Evidences, potential modeling and mineral targeting

Predicting mineralized locations

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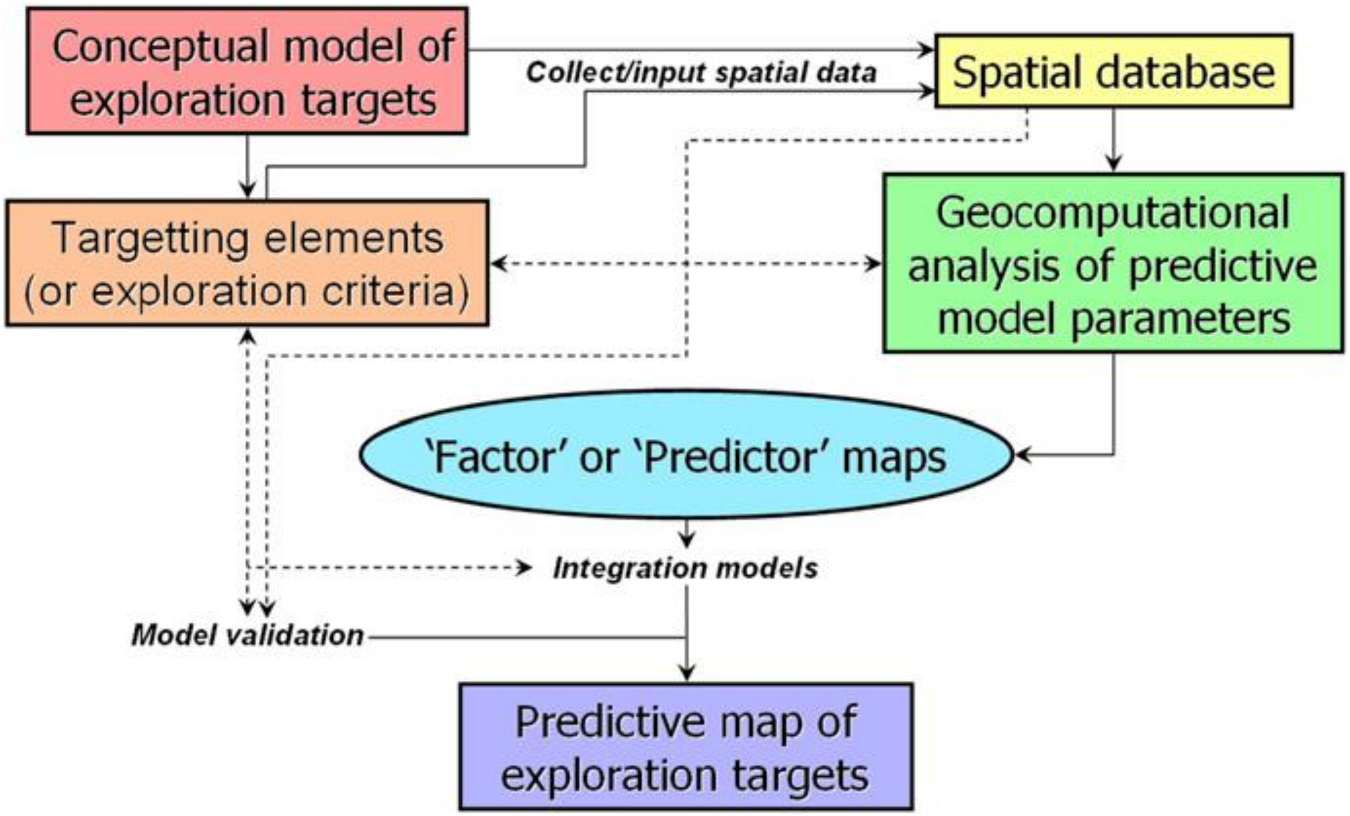
Introduction

Potential modeling helps to:

- appreciate the spatial distribution of mineralized areas
- understand the mineralization process
- establish the physical and chemical processes involved in ore formations
- predict the locations of potential economic mineral resources

In mineral potential modeling, the analytical or empirical relationships between descriptive features and a mineral target express the possibility to predict a given mineral concentration in a location as function of descriptive features.

Intro



Carranza (2011)

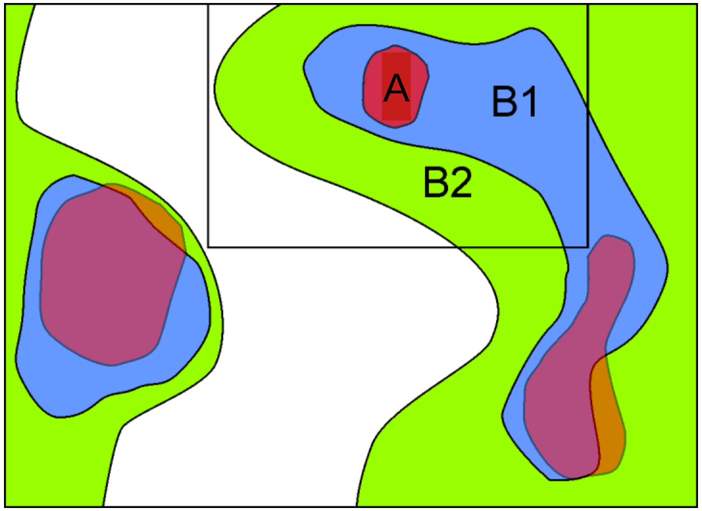
Elements of predictive modeling of mineral exploration targets.

Intro

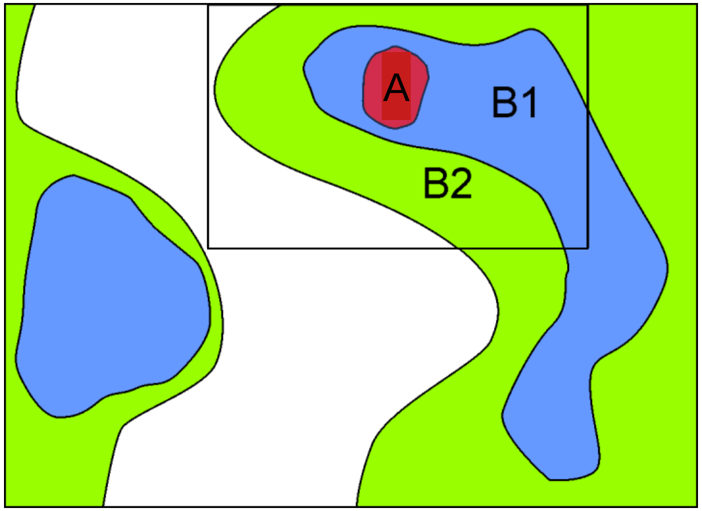
Where to look?

$$P(A) = f(B_i)$$

$P(A)$: Probability to find A
B_i = Known facts



Response theme



Evidential theme
(Multi-class theme)

Basics *(from Schaben H.) 2012*

Conditional probaility:
The conditional probability of $A \in \boldsymbol{A}$ given $B \in A$ with $P(B) > 0$ is defined as:

$$P_B(A) = \frac{P(AB)}{P(B)}$$

Independence of events:
Events $A, B \in \boldsymbol{A}$ are stochastically independent, if

$$P(AB) = P(A)\,P(B)$$

Bayes' rule:
Bayes' rule states that for $A, B \in \boldsymbol{A}$ with, $P(B) > 0$ the corresponding conditional probabilities are related by $P(A) > 0$

$$P(A \mid B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)}\frac{P(AB)}{P(A)} = P(A)\frac{P(B \mid A)}{P(B)}$$

Basics

Conditional independence of events:
A1 and *A2* are called conditionally independent given *B*, if

$$P(A_1 A_2 \mid B) = P(A_1 \mid B)\, P(A_2 \mid B)$$

Since generally

$$P(A_1 A_2 \mid B) = P(A_1 \mid A_2 B)\, P(A_2 \mid B)$$

conditional independence of *A1* and *A2* given B is equivalent to

$$P(A_1 \mid A_2 B) = P(A_1 \mid B)$$

i.e., knowledge of the occurrence of event *A2* does not affect the probability of event *A1* given the occurrence of event B.

Basics

Odds:
For probabilities P(A) $\neq$ 1; A $\in \boldsymbol{A}$, odds are defined as the ratio

$$O(A) = \frac{P(A)}{P({}^{c}A)} = \frac{p}{1-p}, \quad p \in [0,1)$$

The odds in favor of an event or a proposition is the ratio of the probability that the event will happen to the probability that the event will not happen.

Conditional odds:
Since for *P(B)* > 0 the conditional probability P_B is a probability measure on $\boldsymbol{A} \mid B$, the denition of odds applies and yields conditional odds

$$O(A \mid B) = \frac{P(A \mid B)}{P({}^{c}A \mid B)} = \frac{P(A \mid B)}{1 - P(A \mid B)}$$

Basics

Bayes' factor:
If $P(A) \in (0; 1)$; P(B) > 0, then by virtue of Bayes' rule

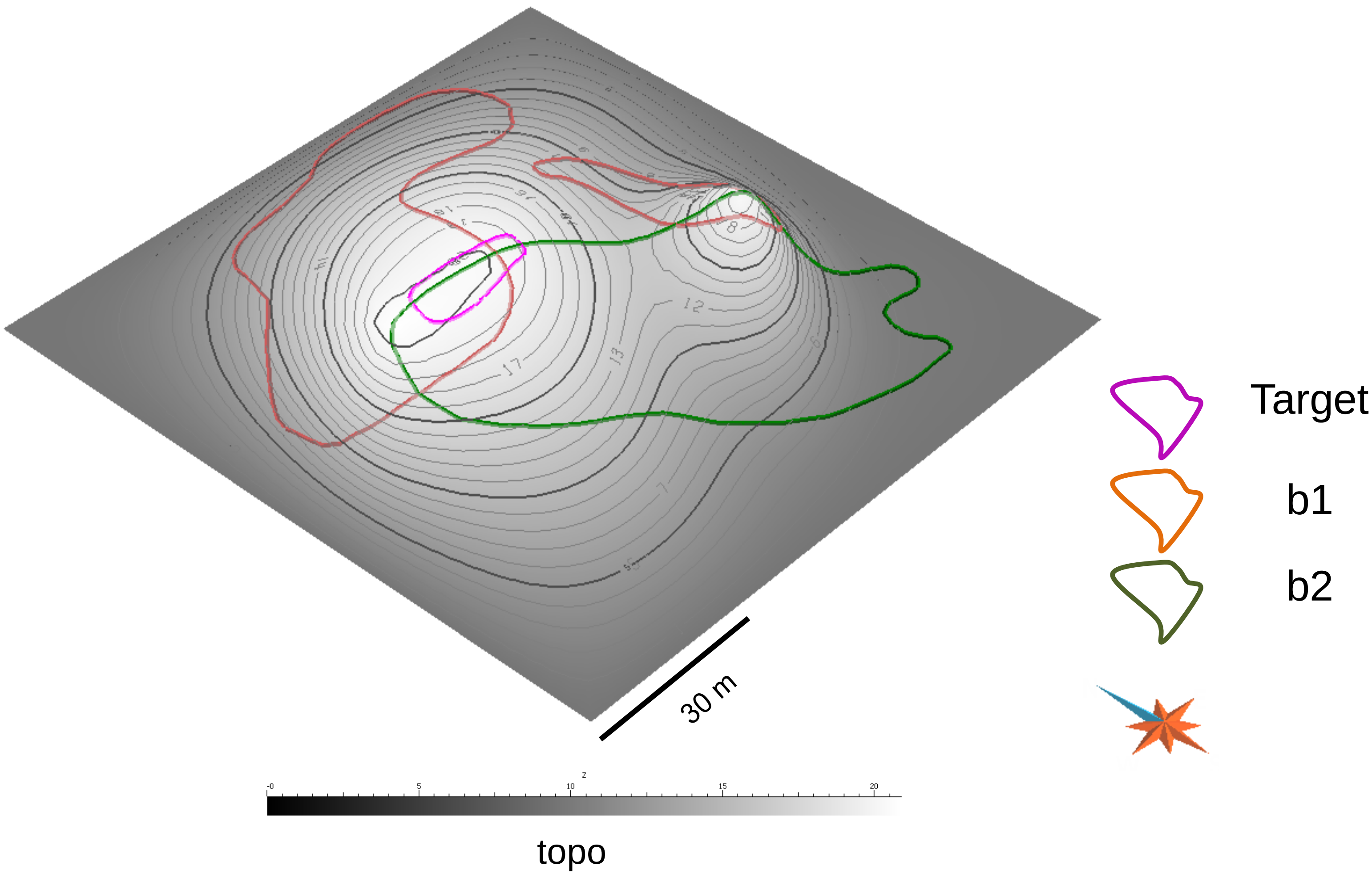
$$\mathrm{O}(A \mid B) = \mathrm{O}(A)\,\frac{P(B \mid A)}{P(B \mid {}^{c}\!A)}$$

Good calls the ratio the "Bayes factor"(Good, 1968, p. 31), and its logarithm, i.e. the "log-factor", the "weight of evidence" (Good, 1950, p. 63; Good, 1968, p. 32).

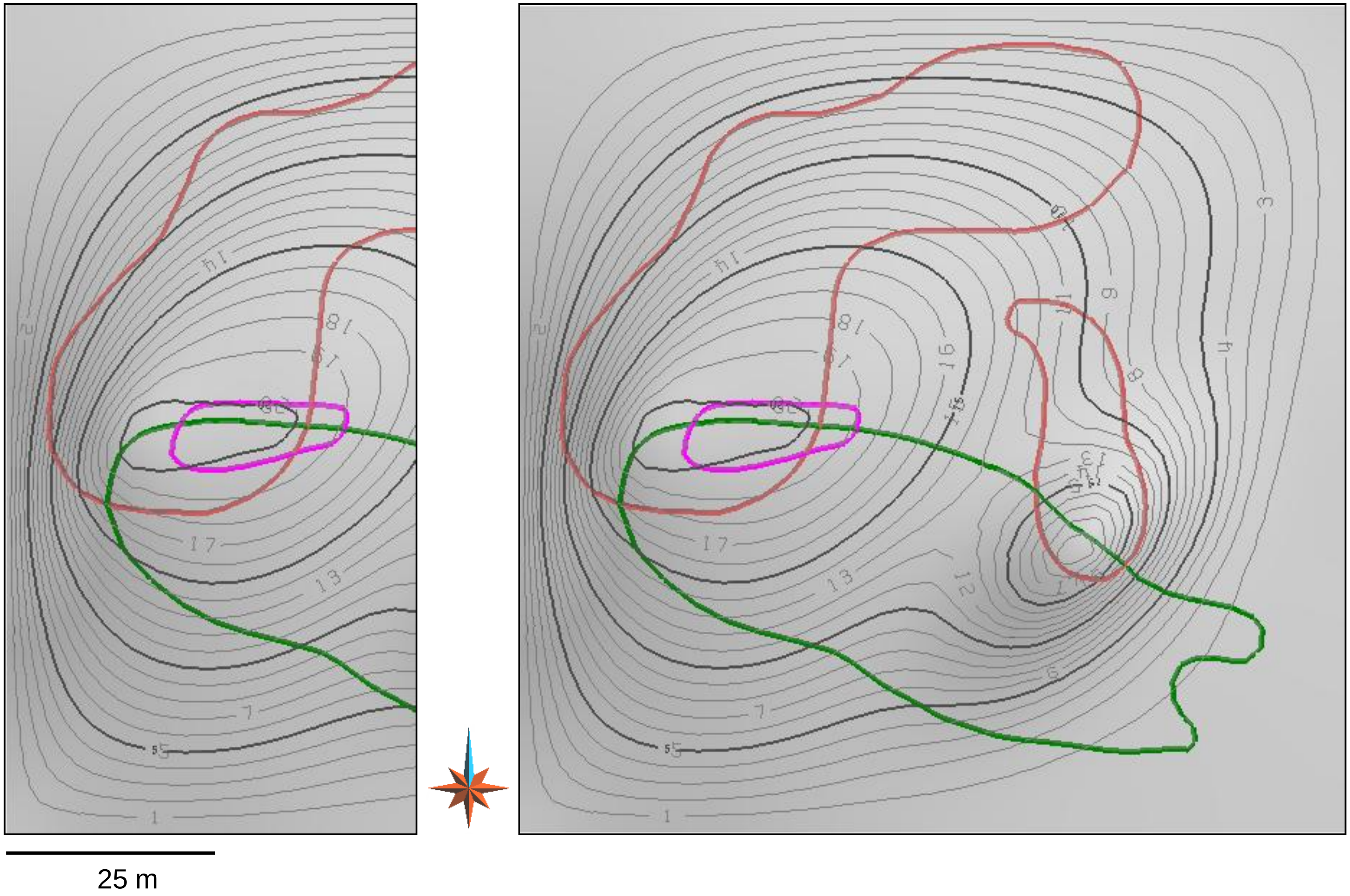
Theorem T23 from Good, 1950, p. 63-64:
Suppose that a series of experiments are performed, with results $E_1, \ldots\ldots\ldots E_n$ and suppose these are independent given H and independent given ${}^{c}H$. Then the resulting factor is equal to the product of the individual factors, and therefore the resulting weight of evidence is equal to the sum of individual weights of evidence.

$$\frac{P(E_1 \ldots E_n \mid H)}{P(E_1 \ldots E_n \mid {}^{c}\!H)} = \frac{P(E_1 \mid H)}{P(E_1 \mid {}^{c}\!H)} \cdots \frac{P(E_n \mid H)}{P(E_n \mid {}^{c}\!H)}$$

Study case



Study case



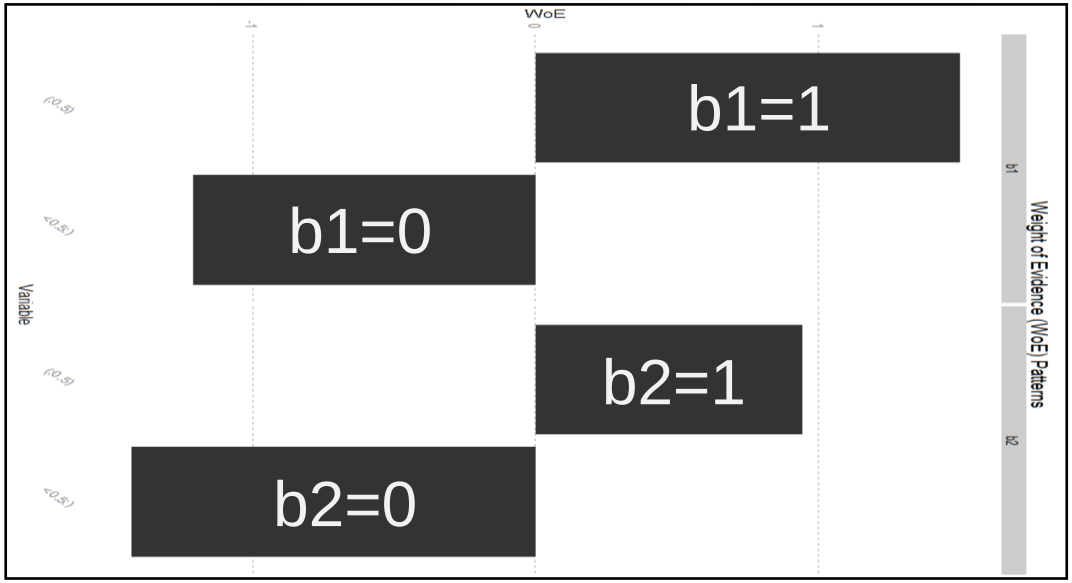
Binary variables

Assuming **conditional independence** of several **binary** evidential random variables B_l, $l = 1, \ldots\ldots m$, given the binary random target variable T yields weights of evidence

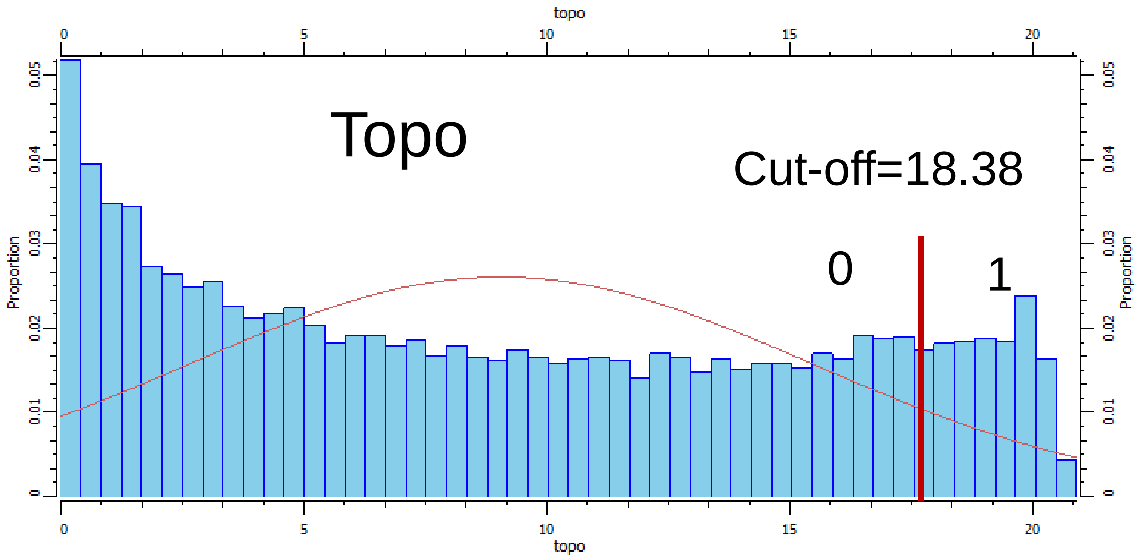
$$W_\ell^+ := \ln \frac{P(\mathrm{B}_\ell = 1 \mid \mathrm{T} = 1)}{P(\mathrm{B}_\ell = 1 \mid \mathrm{T} = 0)}, \quad W_\ell^- := \ln \frac{P(\mathrm{B}_\ell = 0 \mid \mathrm{T} = 1)}{P(\mathrm{B}_\ell = 0 \mid \mathrm{T} = 0)},$$

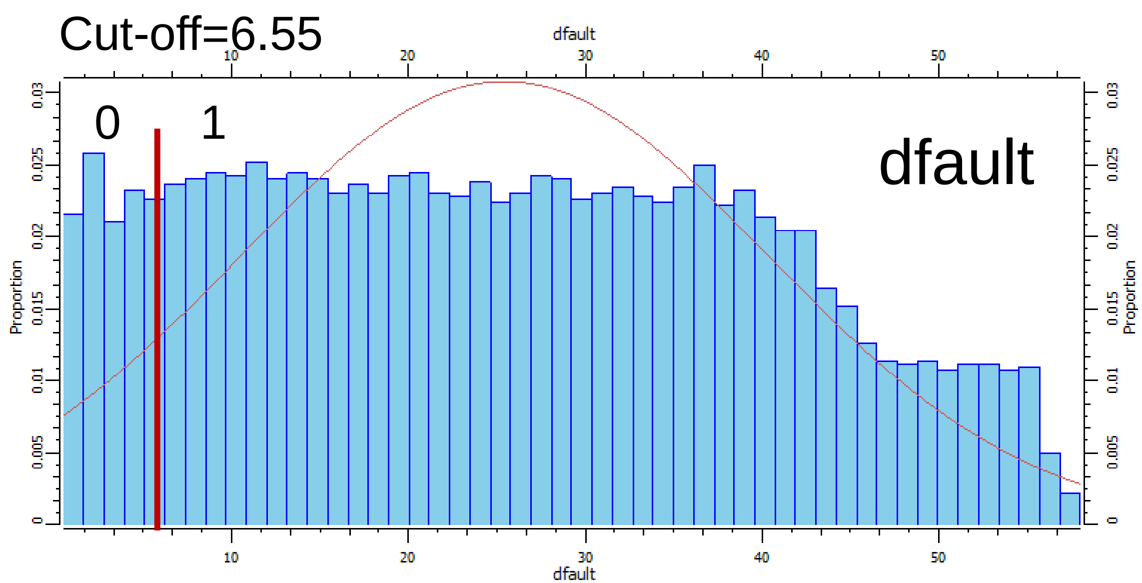
and

$$C_\ell := W_\ell^+ - W_\ell^-, \;\; \ell = 1, \ldots, m,$$



Continuous variables to binary





Continuous variables to binary

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and

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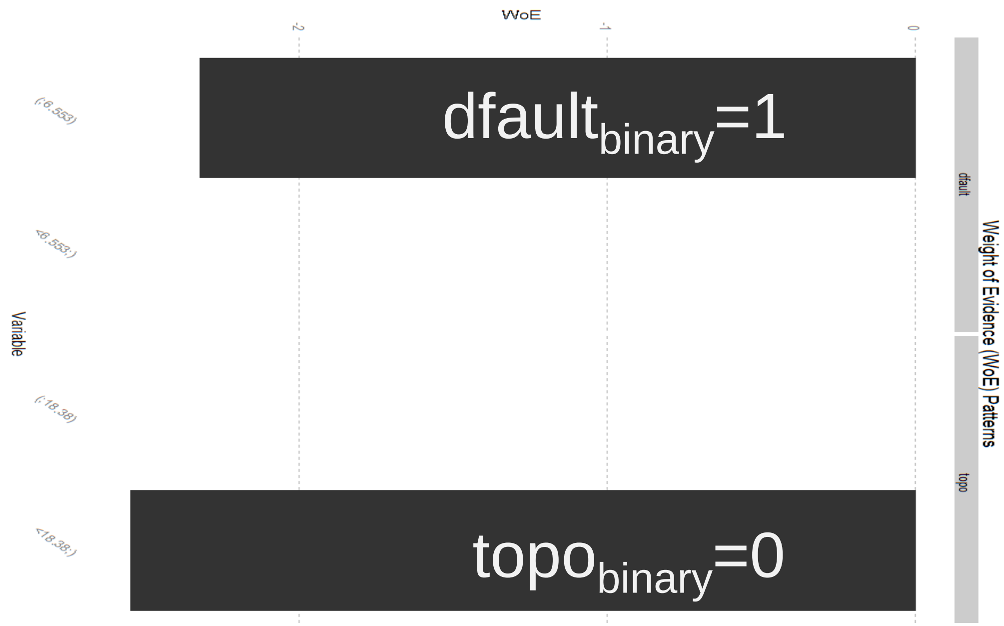
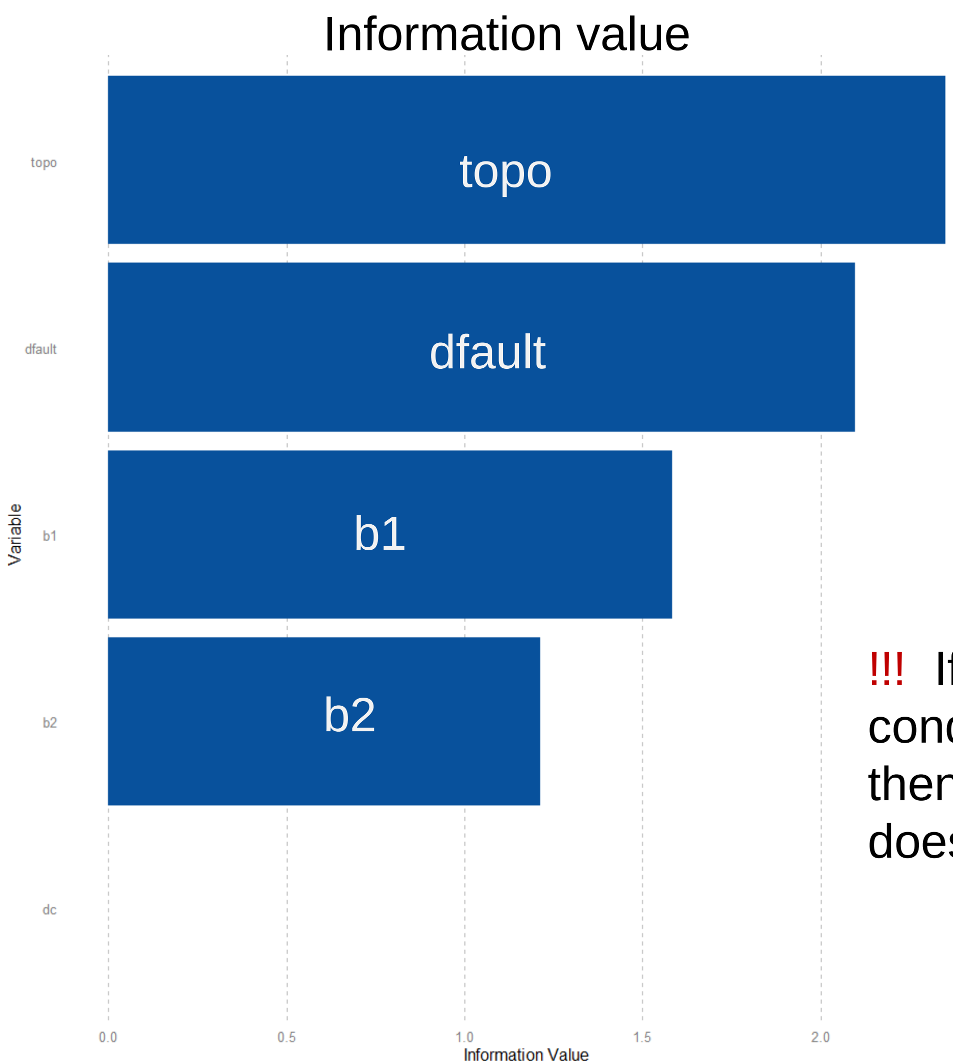


Conditional independence



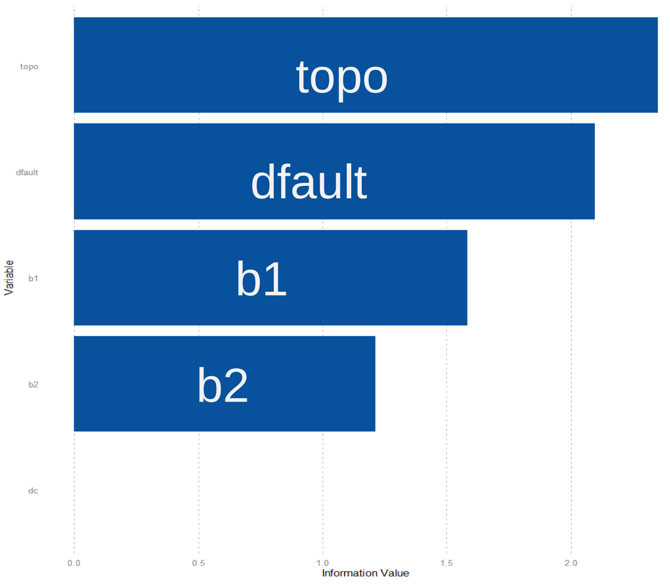
correl.	b1	b2	topo	dfault	d
b1	1.00	-0.05	0.50	-0.24	0.20
b2	-0.05	1.00	0.47	-0.51	0.21
topo	0.50	0.47	1.00	-0.82	0.27
dfault	-0.24	-0.51	-0.82	1.00	-0.23
d	0.20	0.21	0.27	-0.23	1.00

!!! If individual pieces of evidence are not conditionally independent given the target, then the method of weights of evidence does not apply.

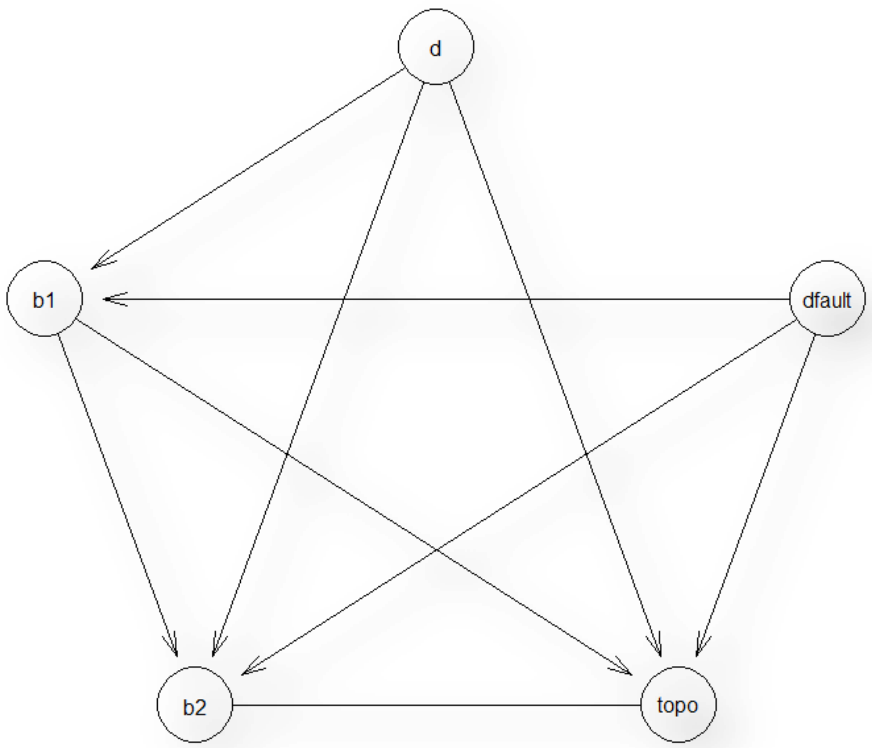
Conditional independence tests

correl.	b1	b2	topo	dfault	d
b1	1.00	-0.05	0.50	-0.24	0.20
b2	-0.05	1.00	0.47	-0.51	0.21
topo	0.50	0.47	1.00	-0.82	0.27
dfault	-0.24	-0.51	-0.82	1.00	-0.23
d	0.20	0.21	0.27	-0.23	1.00

Information value



Incremental association



Logistic regression

Logistic regression (LR) provides an optimum model unrestricted with respect to modeling assumptions (conditional independence) and type of random variables (continuous or discrete) (Schaeben, 2013).

$$log(Odds) = \beta_0 + \beta_l x_1 + \beta_2 x_2 +, ..., +\beta_m x_m$$
$$x_{1m} : \{predictors\}$$
$$\beta_{0m} : \{coefficients\}$$

The logistic function is defined as:

$$\Lambda(z) = \frac{1}{1+\exp(-z)} = \frac{\exp(z)}{1+\exp(z)}, \quad z \in \mathbb{R}.$$

Modeling with LR

The R Project for Statistical Computing

Logistic regression with R:

```
lrM <- glm(d~ b1 + b2 + dfault + topo, family = binomial("logit"), data = proven)
```

Coefficients:	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-50.90	6.47	-7.87	0.00 ***
b1	-1.73	0.43	-4.06	0.00 ***
b2	-0.50	0.40	-1.27	0.20
topo	-0.18	0.08	-2.31	0.02 *
dfault	2.65	0.33	7.98	0.00 ***

$$log(Odds) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 +, ..., +\beta_m x_m$$
$$x_{1m} : \{predictors\}$$
$$\beta_{0m} : \{coefficients\}$$

!!! If individual pieces of evidence are not conditionally independent given the target, then the method of logistic regression applies, but interaction terms are needed to yield the proper complete model.

!!! If (some) pieces of evidence are not binary, then the method of logistic regression still applies, interaction terms are needed, but multi-linear interaction terms may yield only approximations to the proper complete model.

Modeling with LR

lrM_inter <- glm(d ~ b1 + b2 + topo + dfault + b1xb2 + b1xtopo + b1xdfault + b2xtopo + b2xdfault + topoxdfault, family = binomial("logit"), data=proven)

Coefficients:	Estimate	Std. Error	z value	Pr(>|z|)	
b1	30.49	19.11	1.60	0.11	
b2	-7.05	12.20	-0.58	0.56	
topo	2.84	1.33	2.14	0.03	*
dfault	-7.05	3.90	-1.81	0.07	.
b1xb2	4.32	1.39	3.11	0.00	**
b1xtopo	-1.88	1.00	-1.88	0.06	.
b1xdfault	0.34	0.28	1.20	0.23	
b2xtopo	0.08	0.64	0.12	0.90	
b2xdfault	0.48	0.21	2.31	0.02	*
topoxdfault	0.33	0.21	1.61	0.11	

$$log(Odds) = \beta_0 + \beta_l x_1 + \beta_2 x_2 +, ..., +\beta_m x_m$$

$$x_{1m} : \{predictors\}$$

$$\beta_{0m} : \{coefficients\}$$

Building a predictive model

lrM_inter <- glm(d ~ b1 + topo + dfault + b1xb2 + b1xtopo + b1xdfault + topoxdfault, family = binomial("logit"), data=proven)

Coefficients:	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	-41.3061	21.6346	-1.909	0.05623	.
b1	36.019	19.1832	1.878	0.06043	.
topo	2.1006	1.1263	1.865	0.06218	.
dfault	-11.3116	3.4255	-3.302	0.00096	***
b1xb2	0.871	0.4893	1.78	0.07507	.
b1xtopo	-1.9155	0.9939	-1.927	0.05396	.
b1xdfault	-0.3569	0.1748	-2.042	0.04113	*
topoxdfault	0.5911	0.1789	3.304	0.00095	***

$$log(Odds) = \beta_0 + \beta_l x_1 + \beta_2 x_2 +, ..., +\beta_m x_m$$
$$x_{1m} : \{predictors\}$$
$$\beta_{0m} : \{coefficients\}$$

Applying the LR model

lrM_inter <- glm(d ~ b1 + topo + dfault + b1xb2 + b1xtopo + b1xdfault + topoxdfault, family = binomial("logit"), data=study)

Coefficients:	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	-41.3061	21.6346	-1.909	0.05623	.
b1	36.019	19.1832	1.878	0.06043	.
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$$log(Odds) = \beta_0 + \beta_l x_1 + \beta_2 x_2 +, ..., + \beta_m x_m$$
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Predicted probability

Coefficients:	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-41.3061	21.6346	-1.909	0.05623 .
b1	36.019	19.1832	1.878	0.06043 .
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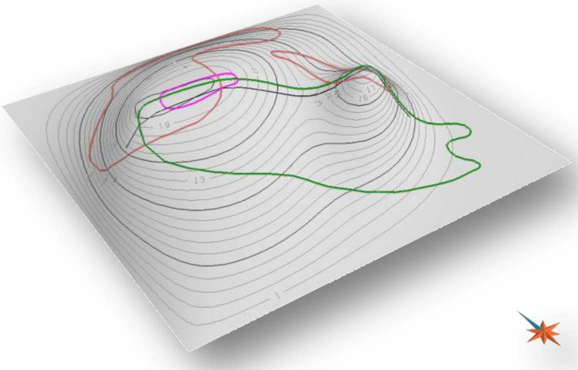
b1xdfault	-0.3569	0.1748	-2.042	0.04113 *
topoxdfault	0.5911	0.1789	3.304	0.00095 ***

Model from proven area



$$log(Odds) = \beta_0 + \beta_l x_1 + \beta_2 x_2 +, \ldots, +\beta_m x_m$$

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$$\Lambda(z) = \frac{1}{1+\exp(-z)} = \frac{\exp(z)}{1+\exp(z)}, \quad z \in \mathbb{R}.$$