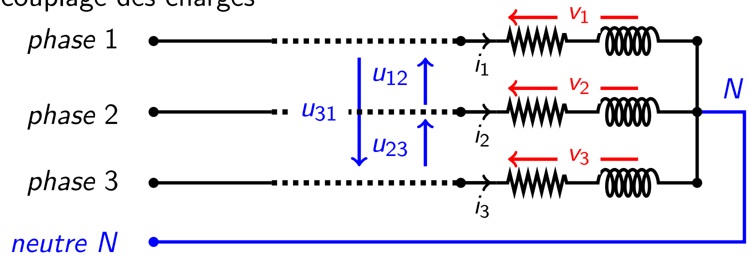


(u_{12}, u_{23}, u_{31}) tensions de ligne $\qquad$ (e_1, e_2, e_3) tensions de phase

$$\begin{bmatrix} u_{12} \\ u_{23} \\ u_{31} \end{bmatrix} = \begin{bmatrix} e_1 - e_2 \\ e_2 - e_3 \\ e_3 - e_1 \end{bmatrix} \quad \rightsquigarrow u_{12} + u_{23} + u_{31} = 0$$

Connections en triphasé : couplage étoile

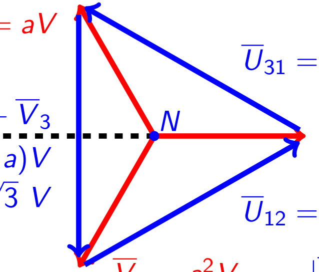
⋆ couplage des charges



(u_{12}, u_{23}, u_{31}) tensions de ligne

(v_1, v_2, v_3) tensions de phase

$\overline{V}_3 = aV$



$\overline{U}_{31} = \overline{V}_3 - \overline{V}_1 = (a-1)V$

$\overline{U}_{23} = \overline{V}_2 - \overline{V}_3$
$= (a^2 - a)V$
$= -j\sqrt{3}\ V$

$\overline{V}_1 = V = 230V$

$\overline{U}_{12} = \overline{V}_1 - \overline{V}_2 = (1-a^2)V$

$\overline{V}_2 = a^2 V$

$|\overline{U}_{12}| = \sqrt{3}V = 400V$

Couplage étoile : tensions

⋆ les tensions de ligne vérifient $u_{12} + u_{23} + u_{31} = 0$

$$\begin{bmatrix} u_{12} \\ u_{23} \\ u_{31} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = [C] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \overline{V}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix} = \overline{V}_d \sqrt{3} e^{j\frac{\pi}{6}} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

⋆ tensions de phase

$[C]$ est non inversible

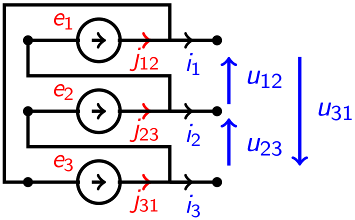
mais si de plus $v_1 + v_2 + v_3 = 0$ *est vérifié* $\Longrightarrow$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \frac{{}^t[C]}{3} \begin{bmatrix} u_{12} \\ u_{23} \\ u_{31} \end{bmatrix}$$

$$\begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix} = \overline{U}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \quad \rightsquigarrow \quad \begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \frac{\overline{U}_d}{\sqrt{3}} e^{-j\frac{\pi}{6}} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

$$v_1 + v_2 + v_3 = 0$$

Δ couplage des sources



$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} u_{12} \\ u_{23} \\ u_{31} \end{bmatrix}$$

égalité des tensions de ligne et de phase
pas de neutre accessible

(i_1, i_2, i_3) courants de ligne

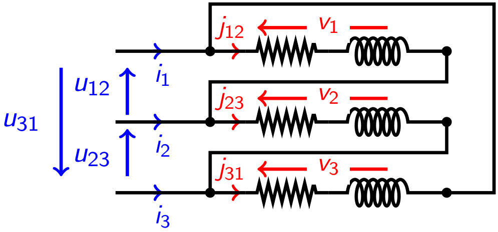
(j_{12}, j_{23}, j_{31}) courants de phase

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} j_{12} - j_{31} \\ j_{23} - j_{12} \\ j_{31} - j_{23} \end{bmatrix} \quad \rightsquigarrow i_1 + i_2 + i_3 = 0$$

Δ couplage des charges

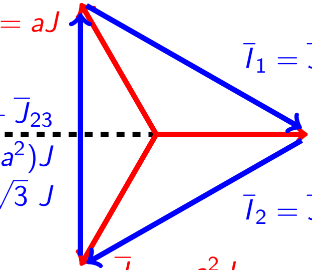
courants de ligne (i_1, i_2, i_3)

courants de phase (j_{12}, j_{23}, j_{31})



(u_{12}, u_{23}, u_{31}) tensions de ligne

(v_1, v_2, v_3) tensions de phase



$\overline{J}_{31} = aJ$

$\overline{I}_1 = \overline{J}_{12} - \overline{J}_{31} = (1-a)J$

$\overline{I}_{23} = \overline{J}_{31} - \overline{J}_{23}$
$= (a - a^2)J$
$= j\sqrt{3}\ J$

$\overline{J}_{12} = J$

$\overline{I}_2 = \overline{J}_{23} - \overline{J}_{12} = (a^2 - 1)J$

$\overline{J}_{23} = a^2 J$

$|\overline{I}_2| = \sqrt{3}J$

Couplage triangle : courants

Δ les courants de ligne vérifient $i_1 + i_2 + i_3 = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} j_{12} - j_{31} \\ j_{23} - j_{12} \\ j_{31} - j_{23} \end{bmatrix} = {}^t[C] \begin{bmatrix} j_{12} \\ j_{23} \\ j_{31} \end{bmatrix}$$

$$\begin{bmatrix} \overline{J}_{12} \\ \overline{J}_{23} \\ \overline{J}_{31} \end{bmatrix} = \overline{J}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \quad \leadsto \quad \begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \end{bmatrix} = \overline{J}_d \sqrt{3} e^{-j\frac{\pi}{6}} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

Δ courants de phase *${}^t[C]$ est non inversible*

mais si de plus $j_{12} + j_{23} + j_{31} = 0$ *est vérifié* $\Longrightarrow \begin{bmatrix} j_{12} \\ j_{23} \\ j_{31} \end{bmatrix} = \frac{[C]}{3} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$

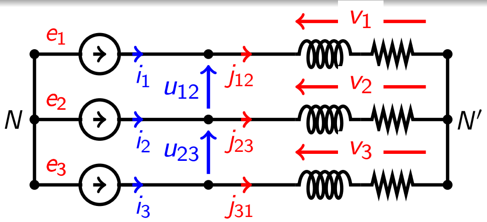
$$\begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \end{bmatrix} = \overline{I}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \quad \leadsto \quad \begin{bmatrix} \overline{J}_{12} \\ \overline{J}_{23} \\ \overline{J}_{31} \end{bmatrix} = \frac{\overline{I}_d}{\sqrt{3}} e^{j\frac{\pi}{6}} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

Connections en triphasé : synthèse

Couplage étoile $\curlywedge$	Couplage triangle Δ
$[u] = [C][v]$	$[u] = [v]$
$[\,i\,] = [\,j\,]$	$[\,i\,] = \ ^t[C][\,j\,]$ ou $[\,i\,] = [C][\,j\,]$
$i_1 + i_2 + i_3 = 0$	$v_1 + v_2 + v_3 = 0$
$U = \sqrt{3}V$	$U = V$
$I = J$	$I = \sqrt{3}J$
$P = 3VJ\cos\varphi = \sqrt{3}UI\cos\varphi$	$P = 3VJ\cos\varphi = \sqrt{3}UI\cos\varphi$
$Q = \sqrt{3}UI\sin\varphi$	$Q = \sqrt{3}UI\sin\varphi$

Calcul des puissances sur 3 charges $1-\varphi$ identiques

$$\begin{bmatrix}\overline{E}_1\\ \overline{E}_2\\ \overline{E}_3\end{bmatrix} = \overline{E}_d \begin{bmatrix}1\\ a^2\\ a\end{bmatrix}$$



$$\begin{bmatrix}\overline{V}_1\\ \overline{V}_2\\ \overline{V}_3\end{bmatrix} = \begin{bmatrix}\overline{Z}\,\overline{J}_{12}\\ \overline{Z}\,\overline{J}_{23}\\ \overline{Z}\,\overline{J}_{31}\end{bmatrix}$$

$$i_1 + i_2 + i_3 = 0 \qquad i_1 = j_{12} \quad i_2 = j_{13} \quad i_3 = j_{31}$$

$$v_{NN'} = e_1 - v_1 = e_2 - v_2 = e_3 - v_3$$

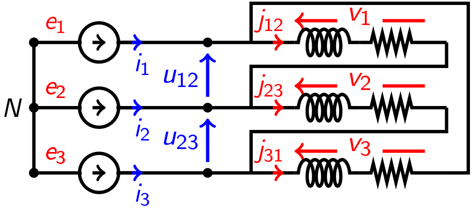
$$3\,\overline{V}_{NN'} = \overline{E}_1 + \overline{E}_2 + \overline{E}_3 - \overline{Z}(\overline{I}_1 + \overline{I}_2 + \overline{I}_3) = 0$$

les neutres ont même potentiel

$$\begin{bmatrix}\overline{E}_1\\ \overline{E}_2\\ \overline{E}_3\end{bmatrix} = \begin{bmatrix}\overline{V}_1\\ \overline{V}_2\\ \overline{V}_3\end{bmatrix} = \overline{Z}\begin{bmatrix}\overline{I}_1\\ \overline{I}_2\\ \overline{I}_3\end{bmatrix}$$

$$\overline{S} = \overline{V}_1\overline{J}_{12}^* + \overline{V}_2\overline{J}_{23}^* + \overline{V}_3\overline{J}_{31}^* = 3\,\frac{|\overline{E}_d|^2}{\overline{Z}^*}$$

$$\begin{bmatrix} \overline{E}_1 \\ \overline{E}_2 \\ \overline{E}_3 \end{bmatrix} = \overline{E}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$



$$\begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \begin{bmatrix} \overline{Z}\,\overline{J}_{12} \\ \overline{Z}\,\overline{J}_{23} \\ \overline{Z}\,\overline{J}_{31} \end{bmatrix}$$

$$u_{12} = v_1 = e_1 - e_2 \qquad u_{23} = v_2 = e_2 - e_3 \qquad u_{31} = v_3 = e_3 - e_1$$

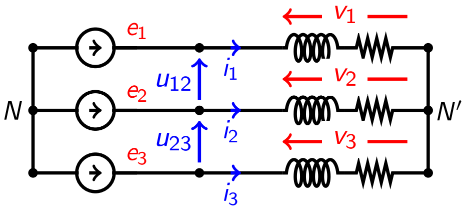
$$i_1 = j_{12} - j_{31} \qquad i_2 = j_{23} - j_{12} \qquad i_3 = j_{31} - j_{23}$$

$$\begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix} = \begin{bmatrix} \overline{E}_1 - \overline{E}_2 \\ \overline{E}_2 - \overline{E}_3 \\ \overline{E}_3 - \overline{E}_1 \end{bmatrix} = \overline{E}_d(1-a^2) \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} = \begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \overline{Z} \begin{bmatrix} \overline{J}_{12} \\ \overline{J}_{23} \\ \overline{J}_{31} \end{bmatrix}$$

$$\begin{aligned} \overline{S} &= \overline{V}_1\overline{J}_{12}^* + \overline{V}_2\overline{J}_{23}^* + \overline{V}_3\overline{J}_{31}^* = 3\,(1-a^2)\overline{E}_d\left[(1-a^2)\frac{\overline{E}_d}{\overline{Z}}\right]^* \\ &= 9\,\frac{|\overline{E}_d|^2}{\overline{Z}^*} \end{aligned}$$

Calcul des puissances sur 3 charges $1-\varphi$ différentes

$$\begin{bmatrix}\overline{E}_1\\\overline{E}_2\\\overline{E}_3\end{bmatrix} = \overline{E}_d \begin{bmatrix}1\\a^2\\a\end{bmatrix}$$



$$\begin{bmatrix}\overline{V}_1\\\overline{V}_2\\\overline{V}_3\end{bmatrix} = \begin{bmatrix}\overline{Z}_1\,\overline{I}_1\\\overline{Z}_2\,\overline{I}_2\\\overline{Z}_3\,\overline{I}_3\end{bmatrix}$$

$$i_1 + i_2 + i_3 = 0 \qquad v_{NN'} = e_1 - v_1 = e_2 - v_2 = e_3 - v_3$$

$$\overline{I}_k = (\overline{E}_k - \overline{V}_{NN'})/\overline{Z}_k \quad \rightsquigarrow \quad \overline{V}_{NN'} = \frac{\sum \overline{E}_k/\overline{Z}_k}{1/\overline{Z}_k} \neq 0$$

$$\begin{bmatrix}\overline{U}_{12}\\\overline{U}_{23}\\\overline{U}_{31}\end{bmatrix} = \begin{bmatrix}\overline{Z}_1 & -\overline{Z}_2 & 0\\0 & \overline{Z}_2 & -\overline{Z}_3\\-\overline{Z}_1 & 0 & \overline{Z}_3\end{bmatrix}\begin{bmatrix}\overline{I}_1\\\overline{I}_2\\\overline{I}_3\end{bmatrix}$$

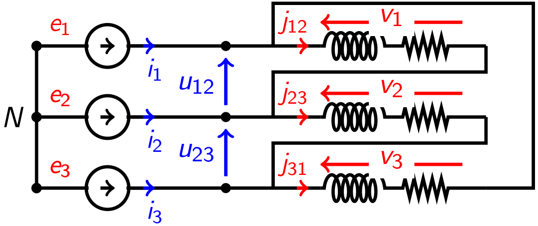
matrice non inversible mais relation supplémentaire $i_1 + i_2 + i_3 = 0$

$$\begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \end{bmatrix} = \frac{1}{\overline{Z}_1\overline{Z}_2 + \overline{Z}_1\overline{Z}_3 + \overline{Z}_2\overline{Z}_3} \begin{bmatrix} \overline{Z}_3 & 0 & -\overline{Z}_2 \\ -\overline{Z}_3 & \overline{Z}_1 & 0 \\ 0 & -\overline{Z}_1 & \overline{Z}_2 \end{bmatrix} \begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix}$$

$$\begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix} = \overline{E}_d(1-a^2) \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

$$\overline{S} = \overline{E}_1\overline{I}_1^* + \overline{E}_2\overline{I}_2^* + \overline{E}_3\overline{I}_3^* = \overline{E}_d \begin{bmatrix} 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \end{bmatrix}^*$$

$$= 3\,|\overline{E}_d|^2 \left(\frac{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3}{\overline{Z}_1\overline{Z}_2 + \overline{Z}_1\overline{Z}_3 + \overline{Z}_2\overline{Z}_3} \right)^*$$



$$\begin{bmatrix} \overline{E}_1 \\ \overline{E}_2 \\ \overline{E}_3 \end{bmatrix} = \overline{E}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

$$\begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \begin{bmatrix} \overline{Z}_1\,\overline{J}_{12} \\ \overline{Z}_2\,\overline{J}_{23} \\ \overline{Z}_3\,\overline{J}_{31} \end{bmatrix}$$

$$u_{12} = v_1 = e_1 - e_2 \qquad u_{23} = v_2 = e_2 - e_3 \qquad u_{31} = v_3 = e_3 - e_1$$

$$i_1 = j_{12} - j_{31} \qquad i_2 = j_{23} - j_{12} \qquad i_3 = j_{31} - j_{23}$$

$$\begin{bmatrix} \overline{U}_{12} \\ \overline{U}_{23} \\ \overline{U}_{31} \end{bmatrix} = \begin{bmatrix} \overline{E}_1 - \overline{E}_2 \\ \overline{E}_2 - \overline{E}_3 \\ \overline{E}_3 - \overline{E}_1 \end{bmatrix} = \overline{E}_d(1-a^2) \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} = \begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \end{bmatrix} = \begin{bmatrix} \overline{Z}_1\overline{J}_{12} \\ \overline{Z}_2\overline{J}_{23} \\ \overline{Z}_3\overline{J}_{31} \end{bmatrix}$$

$$\overline{S} \;=\; \overline{V}_1\overline{J}_{12}^* + \overline{V}_2\overline{J}_{23}^* + \overline{V}_3\overline{J}_{31}^* \;=\; 3\,|\overline{E}_d|^2 \left(\frac{1}{\overline{Z}_1^*} + \frac{1}{\overline{Z}_2^*} + \frac{1}{\overline{Z}_3^*} \right)$$