Répartition de puissances dans un réseau électrique

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2A Energie et Fluides - EFS8AA

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Grandeurs réduites

Système per unit (p.u.) :

se ramener à une puissance apparente nominale S_b

identique pour tout le réseau

$$S_b = \sqrt{3} U_b I_b = 3 V_b I_b$$

$$\underline{v} = \frac{\underline{V}}{V_b} \qquad \underline{i} = \frac{\underline{I}}{I_b} \qquad p = \frac{P}{S_b} \qquad q = \frac{Q}{S_b}$$

partie 1 à 63 kV : $U_{b1} = 63 kV$ $\quad \underline{U}_1 = 69.3 kV$ $\quad \rightsquigarrow \underline{v}_1 = 1.1$

partie 2 à 400 kV : $U_{b2} = 400 kV$ $\quad \underline{U}_2 = 440 kV$ $\quad \rightsquigarrow \underline{v}_2 = 1.1$

Grandeurs réduites

$\star$ impédance $$\underline{z} = \underline{Z}\,\frac{I_b}{V_b} = \underline{Z}\,\frac{S_b}{U_b^2} = \frac{\underline{Z}}{Z_b}$$

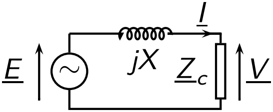
$\rightsquigarrow$ impédance de base $$Z_b = \frac{U_b^2}{S_b}$$

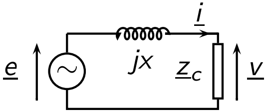
$\star$ admittance $$\underline{y} = \underline{Y}\,\frac{U_b^2}{S_b} = \underline{Y}\,Z_b$$

$\star$ puissances $$\underline{s} = \frac{\underline{S}}{S_b} = \frac{3\;\underline{V}\;\underline{I}^*}{3V_b I_b} = \underline{v}\;\underline{i}^*$$

$$p + jq = \frac{P + jQ}{S_b} = \underline{v}\;\underline{i}^*$$

Grandeurs réduites





$$S_b = 100MVA \qquad U_b = 400kV$$

$$X = 20\Omega \qquad \underline{Z}_c = 1000(1+j) \qquad \underline{V} = 230kV$$

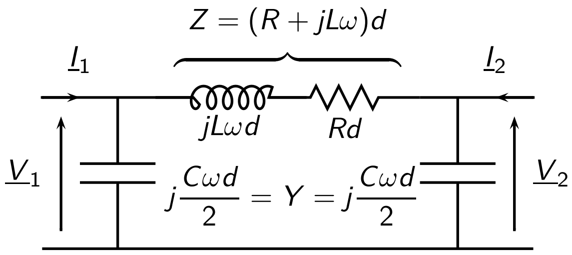
$$\underline{I} = \frac{\underline{V}}{\underline{Z}_c} = 115(1-j) \qquad \underline{E} = \underline{V} + jX\underline{I} = 232\ 10^3 \exp\left(-j9.9\ 10^{-3}\right)$$

$$\underline{S} = 3\underline{V}\,\underline{I}^* = 80(1+j) \times 10^6$$

$$Z_b = \frac{U_b^2}{S_b} = \frac{400^2}{100} \qquad x = \frac{X}{Z_b} = 0.0125 \qquad \underline{z} = \frac{\underline{Z}_c}{Z_b} = 0.62 + 0.62j$$

$$V_b = 230kV \qquad \underline{v} = \frac{\underline{V}}{V_b} = 1 \qquad \underline{i} = \frac{\underline{v}}{\underline{z}} = 0.8 - 0.8j$$

$$\underline{e} = \underline{v} + jx\underline{i} = 1.01 + 0.01j \qquad \underline{s} = \underline{v}\,\underline{i}^* = 0.8 + 0.8j$$

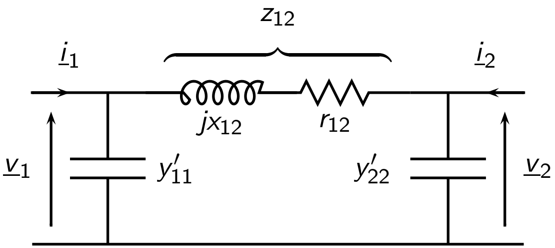


$$\begin{pmatrix} \underline{V}_2 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} 1+ZY & -Z \\ Y(2+ZY) & -(1+ZY) \end{pmatrix} \begin{pmatrix} \underline{V}_1 \\ \underline{I}_1 \end{pmatrix}$$

$$\begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} Y+1/Z & -1/Z \\ -1/Z & Y+1/Z \end{pmatrix} \begin{pmatrix} \underline{V}_1 \\ \underline{V}_2 \end{pmatrix}$$

exemple : $225kV \quad 300A \quad d = 100km$

$$r = 0.06\,\Omega/km \quad L = 1.24\,mH/km \quad C = 8.8\,nF/km$$

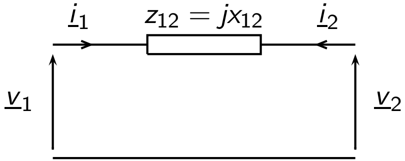


$$\begin{pmatrix} \underline{i}_1 \\ \underline{i}_2 \end{pmatrix} = \begin{pmatrix} y'_{11} + 1/z_{12} & -1/z_{12} \\ -1/z_{12} & y'_{22} + 1/z_{12} \end{pmatrix} \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}$$

$$225kV \quad 100MVA \quad I_n = \frac{S_n}{\sqrt{3}U_n} = 256A \quad Z_b = \frac{U_n^2}{S_b} = \frac{225^2}{100} = 506\Omega$$

$$x_{12} = \frac{L\omega\, d}{Z_b} = 0.077 \quad r_{12} = \frac{r\, d}{Z_b} = 0.012 \quad y'_{11} = y'_{22} = \frac{C}{2}\omega d Z_b = 0.07$$

$\underline{s}_{12} = \underline{v}_1\, \underline{i}_1^* \;\Rightarrow$



$\Leftarrow\; \underline{v}_2\, \underline{i}_2^* = \underline{s}_{21}$

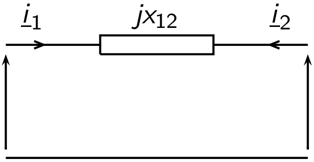
$$\underline{v}_1 - \underline{v}_2 = \underline{z}_{12}\, \underline{i}_1 = -\underline{z}_{12}\, \underline{i}_2 \quad \rightsquigarrow \quad \begin{pmatrix} \underline{i}_1 \\ \underline{i}_2 \end{pmatrix} = \frac{1}{jx_{12}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}$$

$$\underline{s}_{12} = \underline{v}_1\, \underline{i}_1^* = \frac{j}{x_{12}} \left(|\underline{v}_1|^2 - \underline{v}_1 \underline{v}_2^* \right)$$

$$\underline{s}_{21} = \underline{v}_2\, \underline{i}_2^* = \frac{j}{x_{12}} \left(|\underline{v}_2|^2 - \underline{v}_2 \underline{v}_1^* \right)$$

$$\textit{dans la ligne :}\ \underline{s}_{12} + \underline{s}_{21} = j \frac{|\underline{v}_1 - \underline{v}_2|^2}{x_{12}} \quad \rightsquigarrow \quad \Re e(\underline{s}_{12} + \underline{s}_{21}) = 0$$

$\underline{v}_1 = |\underline{v}_1| e^{j\,\theta_1}$



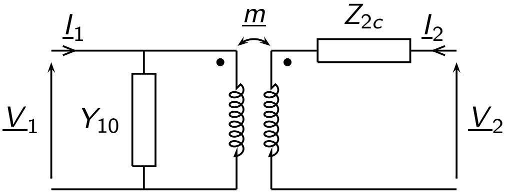
$\underline{v}_2 = |\underline{v}_2| e^{j\,\theta_2}$

$$\underline{s}_{12} = \frac{j}{x_{12}}\,\underline{v}_1\,\left(\underline{v}_1 - \underline{v}_2\right)^* = j\frac{|\underline{v}_1|^2}{x_{12}}\left(1 - \left|\frac{\underline{v}_2}{\underline{v}_1}\right| e^{-j(\theta_2 - \theta_1)}\right)$$

- $\star\;\; p_{12} = \Re e(\underline{s}_{12}) = \dfrac{|\underline{v}_1|\,|\underline{v}_2|}{x_{12}} \sin(\theta_1 - \theta_2)$

- $\star\;\; q_{12} = \Im m(\underline{s}_{12}) = \dfrac{|\underline{v}_1|^2}{x_{12}}\left(1 - \left|\dfrac{\underline{v}_2}{\underline{v}_1}\right| \cos(\theta_1 - \theta_2)\right)$

Transformateur non-idéal (grandeurs non réduites)



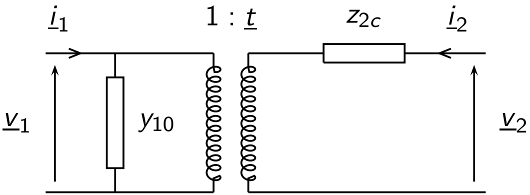
$$Y_{10} = \frac{1}{R_{fe}} + \frac{1}{jL\omega} \qquad Z_{2c} = \rho + j\lambda\omega$$

$$\underline{m}\,\underline{V}_1 = \underline{V}_2 - Z_{2c}\,\underline{I}_2 \qquad \underline{I}_1 = Y_{10}\underline{V}_1 - \underline{m}^*\,\underline{I}_2$$

$|\underline{m}|$ variable, $\arg(\underline{m}) = -k\pi/6$ fixe : régulateur de tension

$|\underline{m}|$ fixe, $\arg(\underline{m})$ variable : transformateur déphaseur

Transformateur non-idéal (grandeurs réduites)



$$\underline{t}\,\underline{v}_1 = \underline{v}_2 - z_{2c}\,\underline{i}_2$$

$$\underline{i}_1 = y_{10}\,\underline{v}_1 - \underline{t}^*\,\underline{i}_2$$

$$\rightsquigarrow \begin{pmatrix} \underline{i}_1 \\ \underline{i}_2 \end{pmatrix} = \begin{pmatrix} y_{10} + \dfrac{|\underline{t}|^2}{z_{2c}} & -\dfrac{\underline{t}^*}{z_{2c}} \\ -\dfrac{\underline{t}}{z_{2c}} & \dfrac{1}{z_{2c}} \end{pmatrix} \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix}$$

$$\underline{t} = \frac{\underline{m}}{M} = \underline{m}\,\frac{U_{b1}}{U_{b2}} \qquad z_{2c} = Z_{2c}\,\frac{S_b}{U_{b2}^2} \qquad y_{10} = Y_{10}\,\frac{U_{b1}^2}{S_b}$$

1 MVA - 100 MVA

$$y_{10} = \frac{1}{r_{fe}} + \frac{1}{jl\omega}$$

$$r_{fe} \simeq 100 - 500\, p.u.$$

$$l\omega \simeq 30 - 50\, p.u.$$

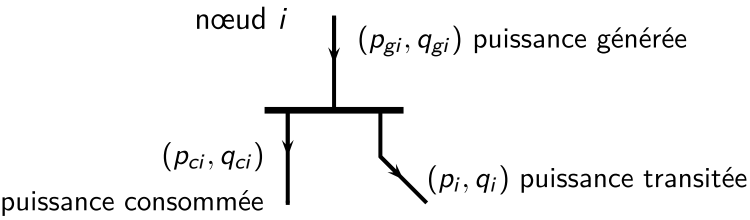
$$i_0 \simeq 0.01 - 0.001\, p.u.$$

$$z_{2c} = \rho + j\lambda\omega$$

$\rho \simeq 0.005 - 0.002\, p.u.$ par enroulement

$\lambda\omega \simeq 0.03 - 0.08\, p.u.$ par enroulement

$\lambda\omega \simeq 0.10 - 0.20\, p.u.$ réseau de transport



bilan de puissance : $\begin{cases} p_i = p_{gi} - p_{ci} \\ q_i = q_{gi} - q_{ci} \end{cases}$

- $\star$ nœud de consommation : charge $\rightsquigarrow$ (p_{ci}, q_{ci}) fixés
- $\star$ nœud de tension : alternateur $\rightsquigarrow$ $|\underline{v}_i|$ fixé
- $\star$ nœud de référence : $\rightsquigarrow$ $\underline{v}_N$ fixé

Nœuds du réseau électrique

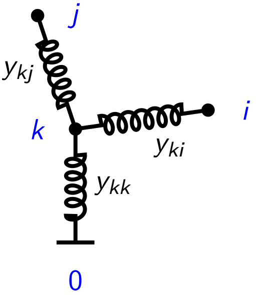
	nombre	données	inconnues
consommation	n_c	(p_i, q_i)	$(\lvert\underline{v}_i\rvert, \arg(\underline{v}_i))$
tension	n_t	$(\lvert\underline{v}_i\rvert, p_i)$	$(\arg(\underline{v}_i), q_i)$
référence	1	$(\lvert\underline{v}_i\rvert, \arg(\underline{v}_i))$	(p_i, q_i)

répartition de puissance : obtenir $(|\underline{v}_i|, \arg(\underline{v}_i))$ en chaque nœud

$2n_c + n_t$ inconnues : n_c $(|\underline{v}_i|, \arg(\underline{v}_i))$ et n_t $(\arg(\underline{v}_i))$

$2n_c + n_t$ équations : n_c (p_i, q_i) et n_t (p_i)

- injecté : $\underline{s}_k = (p_{gk} - p_{ck}) + j(q_{gk} - q_{ck})$



$$\underline{s}_k = \underline{v}_k \underline{i}_k^* = \underline{v}_k \left(\sum_{i=1}^{N} y_{ki} (\underline{v}_k - \underline{v}_i) \right)^*$$

$$\underline{s}_k = \underline{v}_k \left(y_{kk} \underline{v}_k + \sum_{i \neq k} y_{ki} (\underline{v}_k - \underline{v}_i) \right)^*$$

$$\underline{s}_k = |\underline{v}_k|^2 \left(y_{kk} + \sum_{i \neq k} y_{ki} \right)^* - \sum_{i \neq k} y_{ki}^* \underline{v}_k \underline{v}_i^*$$

- transité : $\underline{s}_k = \underline{v}_k \left(\sum_{i=1}^{N} \mathcal{Y}_{ki} \underline{v}_i \right)^*$

⋆ nœud de consommation

$$\underline{s}_k = \underline{v}_k \left(\sum_{i=1}^{N} \mathcal{Y}_{ki} \underline{v}_i \right)^* \quad \rightsquigarrow \quad \begin{array}{l} p_k = -p_{ck} = \Re e(\underline{s}_k) \\ q_k = -q_{ck} = \Im m(\underline{s}_k) \end{array}$$

⋆ nœud de tension

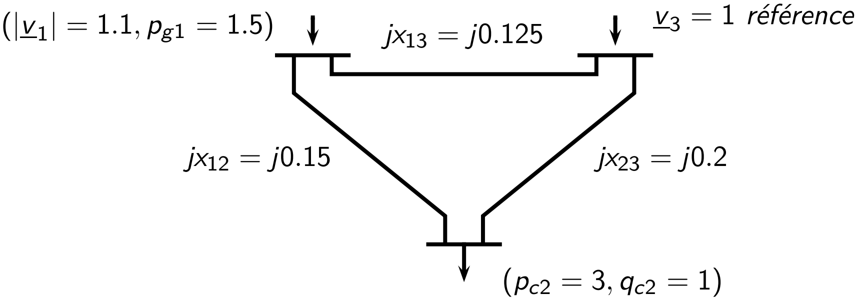
$$\underline{s}_k = \underline{v}_k \left(\sum_{i=1}^{N} \mathcal{Y}_{ki} \underline{v}_i \right)^* \quad \rightsquigarrow \quad p_k = p_{gk} = \Re e(\underline{s}_k)$$

But : calculer les tensions aux nœuds $\underline{v}_i$

$\rightsquigarrow$ $(v_i = |\underline{v}_i|, \theta_i = \arg(\underline{v}_i))$ pour les nœuds de consommation

$\rightsquigarrow$ $(\theta_i = \arg(\underline{v}_i))$ pour les nœuds de tension

en déduire q_i pour les nœuds de tension et (p_i, q_i) pour le nœud référence



inconnues : $(\theta_1, v_2, \theta_2)$

données : $(p_1 = p_{g1} = 1.5, p_2 = -p_{c2} = -3, q_2 = -q_{c2} = -1)$

Exemple à 3 nœuds (lignes sans pertes)

$$\begin{pmatrix} \underline{i}_1 \\ \underline{i}_2 \\ \underline{i}_3 \end{pmatrix} = [\mathcal{Y}] \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \underline{v}_3 \end{pmatrix} = j \begin{pmatrix} \frac{1}{x_{12}} + \frac{1}{x_{13}} & -\frac{1}{x_{12}} & -\frac{1}{x_{13}} \\ -\frac{1}{x_{12}} & \frac{1}{x_{12}} + \frac{1}{x_{23}} & -\frac{1}{x_{23}} \\ -\frac{1}{x_{13}} & -\frac{1}{x_{23}} & \frac{1}{x_{13}} + \frac{1}{x_{23}} \end{pmatrix} \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \underline{v}_3 \end{pmatrix}$$

avec $x_{12} = 0.15, x_{23} = 0.2, x_{13} = 0.125$

$$[\mathcal{Y}] = j \begin{pmatrix} -14.67 & 6.667 & 8. \\ 6.667 & -11.67 & 5 \\ 8. & 5. & -13. \end{pmatrix}$$

Exemple à 3 nœuds (lignes sans pertes)

$$[\underline{s}] = \begin{pmatrix} \underline{s}_1 \\ \underline{s}_2 \\ \underline{s}_3 \end{pmatrix} = \begin{pmatrix} \underline{v}_1\, \underline{i}_1^* \\ \underline{v}_2\, \underline{i}_2^* \\ \underline{v}_3\, \underline{i}_3^* \end{pmatrix} = [\underline{v}] \otimes ([\mathcal{Y}][\underline{v}])^*$$

$$\begin{cases} p_1 = 1.5 = |\underline{v}_1| \; \Re e\left(\mathcal{Y}_{11}^* |\underline{v}_1| + e^{j\theta_1}\left(\mathcal{Y}_{12}|\underline{v}_2| e^{j\theta_2} + \mathcal{Y}_{13}\underline{v}_3\right)^*\right) \\ \\ p_2 + jq_2 = -3.0 - j1.0 = \mathcal{Y}_{22}^* |\underline{v}_2|^2 + |\underline{v}_2| e^{j\theta_2}\left(\mathcal{Y}_{21}|\underline{v}_1| e^{j\theta_1} + \mathcal{Y}_{31}\underline{v}_3\right)^* \end{cases}$$

$$\begin{aligned} p_1 &= |\underline{v}_1| \left(\frac{|\underline{v}_2|}{x_{12}} \sin(\theta_1 - \theta_2) + \frac{|\underline{v}_3|}{x_{13}} \sin(\theta_1) \right) \\ p_2 &= |\underline{v}_2| \left(\frac{|\underline{v}_1|}{x_{12}} \sin(\theta_2 - \theta_1) + \frac{|\underline{v}_3|}{x_{23}} \sin(\theta_2) \right) \\ q_2 &= \frac{|\underline{v}_2|}{x_{12}} \left(|\underline{v}_1| - |\underline{v}_2| \cos(\theta_2 - \theta_1)\right) + \frac{|\underline{v}_2|}{x_{23}} \left(|\underline{v}_2| - |\underline{v}_3| \cos(\theta_2)\right) \end{aligned}$$

Exemple à 3 nœuds (lignes sans pertes)

$$q_2 \simeq \frac{|\underline{v}_2|}{x_{12}}\left(|\underline{v}_1| - |\underline{v}_2|\right) + \frac{|\underline{v}_2|}{x_{23}}\left(|\underline{v}_2| - |\underline{v}_3|\right) \qquad \rightsquigarrow |\underline{v}_2|$$

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \simeq \begin{pmatrix} |\underline{v}_1|\left(\frac{|\underline{v}_2|}{x_{12}} + \frac{|\underline{v}_3|}{x_{13}}\right) & -\frac{|\underline{v}_1||\underline{v}_2|}{x_{12}} \\ -\frac{|\underline{v}_1||\underline{v}_2|}{x_{12}} & |\underline{v}_1|\left(\frac{|\underline{v}_2|}{x_{12}} + \frac{|\underline{v}_3|}{x_{23}}\right) \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

Solution approximée : $(\theta_1, |\underline{v}_2|, \theta_2) = (-0.0243, 0.967, -0.266)$

$$\rightsquigarrow \quad (q_1 = 1.84, p_3 = 1.5, q_3 = -0.84)$$

Solution exacte : $(\theta_1, |\underline{v}_2|, \theta_2) = (-0.0248, 0.928, -0.280)$

$$\rightsquigarrow \quad (q_1 = 2.36, p_3 = 1.5, q_3 = -0.25)$$

Exemple à 3 nœuds (lignes sans pertes)

$$\begin{cases} q_1 = \Im m \left(\mathcal{Y}_{11}^* |\underline{v}_1|^2 + \underline{v}_1 \left(\mathcal{Y}_{12} \underline{v}_2 + \mathcal{Y}_{13} \underline{v}_3 \right)^* \right) = 2.3669 \\ p_3 + jq_3 = \mathcal{Y}_{33}^* |\underline{v}_3|^2 + \underline{v}_3 \left(\mathcal{Y}_{31} \underline{v}_1 + \mathcal{Y}_{32} \underline{v}_2 \right)^* = 1.5 - j0.2550 \end{cases}$$

vérifications

$$\underline{s}_{12} = \underline{v}_1 \left(\frac{\underline{v}_1 - \underline{v}_2}{\underline{z}_{12}} \right)^* = 1.7179 + j1.4842$$

$$\underline{s}_{13} = -0.2179 + j0.8827$$

$$p_1 + jq_1 = \underline{s}_{12} + \underline{s}_{13}$$

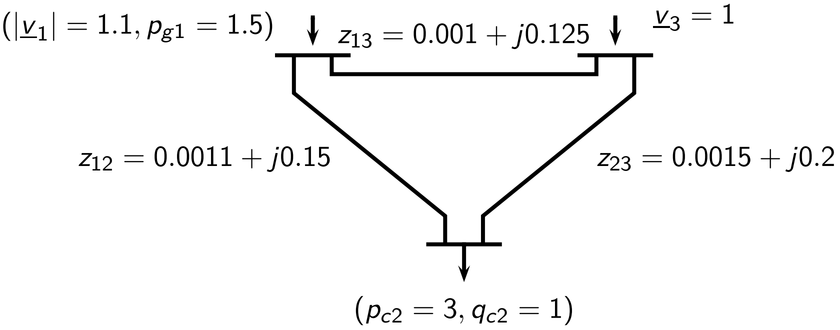
$$\underline{s}_1 = 1.5 + j2.3669 \qquad \underline{s}_{12} + \underline{s}_{21} = \frac{|\underline{v}_1 - \underline{v}_2|^2}{\underline{z}_{12}^*} = j\,0.6389$$

$$\underline{s}_2 = -3 - j \qquad \underline{s}_{13} + \underline{s}_{31} = j\,0.08539$$

$$\underline{s}_3 = 1.5 - j0.2550 \qquad \underline{s}_{23} + \underline{s}_{32} = j\,0.3875$$

$$\underline{s}_1 + \underline{s}_2 + \underline{s}_3 = j\,1.119$$

Exemple à 3 nœuds (lignes avec pertes)

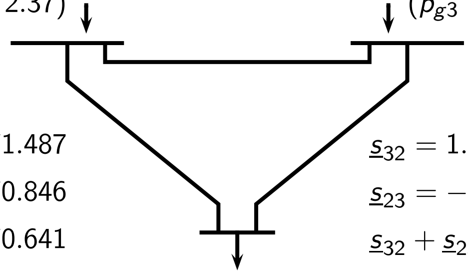


données : $(p_1 = p_{g1} = 1.5, p_2 = -p_{c2} = -3, q_2 = -q_{c2} = -1)$

inconnues : $(\theta_1, v_2, \theta_2) \quad \rightsquigarrow \quad (-0.0260, 0.925, -0.281)$

$$\underline{s}_{31} = 0.222 - j0.799$$
$$\underline{s}_{13} = -0.221 + j0.885$$
$$\underline{s}_{13} + \underline{s}_{31} = 6.9e - 4 + j0.0859$$

$(p_{g1} = 1.5, q_{g1} = 2.37)$ $(p_{g3} = 1.508, q_{g3} = -0.2$



$$\underline{s}_{12} = 1.721 + j1.487$$
$$\underline{s}_{21} = -1.718 - j0.846$$
$${}_2 + \underline{s}_{21} = 0.0047 + j0.641$$

$$\underline{s}_{32} = 1.286 + j0.544$$
$$\underline{s}_{23} = -1.283 - j0.154$$
$$\underline{s}_{32} + \underline{s}_{23} = 0.0029 + j0.390$$

$(p_{c2} = 3, q_{c2} = 1)$

vérifications

$$
\begin{array}{rcccccc}
p_{g1} + jq_{g1} & = & 1.5 + j\,2.37 & = & \underline{s}_1 & = & \underline{s}_{12} + \underline{s}_{13} \\
-p_{c2} - jq_{c2} & = & -3 - j & = & \underline{s}_2 & = & \underline{s}_{21} + \underline{s}_{23} \\
p_{g3} + jq_{g3} & = & 1.508 - j\,0.255 & = & \underline{s}_3 & = & \underline{s}_{31} + \underline{s}_{32} \\
\hline
\end{array}
$$

$$
\begin{aligned}
p_{g1} - p_{c2} + p_{g3} + j(q_{g1} - q_{c2} + q_{g3})& \\
&= \underline{s}_1 + \underline{s}_2 + \underline{s}_3 \\
&= (\underline{s}_{12} + \underline{s}_{12}) + (\underline{s}_{21} + \underline{s}_{23}) + (\underline{s}_{31} + \underline{s}_{32}) \\
&= 0.00829 + j\,1.117
\end{aligned}
$$