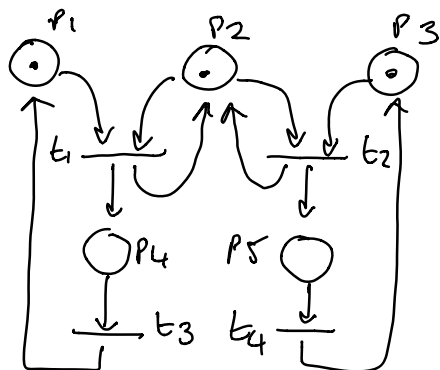




$$W = \begin{pmatrix} -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & +1 \\ +1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \end{pmatrix}$$

1)

$$\begin{pmatrix} -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & +1 \\ +1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ +1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\sigma = t_1 t_2 t_3 t_4 t_1 \quad S = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$2) \quad PRE = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma = t_1 t_2 t_1$$

$$\bullet \quad \pi_{\min} \geq PRE(-, t_2) \quad \pi_{\min} \geq \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

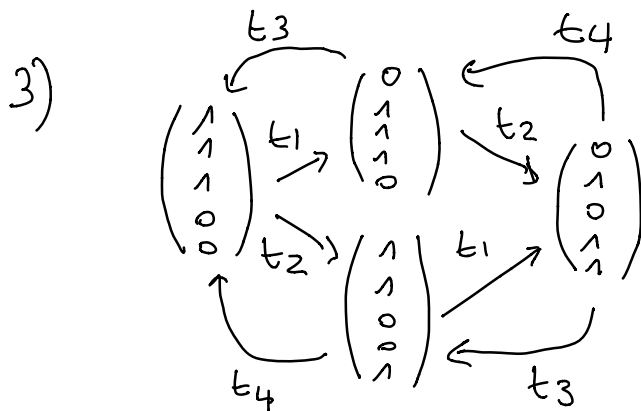
$$\bullet \quad \pi_{\min} + W(-, t_1) \geq PRE(-, t_2)$$

$$\pi_{\min} + \begin{pmatrix} -1 \\ 0 \\ +1 \\ 0 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad \pi_{\min} \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\bullet \quad \pi_{\min} + W(-, t_1 t_2) \geq PRE(-, t_1)$$

$$\pi_{\min} + \begin{pmatrix} -1 \\ 1 \\ +1 \\ +1 \end{pmatrix} \geq \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \pi_{\min} \geq \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\boxed{\pi_{\min} \geq \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}}$$



4)

$$(a \ b \ c \ d \ e) \begin{pmatrix} -1 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & +1 \\ +1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \end{pmatrix}$$

$$\begin{aligned} -a + d &= 0 \\ -c + e &= 0 \\ a - d &= 0 \\ c - e &= 0 \end{aligned}$$

$$\begin{cases} a = d \\ c = e \\ b \text{ quelconque} \end{cases}$$

$$F^T = (1 \ 1 \ 1 \ 1 \ 1)$$

$$F^T \cdot n_0 = (1 \ 1 \ 1 \ 1 \ 1) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = 3$$

$$m(P1) + m(P2) + m(P3) + m(P4) + m(Pi) = 3$$