The Beam Propagation Method

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Useful further readings

F. J. Dyson.
The Radiation Theories of Tomonaga, Schwinger, and Feynman.
Physical Review, 75(3):486–502, February 1949.

YY. N. Kosovtsov.
Formal exact operator solutions to nonlinear differential equations.
Open Access Library, 2009.

M. D. Feit and J. A. Fleck.
Light propagation in graded-index optical fibers.
Applied Optics, 17(24):3990–3998, December 1978.

J.W. Goodman.
Introduction to Fourier Optics.
McGraw-Hill physical and quantum electronics series. W. H. Freeman, 2005.

J. A. Fleck, J. R. Morris, and M. D. Feit.
Time-dependent propagation of high energy laser beams through the atmosphere.
Applied physics, 10(2):129–160, June 1976.

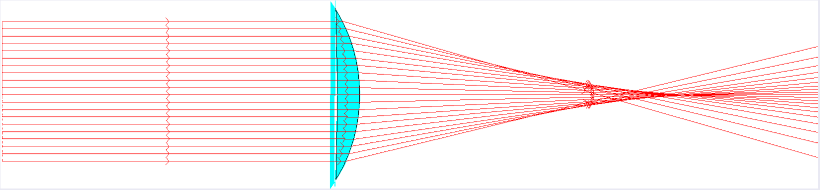
1. The Beam Propagation Method
   - Electromagnetic story
   - Propagation through Discrete Fourier Transform
2. Let us write our own BPM
   - FFT & `python` based free space propagation
   - Refractive index modulation
   - Pack it up together

Beyond the limits of ray tracing

ray tracing is good enough for many applications but...

- Ray tracing use rays...which are just an artifact
- Ray tracing does not account for the phase
- Ray tracing does not allow for interference

Is it a Gaussian Beam ? *example*



Mathematical preamble

Formal exact operator solutions to nonlinear differential equations[1, 2]

Equation

$$\frac{\partial^2 E}{\partial t^2} = E$$

Solution

$$E\left(t + dt\right) = E\left(t\right) e^{dt}$$

Mathematical preamble

Formal exact operator solutions to nonlinear differential equations[1, 2]

Equation

$$\frac{\partial^2 E}{\partial t^2} = AE$$

Solution

$$E\left(t + dt\right) = e^{\sqrt{A}dt} E\left(t\right)$$

Mathematical preamble

Formal exact operator solutions to nonlinear differential equations[1, 2]

Equation

$$\frac{\partial^2 E}{\partial t^2} = A \times E$$

Solution

$$E\left(t + dt\right) = e^{\sqrt{A}dt} \times E\left(t\right)$$

$$A \in \mathbb{R}^n$$

$$\sqrt{A} = R \iff R^2 = A$$

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots + \frac{A^{q-1}}{(q-1)!} + \cdots$$

Mathematical preamble

Formal exact operator solutions to nonlinear differential equations[1, 2]

Equation

$$\frac{\partial^2 E}{\partial t^2} = A(E)$$

Solution

$$E(t+dt) = e^{\sqrt{A}dt}\left(E(t)\right)$$

A is an operator

$$\sqrt{A} = R \iff R^2 = A$$

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots + \frac{A^{q-1}}{(q-1)!} + \cdots$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Time harmonic E

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2} n^2 E = 0$$

Wave propagation

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$$\frac{\partial^2 E}{\partial z^2} = -\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\omega^2}{c^2} n^2 E \right)$$

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$$\frac{\partial^2 E}{\partial z^2} = \left[-\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} n^2\right)\right](E)$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Time harmonic E

$$\frac{\partial^2 E}{\partial z^2} = \left[- \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} n^2 \right) \right] (E)$$

$$\frac{\partial^2 E}{\partial z^2} = \left[- \left(\Delta_\perp E + \frac{\omega^2}{c^2} n^2 \right) \right] (E)$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation | **Time harmonic E**

$$\frac{\partial^2 E}{\partial z^2} = \left[-\left(\Delta_\perp E + \frac{\omega^2}{c^2} n^2\right)\right](E)$$

$$E(z+dz) = \left\{\exp\left(-idz\sqrt{\Delta_\perp + \frac{\omega^2}{c^2} n^2}\right)\right\}(E(z))$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Time harmonic E

$$E\left(z+dz\right)=\left\{\exp\left(-\imath dz\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n^2}\right)\right\}\left(E\left(z\right)\right)$$

Small index variations

$$\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n^2}=\frac{\Delta_{\perp}}{\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n^2}+\frac{\omega}{c}n}+\frac{\omega}{c}n$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Time harmonic E

$$E\left(z+dz\right)=\left\{\exp\left(-i dz\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n^2}\right)\right\}\left(E\left(z\right)\right)$$

Small index variations

$$\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n^2}\approx\frac{\Delta_{\perp}}{\sqrt{\Delta_{\perp}+\frac{\omega^2}{c^2}n_0^2}+\frac{\omega}{c}n_0}+\frac{\omega}{c}n$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Time harmonic E

$$E\left(z+dz\right)=\left\{\exp\left(-\imath dz\sqrt{\Delta_\perp+\frac{\omega^2}{c^2}n^2}\right)\right\}\left(E\left(z\right)\right)$$

Small index variations

$$\sqrt{\Delta_\perp+\frac{\omega^2}{c^2}n^2}\approx\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}+k\frac{n}{n_0}$$

Wave propagation

Helmoltz equation under scalar approximation[3] — *i.e.* polarization assumed constant

Helmoltz equation — Space time harmonic $\mathcal{E}$

$$E\left(z+dz\right)=\left\{\exp\left(-\imath dz\sqrt{\Delta_\perp+\frac{\omega^2}{c^2}n^2}\right)\right\}\left(E\left(z\right)\right)$$

$$\mathcal{E}\left(z+dz\right)=\exp\left(-\imath dz\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}+k\left(\frac{n}{n_0}-1\right)\right]\right)\left(\mathcal{E}\left(z\right)\right)$$

Small index variations

$$\sqrt{\Delta_\perp+\frac{\omega^2}{c^2}n^2}\approx\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}+k\frac{n}{n_0}$$

Operator splitting

Appximated symmetrized notation for numerical evaluation

$$\mathcal{E}\left(z+dz\right)=\exp\left(-idz\left[\frac{\Delta_{\perp}}{k+\sqrt{\Delta_{\perp}+k^{2}}}+k\left(\frac{n}{n_0}-1\right)\right]\right)\left(\mathcal{E}\left(z\right)\right)$$

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Appximated symmetrized notation for numerical evaluation

$$\exp\left(-\imath\ dz\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}+k\left(\frac{n}{n_0}-1\right)\right]\right)$$

Operator splitting

Appximated symmetrized notation for numerical evaluation

$$\exp\left(-i dz\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)\times$$
$$\exp\left(-ikdz\left(\frac{n}{n_0}-1\right)\right)$$

Operator splitting

Appximated symmetrized notation for numerical evaluation

Approximated operator

- Operators do not commute
- Symmetry suitable for numerical solution

$$\exp\left(-i\frac{dz}{2}\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)\times$$
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Operator splitting

Appximated symmetrized notation for numerical evaluation

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$$\mathcal{E}(z+dz) = \exp\left(-i\frac{dz}{2}\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)\times$$

$$\exp\left(-ikdz\left(\frac{n}{n_0}-1\right)\right)\times$$

$$\exp\left(-i\frac{dz}{2}\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)(\mathcal{E}(z))$$

From propagation math to propagation physics

Focusing on one operator

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Recall from wave propagation on slide 5

$$\mathcal{E}(z+dz)=\exp\left(-idz\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}+k\left(\frac{n}{n_0}-1\right)\right]\right)(\mathcal{E}(z))$$

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$$\exp\left(-i\frac{dz}{2}\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)$$

Operator *Propagation in linear medium on a* $\frac{dz}{2}$ *distance*

Recall from wave propagation on slide 5

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From propagation math to propagation physics

$\frac{dz}{2}$ propagation operator

$$\exp\left(-i\frac{dz}{2}\left[\frac{\Delta_\perp}{k+\sqrt{\Delta_\perp+k^2}}\right]\right)$$

Fully customizable thin lens operator

$$\exp\left(-ikdz\left(\frac{n\left(x,y\right)}{n_0}-1\right)\right)$$

From propagation math to propagation physics

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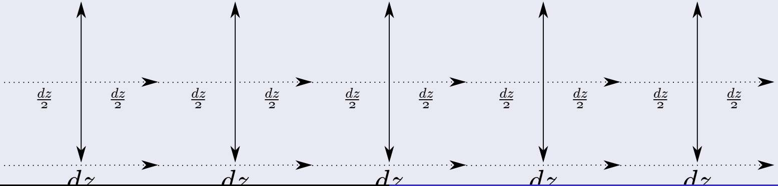
A physically intuitive overview of BPM



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Propagation as a Fourier transform[4, 5]

Envelope equation

- Wave equation: $\Delta E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$
- Scalar harmonic wave equation: $\Delta E + \frac{\omega^2}{c^2} E = 0$
- Envelope equation:
  $\Delta\left[\mathcal{E}(x,y,z)\, e^{-ikz}\right] + k^2 \mathcal{E}(x,y,z)\, e^{-ikz} = 0$

Wave propagation through 2D Discrete Fourier Transform

$$\mathcal{E}_{xy}(z) = \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}} \left[\sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} \mathcal{E}_{mn}(z)\, e^{\frac{2\pi i(mx+ny)}{L}} \right]$$

L is the physical width of the square computational window.

Plug the latter in the former

$$\mathcal{E}_{mn}(z+dz) = \mathcal{E}_{mn}(z)\, e^{\frac{i}{2k}\left(\frac{2\pi}{L}\right)^2 \left(m^2+n^2\right) dz}$$

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Propagation as a Fourier transform[4, 5]

Envelope equation

- Scalar harmonic wave equation: $\Delta E + k^2 E = 0$
- Envelope equation: $e^{-ikz}\left(\Delta\mathcal{E} - 2ik\frac{\partial\mathcal{E}}{\partial z}\right) = 0$

Wave propagation through 2D Discrete Fourier Transform

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Discrete Fourier Transform and Fast Fourier Transform

The Discrete Fourier Transform is the numerical way to compute the Fourier Transform

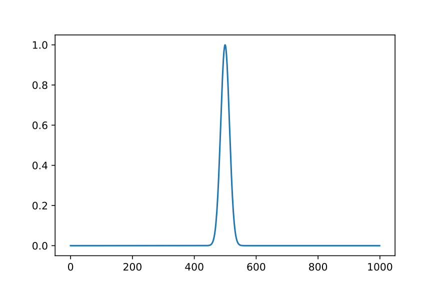
DFT is not FFT

The Fast Fourier Transform algorithm is one particular algorithm that allows to compute the Discrete Fourier Transform

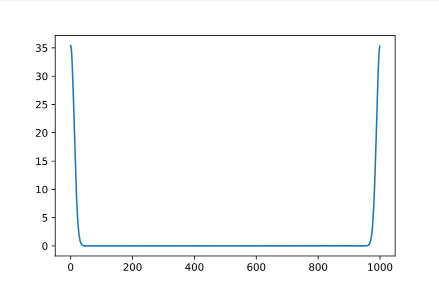
TFD example

The Fourier Transform of a Gaussian is a Gaussian

Example signal



The module of its DFT



Discrete signal — Discrete Fourier Transform

- Same number of points
- A discrete signal is a periodic Fourier Transform
- A discrete Fourier Transform reveals a periodic signal

TFD example

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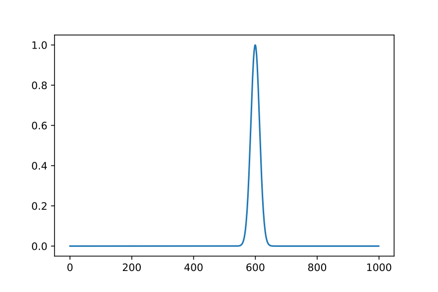
Example signal



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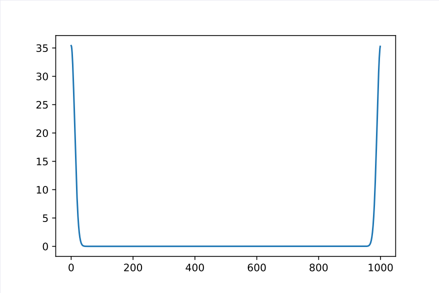

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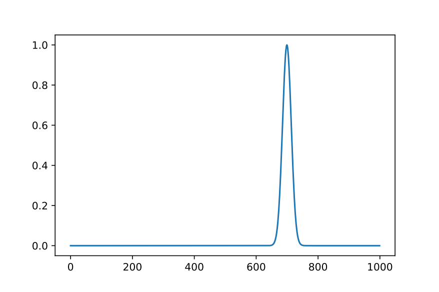

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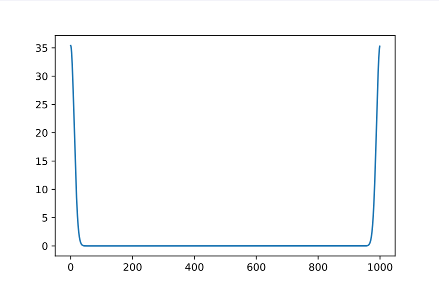

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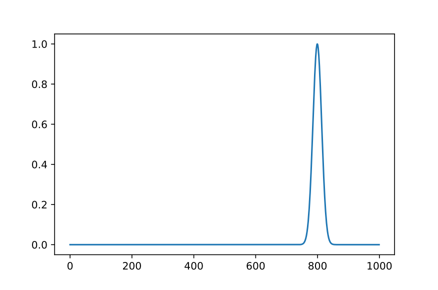

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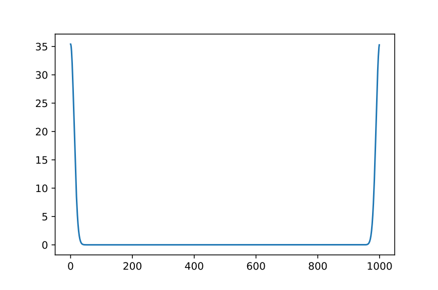

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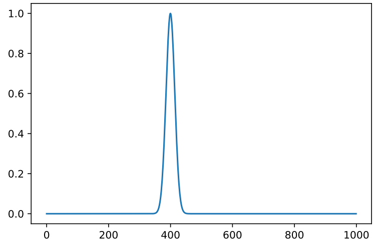

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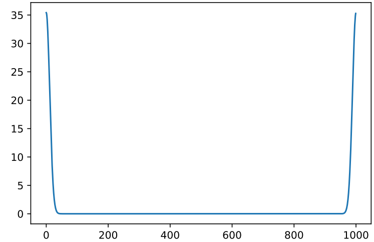

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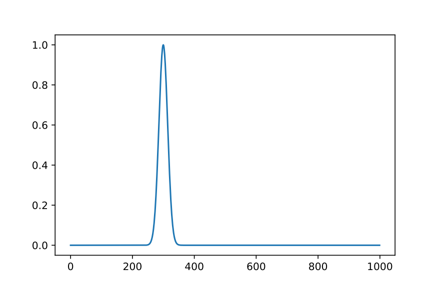

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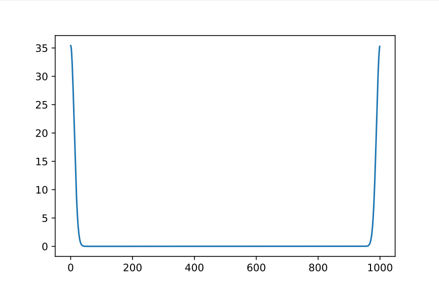

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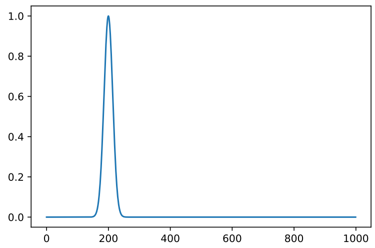

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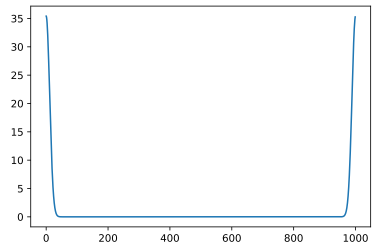

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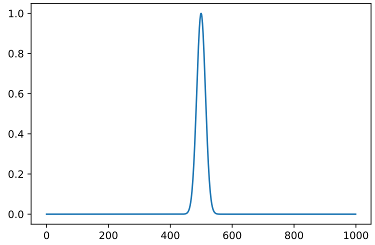

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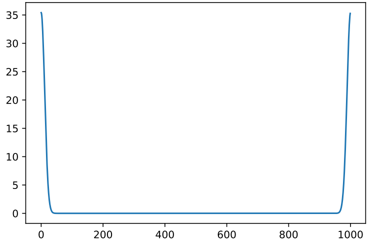

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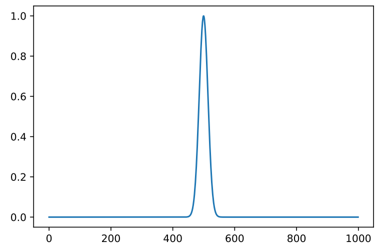
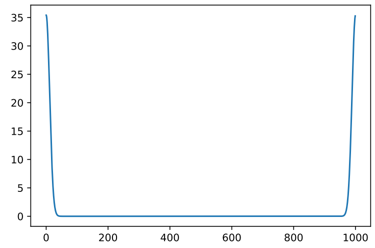


The module of its DFT



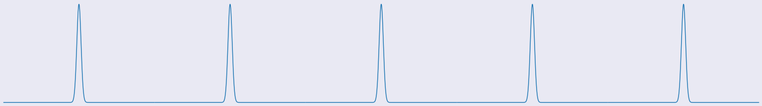
Discrete signal — Discrete Fourier Transform

- Same number of points
- A discrete signal is a periodic Fourier Transform
- A discrete Fourier Transform reveals a periodic signal

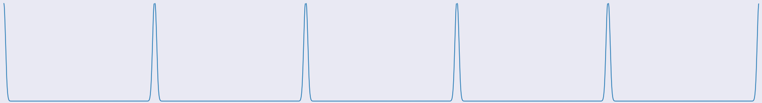
A discrete Fourier Transform reveals a periodic signal

Both signal and Fourier Transform are discrete and periodic

The actual signal as computed



The actual Fourier Transform as computed



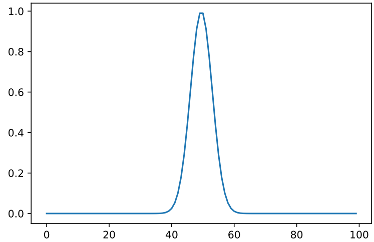
Storing the minimum data — Only one period is stored

- Only one period of the signal and its Fourier Transform is stored
- Do not forget that all of them are taken into account in the computation

Shannon theorem

The sampling frequency must be twice the maximum signal frequency

Example signal



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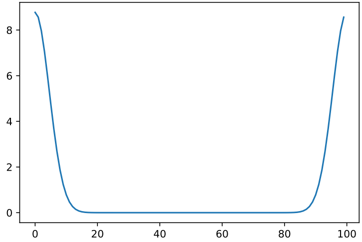

Slow varying approximation

BPM is inaccurate for rapidly varying signals

Shannon theorem

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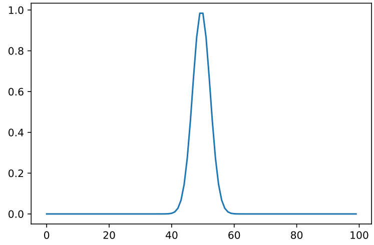
Example signal



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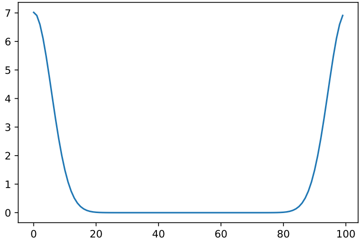

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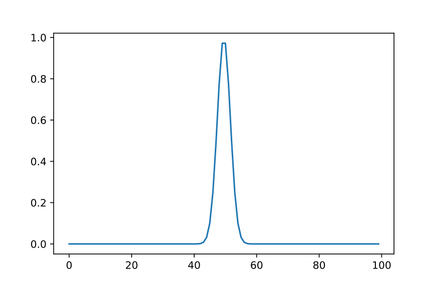

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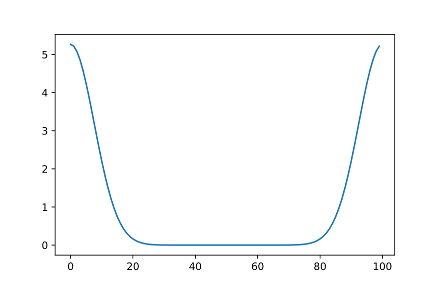

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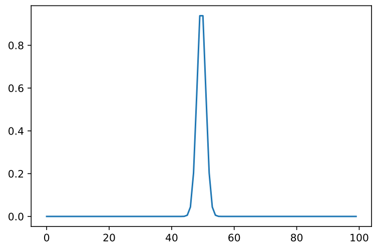

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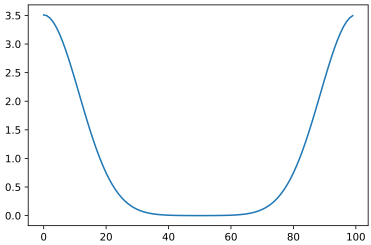

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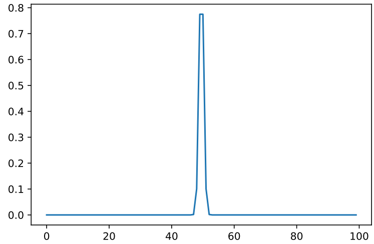

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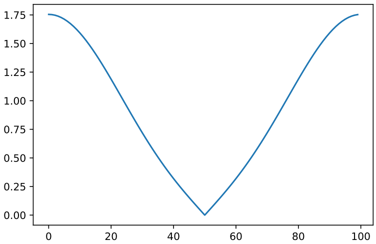

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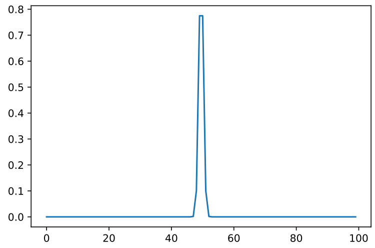

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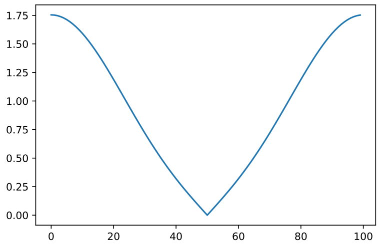

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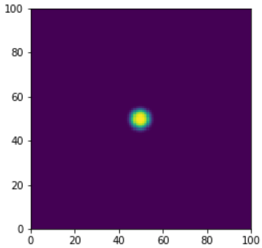
Sampling in 2D

Optics needs 2D. Fourier, sampling, and Shannon are still there !

Example signal



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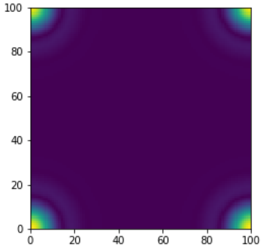
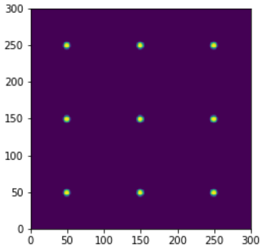


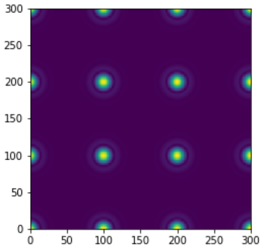
Sampled 2D signal, periodic Fourier Transform

and vice versa

Actual computed signal



Actual computed DFT module



materalia
Pôle de Compétitivité Matériaux
ENLIGHT METZ
CentraleSupélec
UNIVERSITÉ DE LORRAINE

1. The Beam Propagation Method
   - Electromagnetic story
   - Propagation through Discrete Fourier Transform

2. Let us write our own BPM
   - FFT & `python` based free space propagation
   - Refractive index modulation
   - Pack it up together

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Beam Propagation in python

Translate the base equation into python code

Base equation

$$\mathcal{E}_{mn}(z+dz) = \mathcal{E}_{mn}(z)\, e^{\frac{i}{2k}\left(\frac{2\pi}{L}\right)^2\left(m^2+n^2\right)dz}$$

python framework

- Manipulate arrays with the NumPy lib
- Find FFT in NumPy or SciPy

Suggested workflow

- Start with 1D beams
  - Experience 1D FFT
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- Compare your results with what you can get from the LightPipes module

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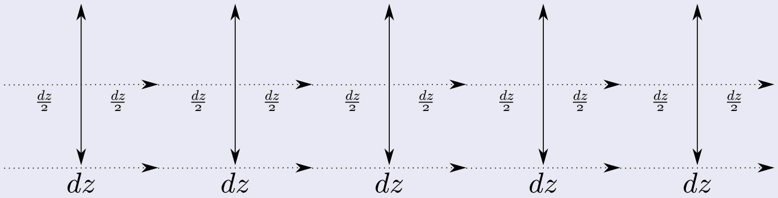
Add linear or non linear index modulation

Recall the physically intuitive overview of BPM



Multiply by a phase object

$$\exp\left(-\imath k dz\left(\frac{n\left(x,y\right)}{n_0}-1\right)\right)$$

Linear or non-linear ?

Nonlinear if $n\left(x,y\right)$ depends on beam intensity

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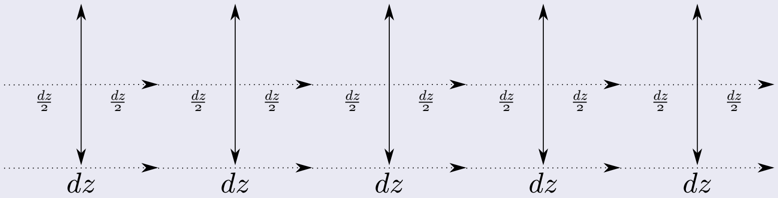

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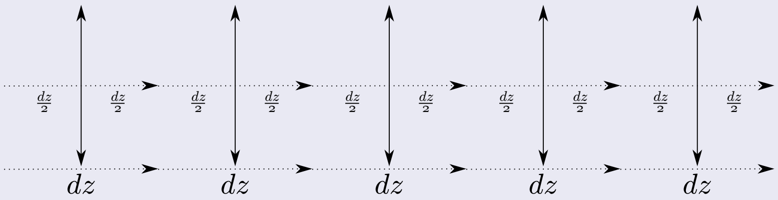
Loop to propagate beam

Recall the physically intuitive overview of BPM



Propagate beam

A loop including both free space (or linear medium) propagation and index modulation should do the trick