

Les équations 1 D du mouvement et de l'énergie – Hypothèse « Fluides parfaits »

Ecoulements de Fanno – Avec pertes de charge, sans transfert de chaleur

$$\rho \frac{DU_i}{Dt} = - \frac{\partial p}{\partial x_i}$$

D : dérivée particulaire

$$\underbrace{\rho \frac{D}{Dt} \left(e + \frac{U_i^2}{2} \right)}_{\text{Variation d'énergie totale}} = - \underbrace{\frac{\partial}{\partial x_i} (p U_i)}_{\text{Travail des forces de pression}} \quad \Rightarrow \quad \rho \frac{De}{Dt} + \underbrace{\rho U_i \frac{DU_i}{Dt}}_{\cancel{-U_i \frac{\partial p}{\partial x_i}}} = \cancel{-U_i \frac{\partial p}{\partial x_i}} - p \frac{\partial U_i}{\partial x_i} \quad \Rightarrow \quad \rho \frac{De}{Dt} = -p \frac{\partial U_i}{\partial x_i}$$

- Introduisons l'enthalpie massique h

$$h = e + \frac{p}{\rho} \quad \Rightarrow \quad \frac{Dh}{Dt} = \frac{De}{Dt} + \frac{1}{\rho} \frac{Dp}{Dt} - \frac{p}{\rho^2} \frac{D\rho}{Dt} \quad \Rightarrow \quad \rho \frac{Dh}{Dt} = \rho \frac{De}{Dt} + \frac{Dp}{Dt} - \frac{p}{\rho} \frac{D\rho}{Dt}$$

- Introduisons l'équation de continuité

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{V}) = 0 \quad \Rightarrow \quad \underbrace{\frac{\partial \rho}{\partial t} + \vec{V} \cdot \overrightarrow{\text{grad}} \rho}_{\frac{D\rho}{Dt}} + \rho \text{div}(\vec{V}) = 0 \quad \Rightarrow \quad \frac{D\rho}{Dt} = -\rho \text{div}(\vec{V})$$

$$\rho \frac{Dh}{Dt} = \rho \frac{De}{Dt} + \frac{Dp}{Dt} + \underbrace{\frac{p}{\rho} \rho \text{div} \vec{V}}_{p \frac{\partial U_i}{\partial x_i}}$$

$$\rho \frac{Dh}{Dt} = \underbrace{\rho \frac{De}{Dt}}_{\substack{\rho \frac{De}{Dt} = -p \frac{\partial U_i}{\partial x_i} \\ \text{Eqn. de l'énergie}}} + \frac{Dp}{Dt} + \underbrace{\frac{p}{\rho} \rho \operatorname{div} \vec{V}}_{p \frac{\partial U_i}{\partial x_i}} \quad \longrightarrow \quad \boxed{\rho \frac{Dh}{Dt} = \frac{Dp}{Dt}}$$

- **Conclusions**

Equation de l'énergie

$$\boxed{\rho \frac{Dh}{Dt} = \frac{Dp}{Dt}}$$

Equation du mouvement

$$\boxed{\rho \frac{DU_i}{Dt} = - \frac{\partial p}{\partial x_i}}$$

- **Pour un écoulement monodimensionnel** $\vec{V} = U \vec{x}$

Equation du mouvement

$$\boxed{\rho U \frac{dU}{dx} = - \frac{dp}{dx}}$$

Equation de l'énergie

$$\rho \cancel{U} \frac{dh}{dx} = \cancel{U} \frac{dp}{dx} \quad \longrightarrow \quad \boxed{\rho \frac{dh}{dx} = \frac{dp}{dx}}$$

- Continuité

$$\frac{d(\rho U)}{dx} = 0$$

- présence d'une perte de charge
- $dQ=0$, pas d'apport de chaleur extérieur.

- Dynamique

$$\rho U \frac{dU}{dx} = -\frac{dP}{dx} - \lambda \frac{1}{D} \frac{1}{2} \rho U^2$$

- Energie

$$\rho C_p \frac{dT}{dx} = -\rho U \frac{dU}{dx} + \rho \frac{dQ}{dx}$$

Division des équations par : ρU^2

$$d\rho U + \rho dU = 0 \quad \Rightarrow \quad \boxed{\frac{d\rho}{\rho} + \frac{dU}{U} = 0}$$

$$\rho U \frac{dU}{dx} = -\frac{dP}{dx} - \lambda \frac{1}{2D} \rho U^2 \quad \xrightarrow{/\rho U^2} \quad \frac{1}{U} \frac{dU}{dx} = -\frac{1}{\rho U^2} \frac{dP}{dx} - \lambda \frac{1}{2D} \quad \Rightarrow \quad \boxed{\frac{dU}{U} + \frac{1}{\gamma M^2} \frac{dp}{p} + \frac{\lambda}{2D} dx = 0}$$

$\rho = \frac{p}{rT} = \frac{\gamma p}{c^2} \quad \text{et} \quad M = \frac{V}{c}$

$$\rho C_p \frac{dT}{dx} = -\rho U \frac{dU}{dx} \quad \xrightarrow{/\rho U^2 \text{ et } C_p = \frac{\gamma r}{\gamma - 1}} \quad \frac{\gamma r}{\gamma - 1} \frac{1}{U^2} \frac{dT}{dx} = -\frac{1}{U} \frac{dU}{dx} \quad \Rightarrow \quad \boxed{\frac{1}{(\gamma - 1) M^2} \frac{dT}{T} + \frac{dU}{U} = 0}$$

$M = \frac{V}{c}$