Nikolaus Lobatschewskii,

Geometrische Untersuchungen der Theorie der Parallellinien

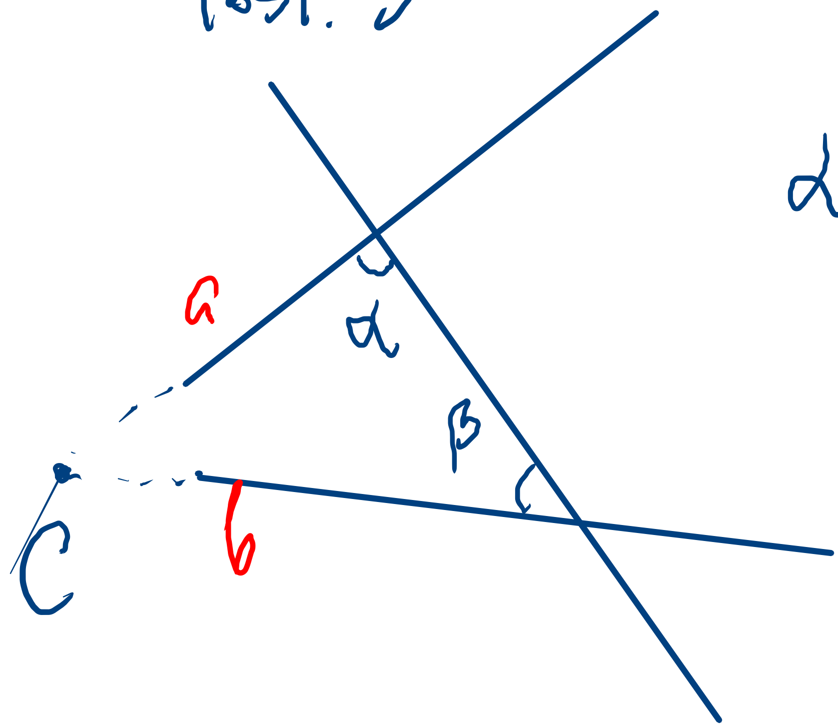
Berlin 1840

Николай Иванович Лобачевский (1792-1856)

N. I. Lobachevsky

Euclide, Éléments

Post. 5

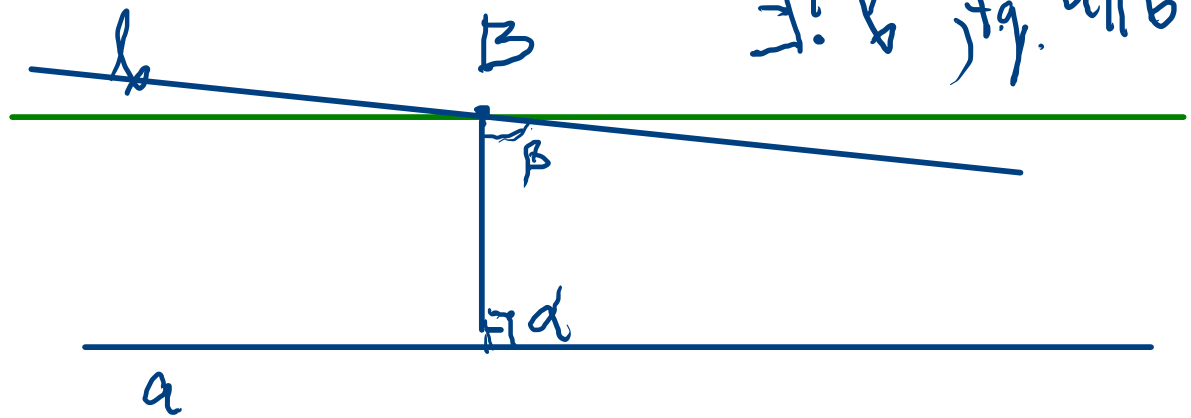


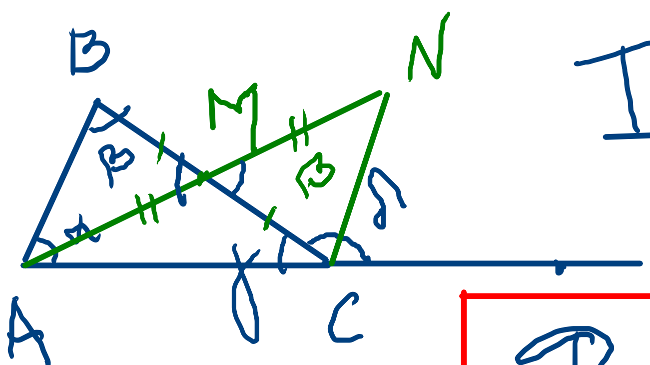
$\alpha+\beta < \pi \ (= 2d = 180^\circ)$

(1795)

Playfair Axiome $\Longleftrightarrow$ P5

$\exists !\, b$ tq. $a \parallel b$ et $B \in b$



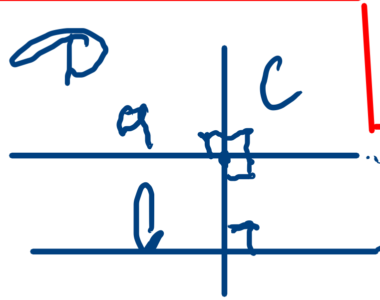


Th. d'angle externe

$\delta > \alpha$

$\delta > \beta$

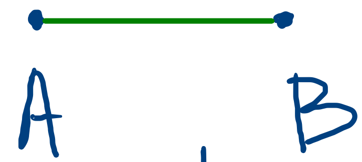
Preuve : $\triangle ABM = \triangle MNC$



Corr. :

Si $a \perp c$, $b \perp c$ alors $a \parallel b$

Cf. P1–P3

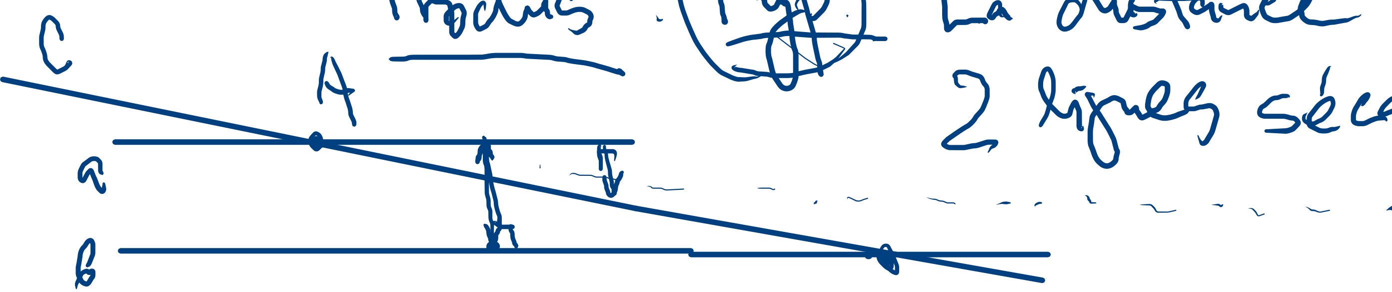


Euclide n'applique pas P5 jusqu'à **I.29 !**

E. Bonola. La geometria non-euclidea, 1906

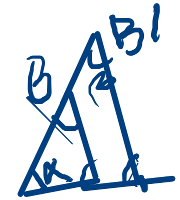
[Essays de prouver P5]

Proclus : Hyp. La distance entre les points de 2 lignes sécantes n'est **pas bornée**



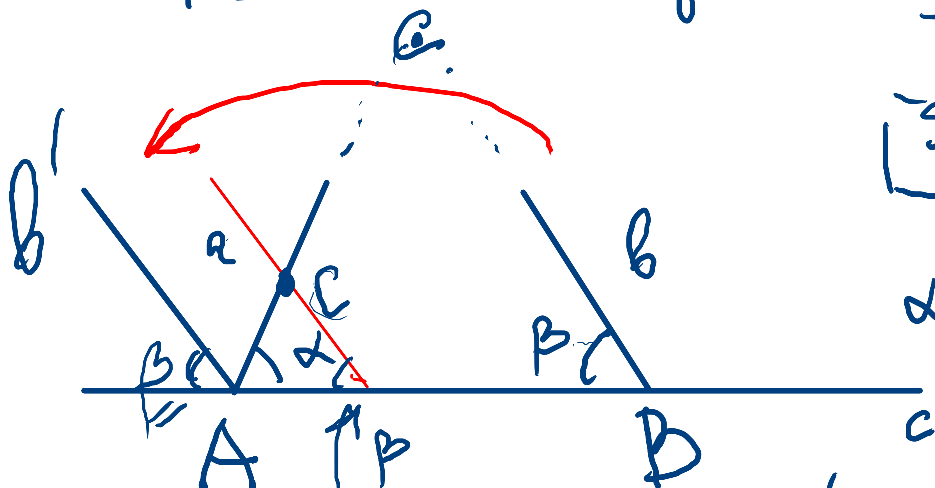
$a \parallel b$

John Wallis (1616-1703)

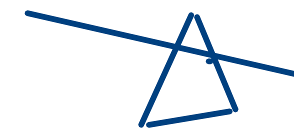


$\Delta AB'C' \sim \Delta ABC$

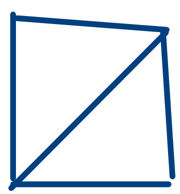
hyp il existe des triangles similaires

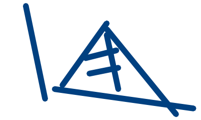


[Scaling]



$\alpha + \beta < \pi$



[géométrie sphérique] || 

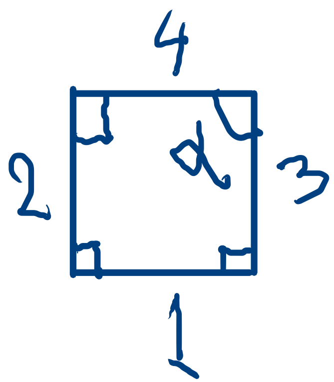
Ptolemé

Gerolamo Saccheri,

Euclides ab omni naevo vindicatus

Milano 1733

3 hypothèses



$\alpha = \frac{\pi}{2}$

$\alpha > \frac{\pi}{2}$ – impossible

$\alpha < \frac{\pi}{2}$ – ?

Johann Heinrich Lambert, (1728-1777)

Theorie der Parallellinien 1766 (publié à 1786)

„sphère de rayon imaginaire"

$$S_{\Delta}=(\alpha+\beta+\gamma-\pi)R^2 \qquad R=1 \quad \text{géom. sphérique}$$

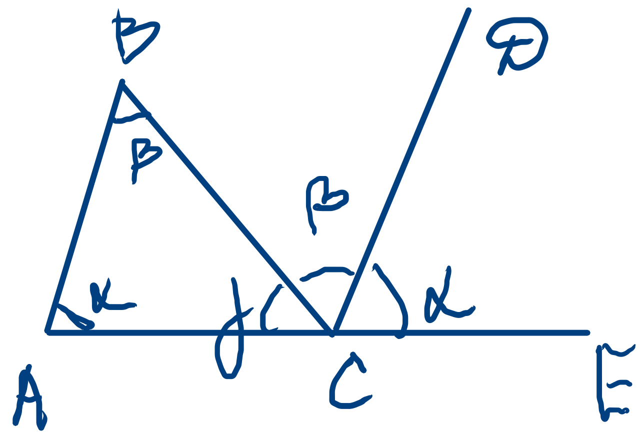
$$R=\sqrt{-1}=i \quad - \quad \text{géom hyperbolique (Lobachevsky)}$$

$$i^2=1$$

Cf. Im. Kant <u>Kritik der reinen Vernunft</u>

1781/87

α



$$\boxed{\alpha+\beta+\gamma=\pi}$$

Constr. auxillière:

<u>$CD \parallel AB$</u>

Gauß → Bessel 1829

„je ne veut pas des cries
des Boeotians" (réf. à la guerre
de Corinth 395 B.C.)

János 'Bolyai' (1802 – 1860)

Appendix 1832
au manuel de maths édité par son père
Farkas
(Wolfgang)

F. Minding (1806 – 1885)

Bemerkung über die Abwicklung krümmer linien von Fläshen