

Alternateur Synchrone

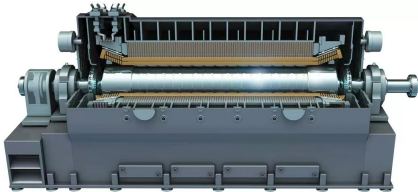
S. Dufour

2A Energie - ENRS8AB

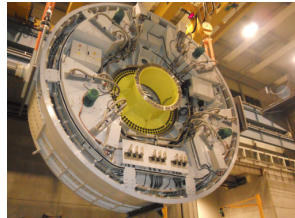
12 mai 2025



Turbogénérateur (vapeur)

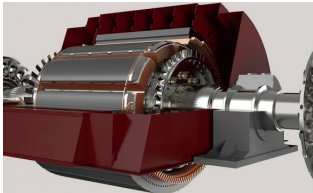


Alternateur éolien

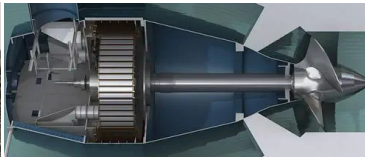


Générateurs hydrauliques

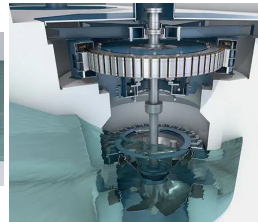
turbine Pelton



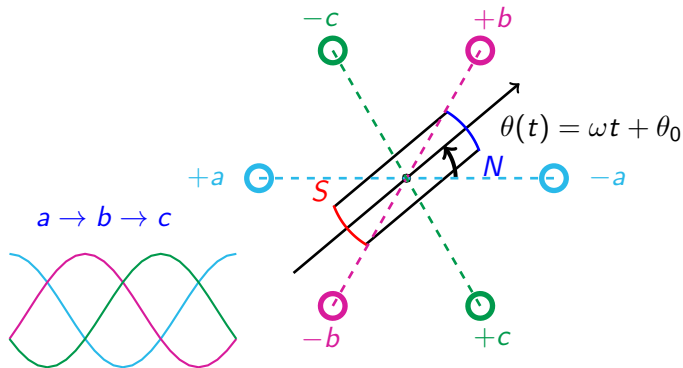
groupe bulbe (hydro)



turbine Kaplan



Alternateur élémentaire triphasé



Système triphasé direct

$$e_a(t) = E\sqrt{2}\cos(\omega t + \theta_0)$$

$$e_b(t) = E\sqrt{2}\cos(\omega t + \theta_0 - 2\pi/3)$$

$$e_c(t) = E\sqrt{2}\cos(\omega t + \theta_0 + 2\pi/3)$$

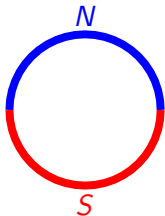
Vitesse de rotation de l'alternateur :

$$\Omega = \omega = 2\pi f$$

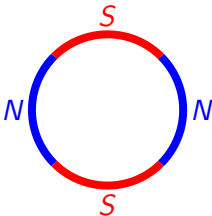
$$f = 50\text{Hz} \quad \leadsto \quad \Omega = 3000\text{tr}/\text{mn}$$

Polarité

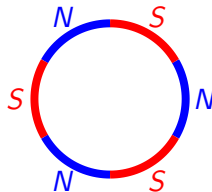
$p = 1$: 1 paire de pôles
2 pôles



$p = 2$
4 pôles



$p = 3$
6 pôles



angle mécanique : $\theta_m \in [0, 2\pi/p]$

angle électrique : $\theta \in [0, 2\pi]$

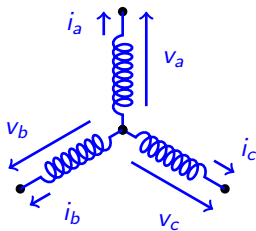
$$\theta = p \theta_m$$

Vitesse de synchronisme :

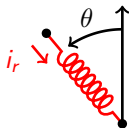
$$\dot{\theta}_m = \Omega = \frac{\omega}{p} = \frac{2\pi f}{p}$$

Alternateur Synchrone

Convention générateur : courants (sortants) vers le réseau



stator :
courants triphasés



rotor : $i_r = I_e$ (courant d'excitation)
courant continu

$$\theta(t) = p \theta_m(t)$$

θ : angle électrique

θ_m : angle mécanique

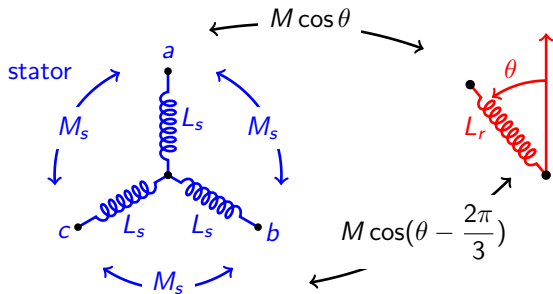
$2p$: nombre de pôles

$\omega = 2\pi f$: pulsation

système direct équilibré

$$[i_s] = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = I\sqrt{2} \begin{bmatrix} \cos(\omega t - \varphi) \\ \cos(\omega t - \varphi - \frac{2\pi}{3}) \\ \cos(\omega t - \varphi + \frac{2\pi}{3}) \end{bmatrix} = \sqrt{2} \Re \left(\bar{I} e^{j\omega t} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \right)$$

Alternateur synchrone : inductances



rotor

$L = L_s - M_s$ inductance phase stator

L_s auto-inductance phase stator

M_s inductance mutuelle phase stator

L_r auto-inductance rotorique

M inductance mutuelle stator-rotor

flux phase stator

$$\varphi_a = -L i_a + M \cos \theta i_r$$

$$\varphi_b = -L i_b + M \cos(\theta - \frac{2\pi}{3}) i_r$$

$$\varphi_c = -L i_c + M \cos(\theta + \frac{2\pi}{3}) i_r$$

flux rotor flux

$$\varphi_r = L_r i_r - M \cos \theta i_a - M \cos(\theta - \frac{2\pi}{3}) i_b - M \cos(\theta + \frac{2\pi}{3}) i_c$$

Alternateur synchrone : stator

hypothèse : $\dot{\theta}_m = \Omega = \frac{\omega}{p}$ vitesse de rotation constante (vitesse de synchronisme)

$$\theta = p \theta_m = p \Omega t + \theta_0 = \omega t + \theta_0 \quad \theta_0 : \text{angle vérifiant une condition aux limites périodiques}$$

$$[\varphi_s] = \begin{bmatrix} \varphi_a \\ \varphi_b \\ \varphi_c \end{bmatrix} = -L \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + M \begin{bmatrix} \cos \omega t + \theta_0 \\ \cos (\omega t + \theta_0 - \frac{2\pi}{3}) \\ \cos (\omega t + \theta_0 + \frac{2\pi}{3}) \end{bmatrix} i_r$$

$$\overline{\Phi} = -L \overline{I} + \frac{M I_e}{\sqrt{2}} e^{j\theta_0} \quad \delta = \theta_0 + \frac{\pi}{2} : \text{angle interne}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = -R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_a \\ \varphi_b \\ \varphi_c \end{bmatrix}$$

$$\overline{V} = -(R + jL\omega) \overline{I} + \underbrace{\frac{M\omega}{\sqrt{2}} e^{j\delta} I_e}_{\overline{E} \text{ force électromotrice}}$$

Alternateur synchrone : rotor

hypothèse : $\dot{\theta}_m = \Omega = \frac{\omega}{p}$ vitesse de rotation constante (vitesse de synchronisme)

$$\theta = p \theta_m = p \Omega t + \theta_0 = \omega t + \theta_0$$

★ Flux du rotor : *constant en fonction du temps*

$$-M \left[i_a \cos \theta + i_b \cos \left(\theta - \frac{2\pi}{3} \right) + i_c \cos \left(\theta + \frac{2\pi}{3} \right) \right] = -\frac{3}{2} M I \sqrt{2} \cos(\theta_0 + \varphi)$$

$$\varphi_r = L_r I_e - \frac{3}{2} M I \sqrt{2} \cos(\theta_0 + \varphi)$$

★ Equation de circuit électrique pour le rotor :

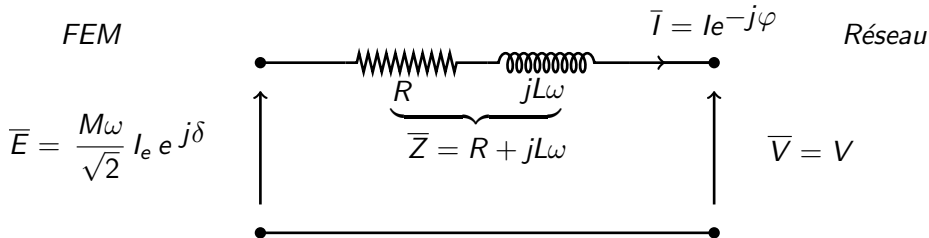
alimentation DC

R_r : résistance rotor

$$V_r = R_r I_e + \frac{d\varphi_r}{dt} = R_r I_e$$

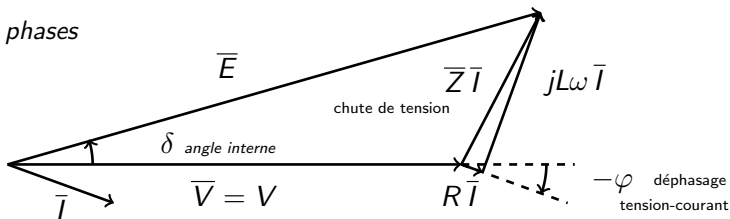
$\leadsto I_e$ courant d'excitation continu

Alternateur Synchrone : schéma équivalent



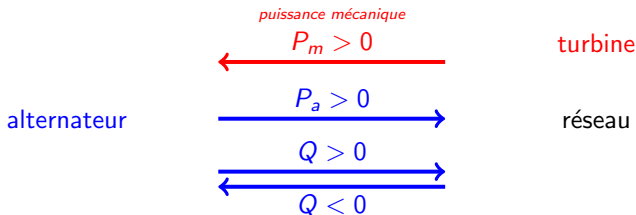
$$\bar{E} = \bar{V} + (R + jL\omega)\bar{I} = \bar{V} + \bar{Z}\bar{I}$$

Diagramme des phases



choix de l'origine des phases : tension de phase réseau

Alternateur Synchrone : puissances



Puissance apparente :

$$\bar{S} = 3\bar{V}\bar{I}^* = 3\bar{E}\bar{I}^* - 3(R + jL\omega)I^2$$

Puissance active :

$$P_a = \Re(\bar{S}) = 3\Re(\bar{E}\bar{I}^*) - 3RI^2 = P_m - P_{Js}$$

Puissance réactive :

$$Q = \Im(\bar{S}) = 3\Im(\bar{E}\bar{I}^*) - 3L\omega I^2$$

Couple (turbine) :

$$\Gamma_m = +3 \frac{p}{\omega} \Re[\bar{E}\bar{I}^*] \simeq \frac{3p}{\omega} \frac{EV}{|\bar{Z}|} \sin \delta$$

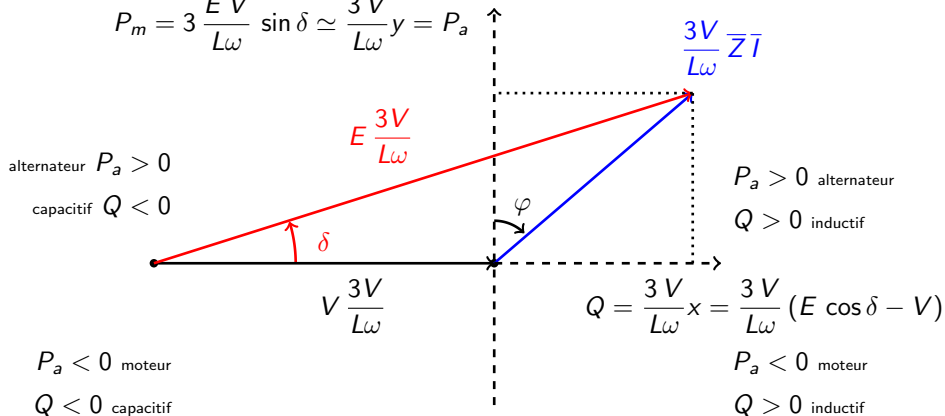
Alternateur Synchrone : diagramme simplifié des puissances

Hypothèse : $R \simeq 0 \rightsquigarrow \bar{Z} \simeq j L \omega$

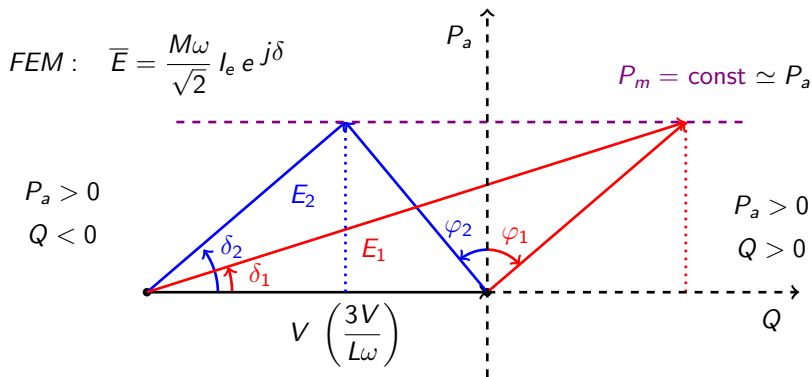
Chute de tension : $\bar{Z} \bar{I} = x + jy$

Puissance apparente : $\bar{S} = P_a + jQ = 3 \frac{V}{\bar{Z}^*} (\bar{Z} \bar{I})^* = \frac{3V}{L\omega} (y - jx) = 3V I e^{j\varphi}$
 envoyée au réseau

$$P_m = 3 \frac{E V}{L\omega} \sin \delta \simeq \frac{3V}{L\omega} y = P_a$$



Contrôle : courant d'excitation



$E_2(I_{e2}) \rightsquigarrow$

$$\delta_2 = \arcsin\left(\frac{L\omega P_m}{3V E_2}\right)$$

$$Q_2 = \frac{3V}{L\omega} (E_2 \cos \delta_2 - V) < 0$$

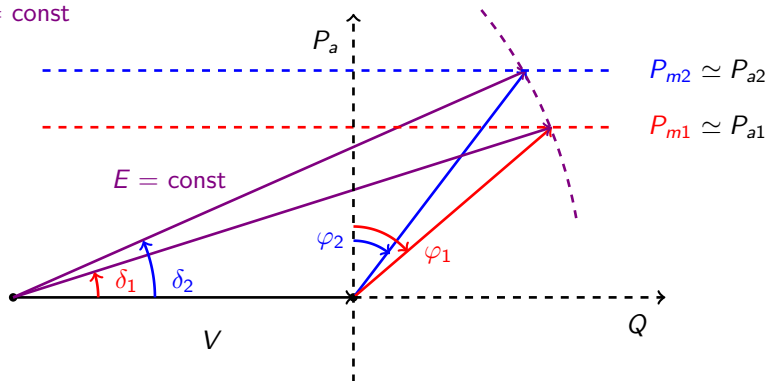
$E_1(I_{e1}) \rightsquigarrow$

$$\delta_1 = \arcsin\left(\frac{L\omega P_m}{3V E_1}\right)$$

$$Q_1 = \frac{3V}{L\omega} (E_1 \cos \delta_1 - V) > 0$$

Contrôle : puissance de la turbine

$I_e = \text{const}$

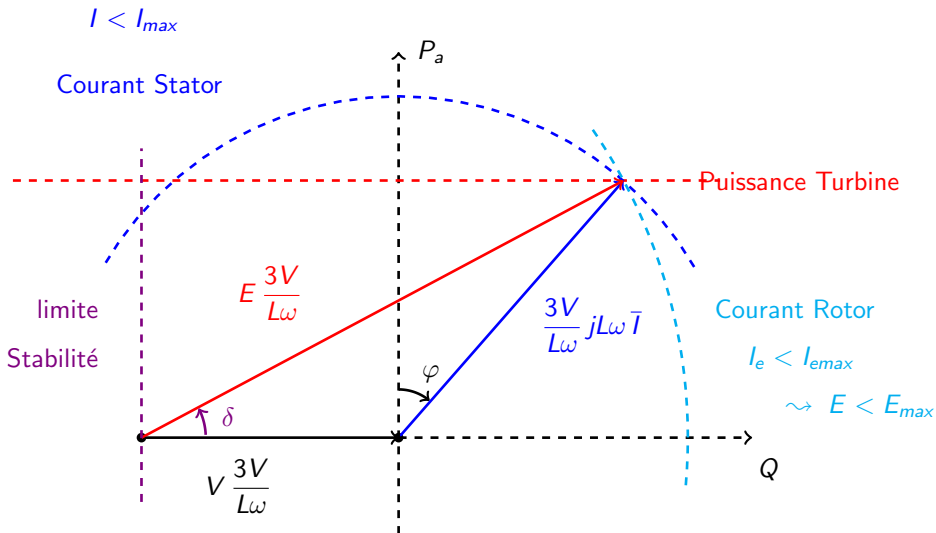


$$P_{m2} \leadsto \delta_2 = \arcsin\left(\frac{L\omega P_{m2}}{3 V E}\right) \quad P_{m1} \leadsto \delta_1 = \arcsin\left(\frac{L\omega P_{m1}}{3 V E}\right)$$

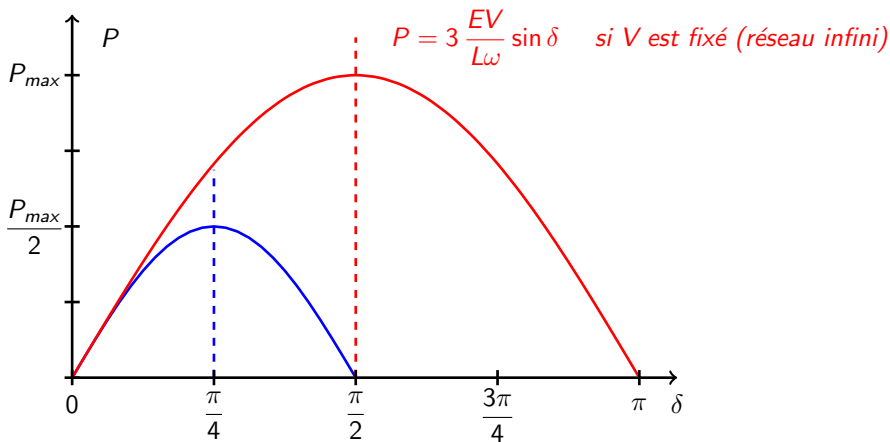
$$Q_2 = \frac{3 V}{L\omega} (E \cos \delta_2 - V) \quad Q_1 = \frac{3 V}{L\omega} (E \cos \delta_1 - V)$$

$$P_{m2} > P_{m1} \ \& \ E = \text{const} \rightarrow \delta_2 > \delta_1 \rightarrow Q_2 < Q_1$$

Diagramme des puissances : limites physiques



Puissance active fournie par l'alternateur



$$P = 3 \frac{E^2}{2L\omega} \sin(2\delta) \quad \text{si } V \text{ n'est pas fixé (alternateur autonome)}$$

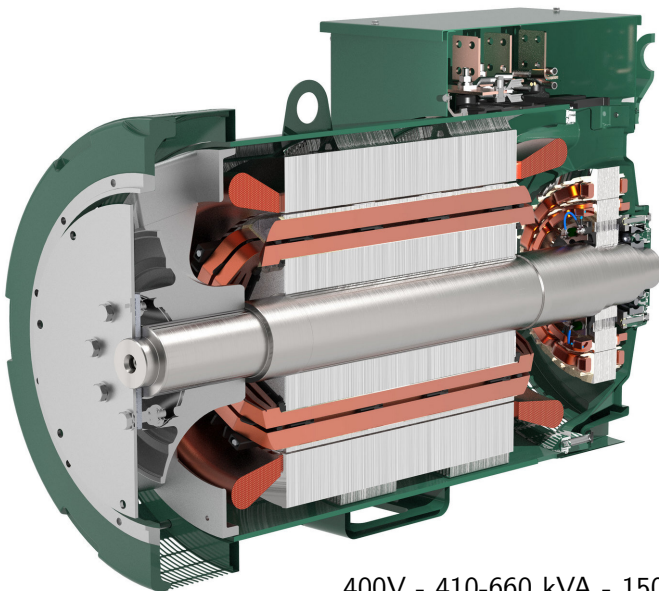
↪ transmission max de la moitié de la puissance !

$$\text{alternateur : } \eta = \frac{P_a}{P_m}$$

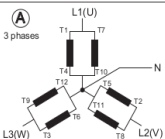
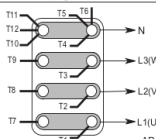
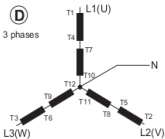
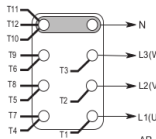
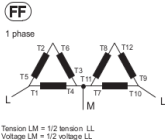
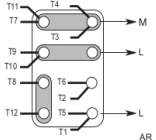
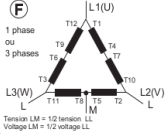
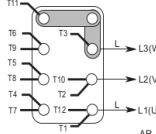
$$\text{pertes : } P_{Js} = 3RI^2, \quad P_{Jr} = R_r I_e^2, \quad P_{fe}, \quad p_{meca}$$

$$\eta = \frac{P_a}{P_a + 3RI^2 + R_r I_e^2 + P_{fe} + p_{meca}}$$

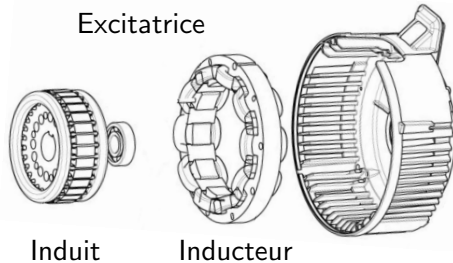
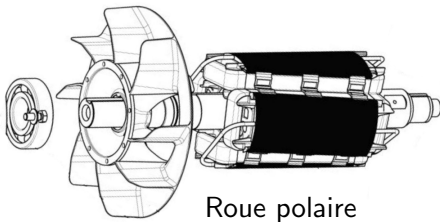
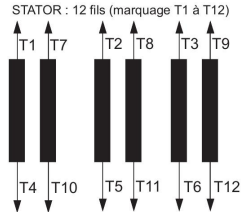
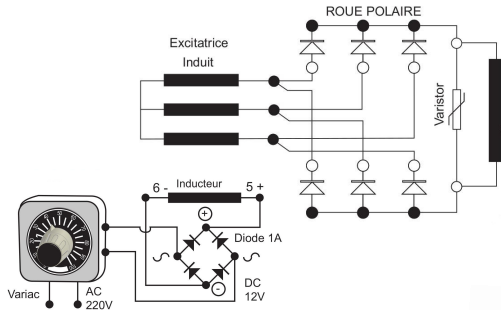
	$\cos \varphi = 0.8$	$\cos \varphi = 1$
10-20 kVA	$\eta = 86 - 87\%$	$\eta = 90 - 91\%$
25-60 kVA	$\eta = 89 - 91\%$	$\eta = 91 - 93\%$
70-200 kVA	$\eta = 93 - 94\%$	$\eta = 95 - 95.5\%$
400-600kVA	$\eta = 95 - 96\%$	$\eta = 96 - 97\%$
730-1000kVA	$\eta = 95 - 96\%$	$\eta = 96 - 97\%$



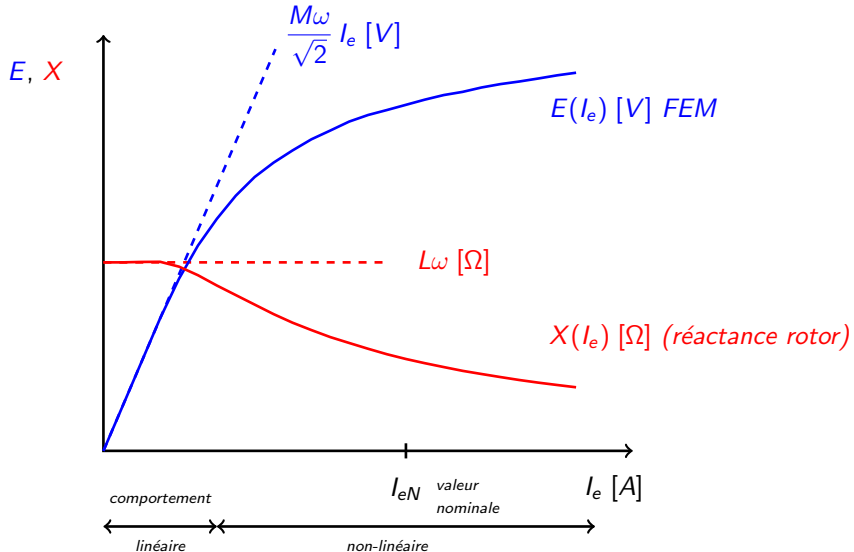
400V - 410-660 kVA - 1500 tr/mn

Code connexions	Tension LL			Couplage usine
(A) 3 phases 	Bobinage	50 Hz	60 Hz	
	6	190 - 208	190 - 240	
	7	220 - 230	-	
	8	-	190 - 208	
Détection de tension R220 : 0 => (T8) / 110 V => (T11) Détection de tension D350 : U => T1 / V => T2 / W => T3				
(D) 3 phases 	Bobinage	50 Hz	60 Hz	
	6	380 - 415	380 - 480	
	7	440 - 460	-	
	8	-	380 - 416	
Détection de tension R220 : 0 => (T8) / 110 V => (T11) Détection de tension D350 : U => T1 / V => T2 / W => T3				
(FF) 1 phase 	Bobinage	50 Hz	60 Hz	
	6	220 - 240	220 - 240	
	7	250 - 260	-	
	8	200	220 - 240	
Détection de tension R220 : 0 => (T1) / 110 V => (T4) Détection de tension D350 : V => T1 / W => T10				
(F) 1 phase ou 3 phases 	Bobinage	50 Hz	60 Hz	
	6	220 - 240	220 - 240	
	7	250 - 260	-	
	8	200	220 - 240	
Détection de tension R220 : 0 => (T8) / 110 V => (T11) Détection de tension D350 : U => T1 / V => T2 / W => T3				

Connexions du rotor (Nidec)

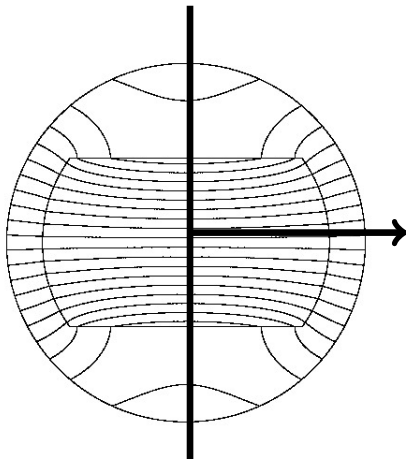


Limite du modèle : saturation du circuit magnétique

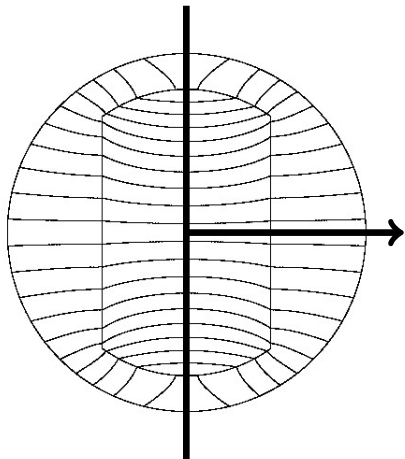


Alternateur synchrone à pôles saillants

Axe d ($\theta = 0$)



Axe q ($\theta = \pi/2$)



$$X_d > X_q$$

Alternateur à pôles saillants : composantes (d, q)

$$E = \bar{V} + jX_d I_d + jX_q I_q$$

$$\bar{V} = V_d + jV_q = V \sin \delta + jV \cos \delta$$

$$I_d = I \cos \psi = I \cos(\delta - \varphi) = \frac{V \sin \delta}{X_q}$$

$$R_s \simeq 0$$

$$I_q = I \sin \psi = I \sin(\delta - \varphi) = \frac{E - V \cos \delta}{X_d}$$

$$\bar{S} = 3(V_d + jV_q)(I_d + jI_q)^* = 3(V_d I_d + V_q I_q) + j3(V_q I_d - V_d I_q)$$

$$P = 3(V_d I_d + V_q I_q) = 3 \frac{EV}{X_d} \sin \delta + 3 \frac{V^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin(2\delta)$$

$$Q = 3(V_q I_d - V_d I_q) = 3 \frac{EV}{X_d} \cos \delta - 3V^2 \left(\frac{(\sin \delta)^2}{X_q} + \frac{(\cos \delta)^2}{X_d} \right)$$

Alternateur à pôles saillants : puissance de sortie

$$P = 3 \frac{EV}{X_d} \sin \delta + 3 \frac{V^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin(2\delta)$$

