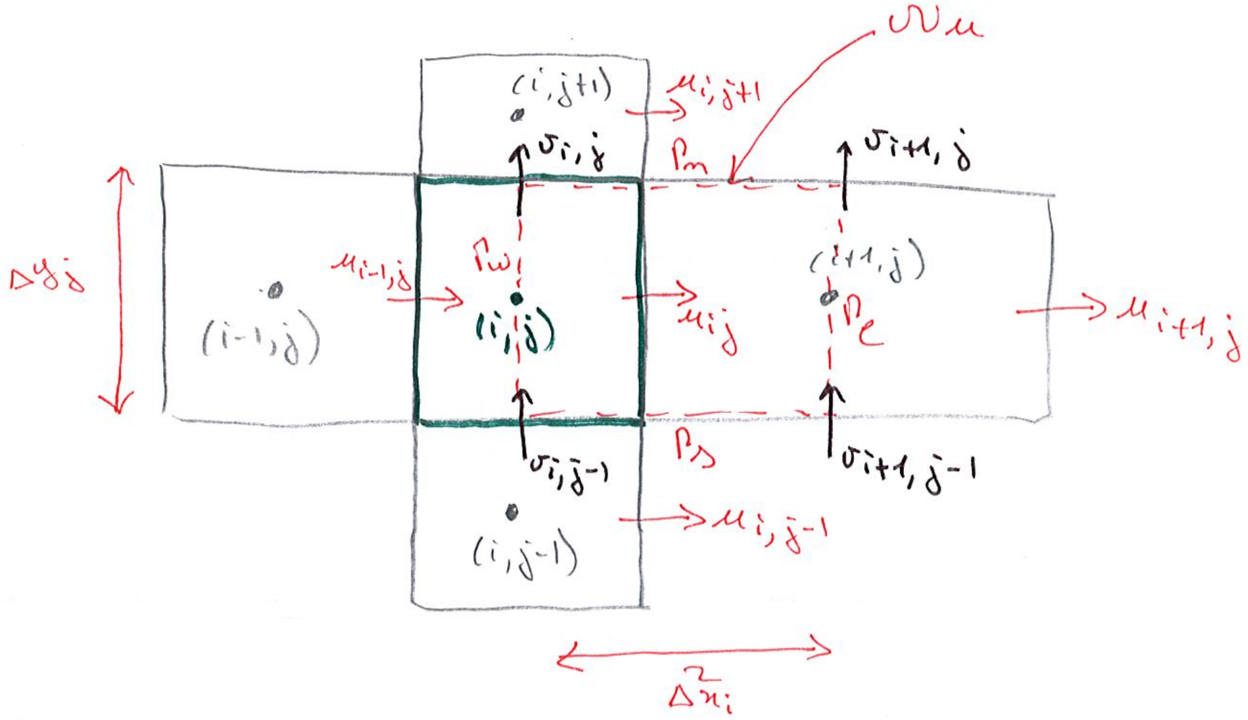
TD - Volumes finis

1

Problème I

$$F_x^c = \rho \vec{v} u$$

sur maillages décalés.



Partie I | Loin du bord. $\rho = 1$.

1) a)

$$\int_{\Gamma_u} F_x^c \cdot \underline{n} \, d\Gamma = \int_{\Gamma_w} u \vec{v}\cdot n \, d\Gamma + \int_{\Gamma_e} u \vec{v}\cdot n \, d\Gamma + \int_{\Gamma_s} u \vec{v}\cdot \vec{n} \, d\Gamma + \int_{\Gamma_n} u \vec{v}\cdot \vec{n} \, d\Gamma$$

ex $\int_{\Gamma_w} u \vec{v}\cdot n \, d\Gamma =$

Approximation sur l'inconnue u:

$$\int_{\Gamma_u} \underline{F}_x^c \cdot \underline{n} \stackrel{?}{=} u_w \int_{\Gamma_w} \vec{v}\cdot\vec{n}\, d\Gamma + u_e \int_{\Gamma_e} \vec{v}\cdot\vec{n}\, d\Gamma + u_s \int_{\Gamma_s} \vec{v}\cdot\vec{n}\, d\Gamma + u_n \int_{\Gamma_n} \vec{v}\cdot\vec{n}\, d\Gamma$$

On a fait apparaître les débits massiques:

$$\int_{\Gamma_w} \vec{v}\cdot\vec{n}\, d\Gamma = -\int_{\Gamma_w} u\, dy \equiv -\bar{u}_w$$

$$\int_{\Gamma_e} \vec{v}\cdot\vec{n}\, d\Gamma = \int_{\Gamma_e} u\, dy \equiv \bar{u}_e$$

$$\int_{\Gamma_n} \vec{v}\cdot\vec{n}\, d\Gamma = \int_{\Gamma_n} v\, dx = \bar{v}_n$$

$$\int_{\Gamma_s} \vec{v}\cdot\vec{n}\, d\Gamma = -\int_{\Gamma} v\, dx = -\bar{v}_s .$$

Le flux discret devient alors:

$$\int_{\Gamma_u} \underline{F}_x^c \cdot \underline{n} \stackrel{?}{=} \bar{u}_e u_e - \bar{u}_w u_w + \bar{v}_n u_n - \bar{v}_s u_s . \qquad (1)$$

b)

$$u_e = \frac{1}{2}\left(u_{ij} + u_{i+1,j}\right)$$

$$u_w = \frac{1}{2}\left(u_{i-1,j} + u_{ij}\right)$$

$$u_n = \frac{1}{2}\left(u_{i,j+1} + u_{ij}\right) \quad \leftarrow \text{arnok}$$

$$u_s = \frac{1}{2}\left(u_{i,j} + u_{i,j-1}\right) \quad \leftarrow \text{arbnoh}$$

on obtient alors en reportant dans (1) :

$$\overline{F}_n^{\,c} = \underbrace{\frac{1}{2}\,\overline{u}_e}_{\overline{AE}}\, u_{i+1,j} + \underbrace{\left(-\frac{1}{2}\,\overline{u}_w\right)}_{\overline{AW}} u_{i-1,j}$$

$$+ \underbrace{\frac{1}{2}\left(\overline{u}_e - \overline{u}_w + \overline{v}_n - \overline{v}_s\right)}_{\overline{AP}} u_{i,j}$$

$$\cancel{+\frac{1}{2}\,\overline{v}_n - \frac{1}{2}\,\overline{v}_s}\,.$$

$$+ \underbrace{\frac{1}{2}\,\overline{v}_n}_{\overline{AN}}\, u_{i,j+1} + \underbrace{\left(-\frac{1}{2}\,\overline{v}_s\right)}_{AS} u_{i,j-1}$$

2) a) $\overline{u}_e = \frac{1}{2}\,\overline{u}_{i+1,j} + \frac{1}{2}\,\overline{u}_{i,j}$

$\overline{u}_w = \frac{1}{2}\,\overline{u}_{i-1,j} + \frac{1}{2}\,\overline{u}_{i,j}$

$\overline{v}_n = \frac{1}{2}\,\overline{v}_{i,j} + \frac{1}{2}\,\overline{v}_{i+1,j}$ à vérifier

$\overline{v}_s = \frac{1}{2}\,\overline{v}_{i,j-1} + \frac{1}{2}\,\overline{v}_{i+1,j-1}$ à vérifier.

b) $\overline{u}_{i,j} = \int_{(\Gamma_{i,j})_e} u\, dy \overset{?}{=} \Delta y_j\, u_{i,j}$

$\overline{u}_{i+1,j} = \int u\, dy \overset{?}{=} \Delta y_j\, u_{i+1,j}$

$\overline{u}_{i-1,j} = \ldots \overset{?}{=} \Delta y_j\, u_{i-1,j}$

$\overline{v}_{i,j} = \int v\, dx \overset{?}{=} \Delta x_i\, v_{i,j}$

$\overline{v}_{i+1,j} = \Delta x_{i+1}\, v_{i+1,j}$; $\overline{v}_{i,j-1} = \Delta x_i\, v_{i,j-1}$

$\overline{v}_{i+1,j-1} = \Delta x_{i+1}\, v_{i+1,j-1}$

3)

$$\overline{AE} = \frac{1}{2}\bar{u}_e \overset{?}{=} \overbrace{\frac{\Delta y_j}{4}\left(u_{i+1,j} + u_{i,j}\right)}$$

$$\overline{AW} = -\frac{1}{2}\bar{u}_w \overset{?}{=} \overbrace{-\frac{\Delta y_j}{4}\left(u_{i-1,j} + u_{i,j}\right)}^{AW}$$

$$\overline{AN} = \frac{1}{2}\bar{v}_n \overset{?}{=} \overbrace{\frac{1}{4}\left(\Delta x_i v_{i,j} + \Delta x_{i+1} v_{i+1,j}\right)}^{AN}$$

$$\overline{AS} = -\frac{1}{2}\bar{v}_s \overset{?}{=} -\frac{1}{4}\left(\overbrace{\Delta x_i v_{i,j-1} + \Delta x_{i+1} v_{i+1,j-1}}^{AS}\right)$$

$$\overline{AP} = \frac{1}{2}\left(\bar{u}_e - \bar{u}_w + \bar{v}_n - \bar{v}_s\right)$$

$$\overset{?}{=} \left(\underbrace{AE + AW + AN + AS}_{AP}\right)$$

$<$

$v_{i+1,j-1}$

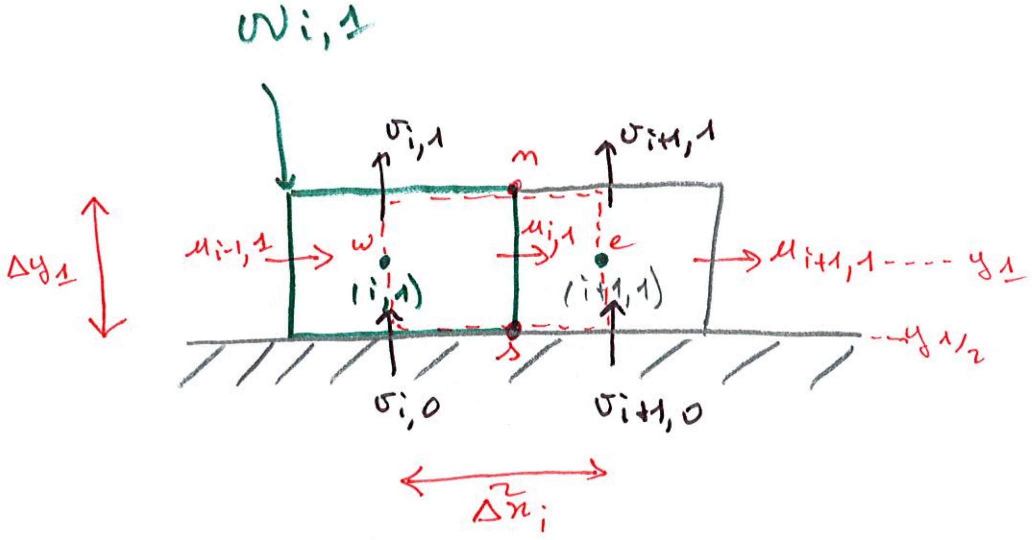
4)

$$AP = \frac{1}{4}\left[\Delta y_j\left(u_{i+1,j} - u_{ij}\right) + \Delta x_{i+1}\left(v_{i+1,j} - v_{i,j}\right)\right]$$

$$+ \frac{1}{4}\left[\Delta y_j\left(u_{ij} - u_{\cancel{i,j-1}\atop i-1,j}\right) + \Delta x_i\left(v_{i,j} - v_{i,j-1}\right)\right]$$

$= 0$ si la continuité est vérifiée dans chaque cellule.

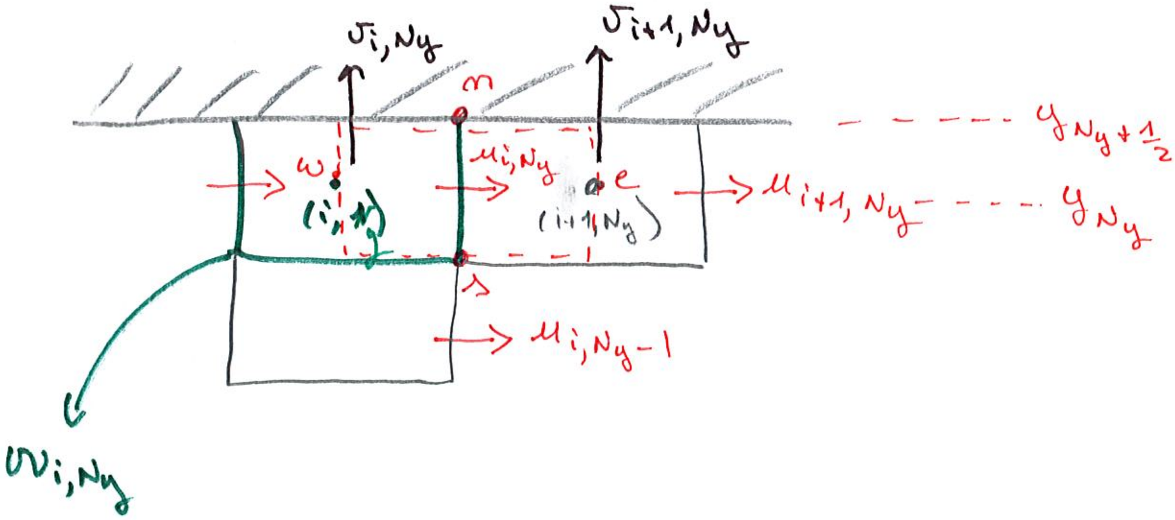
$\omega_{i+1,j}$ et $\omega_{i,j}$



1) Pour $j = 1$.

il faut effectuer $u_s = u_{i,0}$.

les autres restent valable.



pour $j = N_y$

il faut effectuer $u_n = u_{i,N_y+1}$.

les autres restent valables.

2) $$u_s = \frac{1}{2}(u_{i,j} + u_{i,j-1})\,\mathcal{E}_u^S(j) + (1 - \mathcal{E}_u^S(j))\,u_{i,j-1}$$

$$= \frac{\mathcal{E}_u^S(j)}{2}\,u_{i,j} + \left(1 - \frac{\mathcal{E}_u^S(j)}{2}\right)u_{i,j-1}$$

$$u_n = \frac{1}{2}(u_{i,j} + u_{i,j+1})\,\mathcal{E}_u^N(j) + (1 - \mathcal{E}_u^N(j))\,u_{i,j+1}$$

$$= \frac{\mathcal{E}^N}{2}(j)\,u_{i,j} + \left(1 - \frac{\mathcal{E}}{2}^N(j)\right)u_{i,j+1}$$

3) On en déduit l'expression de F_u^C

$$F_u^C = \frac{1}{2}\,\bar{u}_e^h\,u_{i+1,j} + \left(-\frac{1}{2}\,\bar{u}_w^h\right)u_{i-1,j}$$

$$+ \frac{1}{2}\left(\bar{u}_e^h - \bar{u}_w^h + \mathcal{E}_u^N(j)\,\bar{v}_n - \mathcal{E}_u^S(j)\,\bar{v}_s\right)u_{i,j}$$

$$+ \left(1 - \frac{\mathcal{E}^N}{2}(j)\right)\bar{v}_n^h\,u_{i,j+1} + \left(\frac{\mathcal{E}^S(j)}{2} - 1\right)\bar{v}_s^h\,u_{i,j-1}.$$