Les équations 1 D du mouvement et de l'énergie – Hypothèse « Fluides parfaits »

$$\rho \frac{DU_i}{Dt} = -\frac{\partial p}{\partial x_i}$$

D : dérivée particulaire

$$\underbrace{\rho \frac{D}{Dt}\left(e + \frac{U_i^2}{2}\right)}_{\text{Variation d'énergie totale}} = \underbrace{-\frac{\partial}{\partial x_i}\left(pU_i\right)}_{\text{Travail des forces de pression}} \Rightarrow \rho \frac{De}{Dt} + \underbrace{\rho U_i \frac{DU_i}{Dt}}_{\cancel{-U_i \frac{\partial p}{\partial x_i}}} = \cancel{-U_i \frac{\partial p}{\partial x_i}} - p \frac{\partial U_i}{\partial x_i} \Rightarrow \rho \frac{De}{Dt} = -p \frac{\partial U_i}{\partial x_i}$$

- Introduisons l'enthalpie massique *h*

$$h = e + \frac{p}{\rho} \Rightarrow \frac{Dh}{Dt} = \frac{De}{Dt} + \frac{1}{\rho}\frac{Dp}{Dt} - \frac{p}{\rho^2}\frac{D\rho}{Dt} \Rightarrow \rho \frac{Dh}{Dt} = \rho \frac{De}{Dt} + \frac{Dp}{Dt} - \frac{p}{\rho}\frac{D\rho}{Dt}$$

- Introduisons l'équation de continuité

$$\frac{\partial \rho}{\partial t} + div\left(\rho \vec{V}\right) = 0 \Rightarrow \underbrace{\frac{\partial \rho}{\partial t} + \vec{V}.\overrightarrow{grad}\,\rho}_{\frac{D\rho}{Dt}} + \rho div\left(\vec{V}\right) = 0 \Rightarrow \frac{D\rho}{Dt} = -\rho div\left(\vec{V}\right)$$

$$\rho \frac{Dh}{Dt} = \rho \frac{De}{Dt} + \frac{Dp}{Dt} + \underbrace{\frac{p}{\rho}\rho div\vec{V}}_{p\frac{\partial U_i}{\partial x_i}}$$

$$\rho\frac{Dh}{Dt} = \underset{\substack{\rho\frac{De}{Dt}=-p\frac{\partial U_i}{\partial x_i} \\ \text{Eqn. de l'énergie}}}{\rho\frac{De}{Dt}} + \frac{Dp}{Dt} + \underbrace{\frac{p}{\rho}\rho\, div\vec{V}}_{p\frac{\partial U_i}{\partial x_i}} \Rightarrow \boxed{\rho\frac{Dh}{Dt} = \frac{Dp}{Dt}}$$

- **Conclusions**

Equation de l’énergie $\boxed{\rho\frac{Dh}{Dt} = \frac{Dp}{Dt}}$

Equation du mouvement $\boxed{\rho\frac{DU_i}{Dt} = -\frac{\partial p}{\partial x_i}}$

- **Pour un écoulement monodimensionnel** $\vec{V} = U\vec{x}$

Equation du mouvement

$$\boxed{\rho U\frac{dU}{dx} = -\frac{dp}{dx}}$$

Equation de l’énergie

$$\rho\cancel{U}\frac{dh}{dx} = \cancel{U}\frac{dp}{dx} \Rightarrow \boxed{\rho\frac{dh}{dx} = \frac{dp}{dx}}$$

Ecoulements de Fanno – Avec pertes de charge, sans transfert de chaleur

- **Continuité**

$$\frac{d(\rho U)}{dx}=0$$

- **Dynamique**

$$\rho U\frac{dU}{dx}=-\frac{dP}{dx}-\lambda\frac{1}{D}\frac{1}{2}\rho U^2$$

- **Energie**

$$\rho C_p\frac{dT}{dx}=-\rho U\frac{dU}{dx}+\cancel{\rho\frac{dQ}{dx}}$$

(hypothèse de travail)

- présence d'une perte de charge
- *dQ=0*, pas d'apport de chaleur extérieur.

Division des équations par : ρU^2

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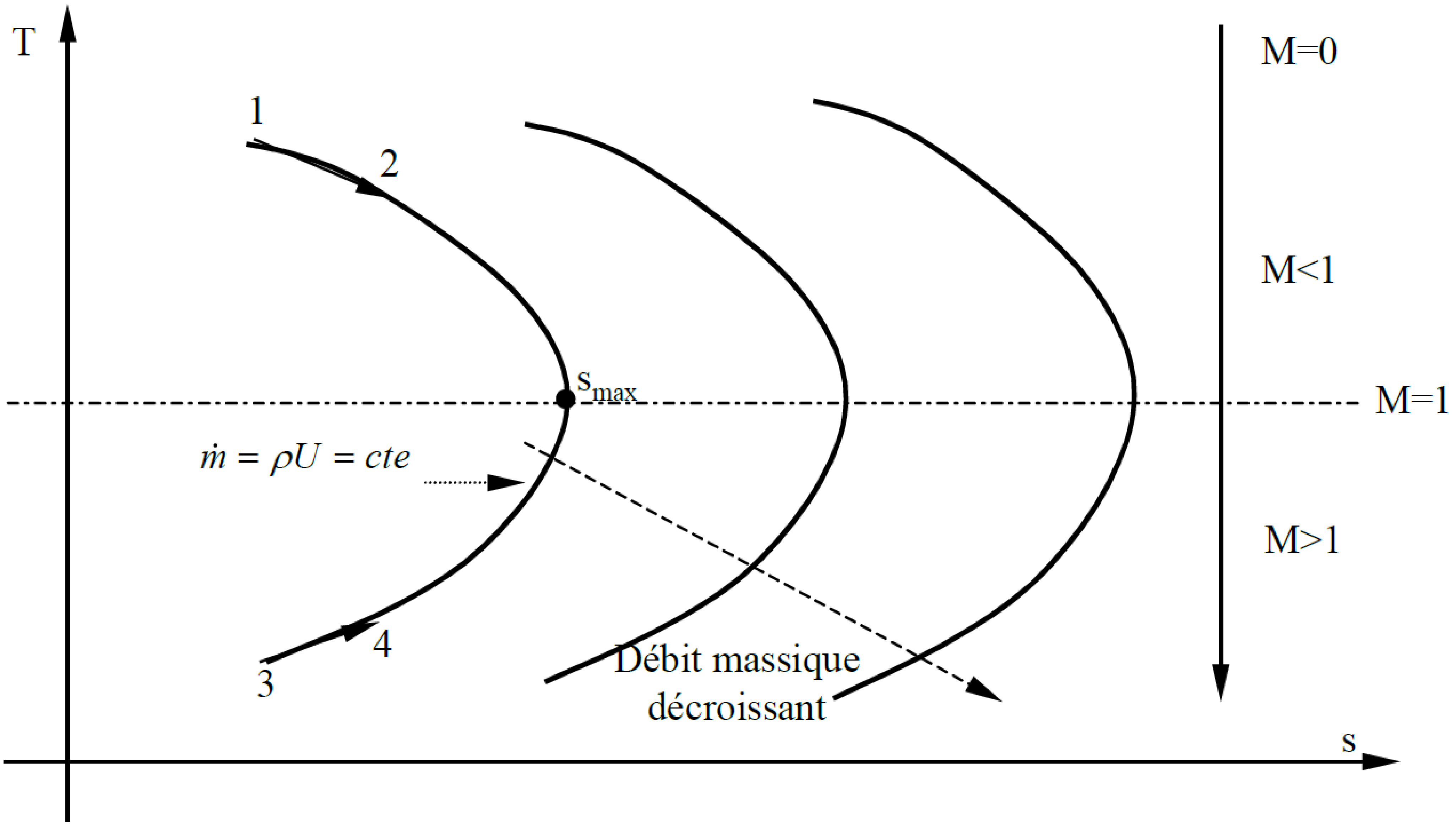
$$d\rho U+\rho dU=0 \Rightarrow \boxed{\frac{d\rho}{\rho}+\frac{dU}{U}=0}$$

$$\rho U\frac{dU}{dx}=-\frac{dP}{dx}-\lambda\frac{1}{2D}\rho U^2 \xrightarrow[/\rho U^2]{} \frac{1}{U}\frac{dU}{dx}=-\frac{1}{\rho U^2}\frac{dP}{dx}-\lambda\frac{1}{2D} \Rightarrow \boxed{\frac{dU}{U}+\frac{1}{\gamma M^2}\frac{dp}{p}+\frac{\lambda}{2D}dx=0}$$

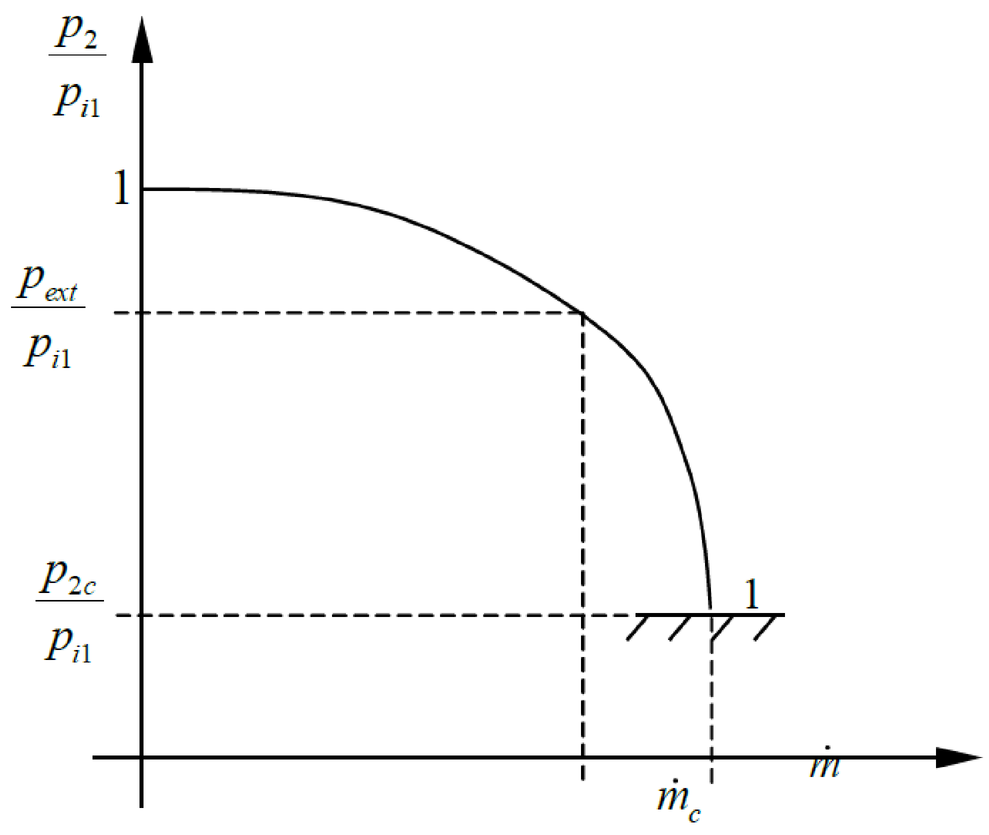
$$\rho=\frac{p}{rT}=\frac{\gamma p}{c^2} \text{ et } M=\frac{V}{c}$$

$$\rho C_p\frac{dT}{dx}=-\rho U\frac{dU}{dx} \xrightarrow[/\rho U^2 \text{ et } C_p=\frac{\gamma r}{\gamma-1}]{} \frac{\gamma r}{\gamma-1}\frac{1}{U^2}\frac{dT}{dx}=-\frac{1}{U}\frac{dU}{dx} \Rightarrow \boxed{\frac{1}{(\gamma-1)M^2}\frac{dT}{T}+\frac{dU}{U}=0}$$

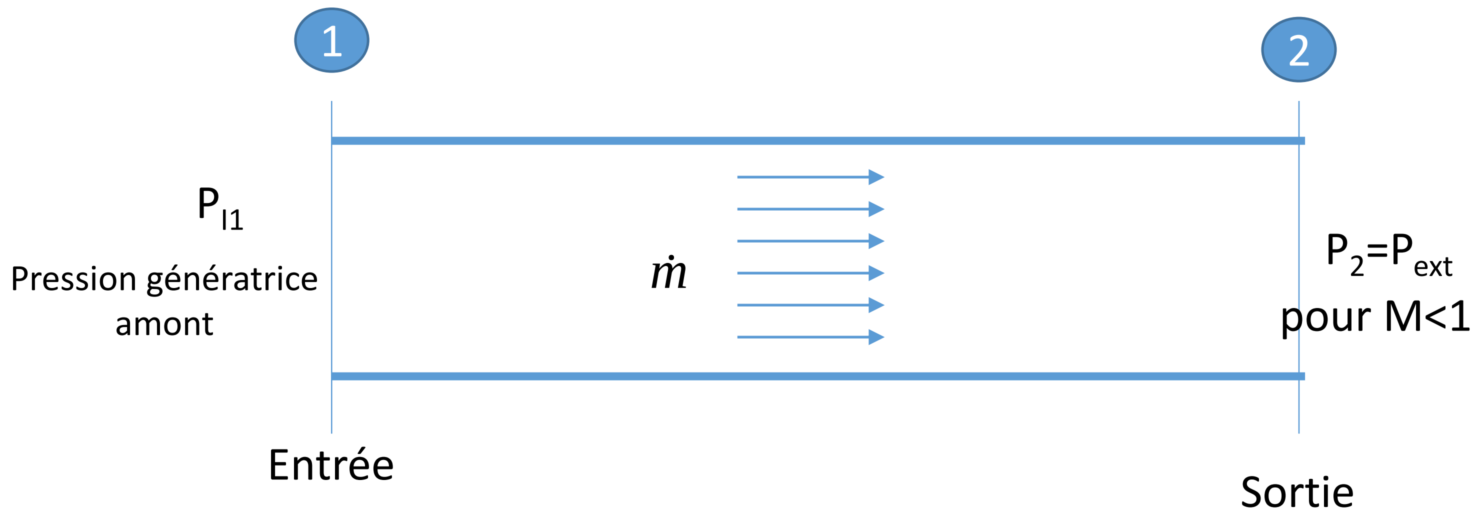
$$M=\frac{V}{c}$$



T
1
2
M=0
M<1
s_{max}
M=1
$\dot{m} = \rho U = cte$
M>1
4
3
Débit massique
décroissant
s



$\frac{p_2}{p_{i1}}$
1
$\frac{p_{ext}}{p_{i1}}$
$\frac{p_{2c}}{p_{i1}}$
1
$\dot{m}_c$
$\dot{m}$



1
2
P_{I1}
Pression génératrice amont
$\dot{m}$
$P_2=P_{ext}$
pour M<1
Entrée
Sortie