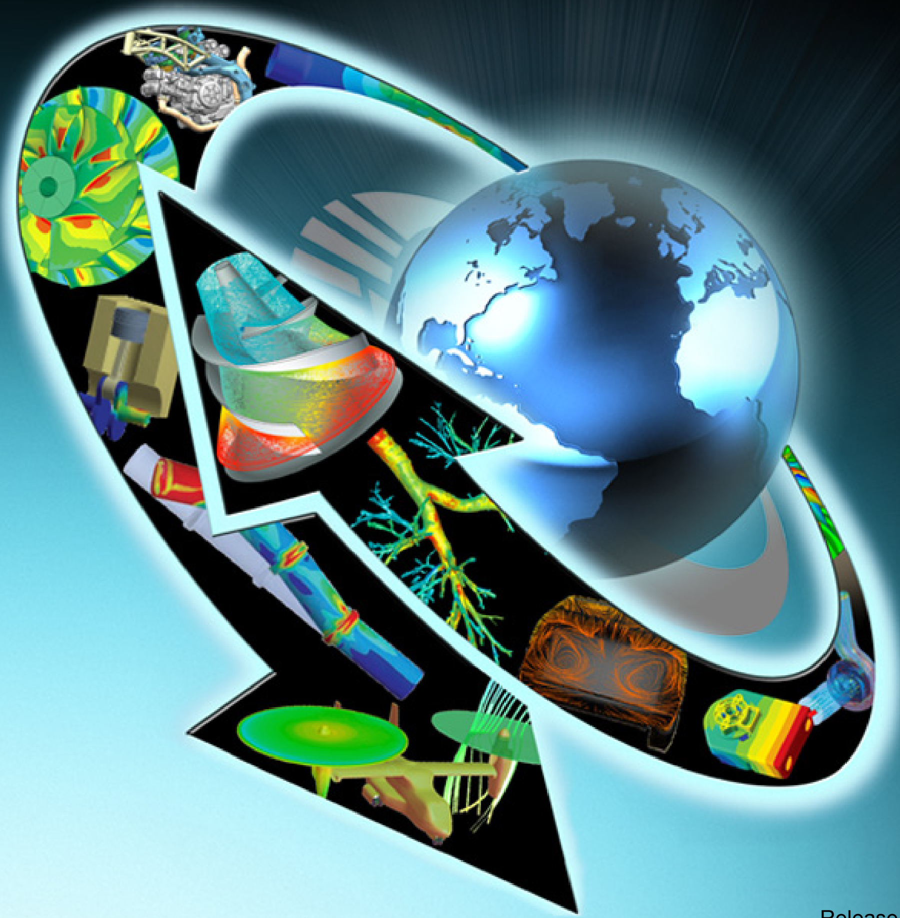
Customer Training Material

Lecture 3

Forced Convection



Heat Transfer Modeling using ANSYS FLUENT

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Release 13.0
December 2010

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- Introduction – Heat Transfer Coefficient
- Laminar and Turbulent Boundary Layers
- Modeling Heat Transfer – The Reynolds Analogy
- Turbulence Modeling and Dynamic and Thermal Wall Functions
- Case Study -- Modeling Heat Transfer for Non-Equilibrium and Complex Flows
- Post Processing

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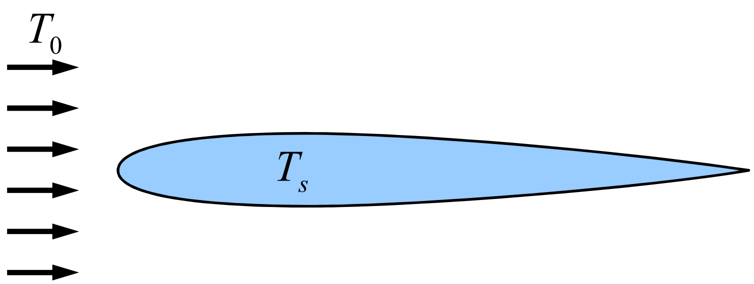
Heat Transfer Coefficient

- **Influence of:**
  - **Geometry, fluid properties, etc.**
  - **Importance of the boundary layer**



Mechanism	Fluid	h ($W/m^2 \cdot K$)
Natural Convection	Gases	5 – 30
	Water	100 – 1000
Forced Convection	**Gas**	**10 – 300**
	Water	**300 – 12,000**
	Oil	**50 – 1,700**
	Liquid metal	**6,000 – 110,000**
Phase Change	Boiling	3,000 – 60,000
	Condensation	5,000 – 110,000

- **Local heat flux** $$\phi\big|_{y=0}(x) = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} = h(x)\left(T_s - T_0\right)$$

- **Mean heat flux** $$\overline{\phi}\big|_{y=0}(x) = \frac{1}{L}\int_0^L h(x)\left(T_p - T_0\right)dx = \overline{h}\left(T_p - T_0\right)$$

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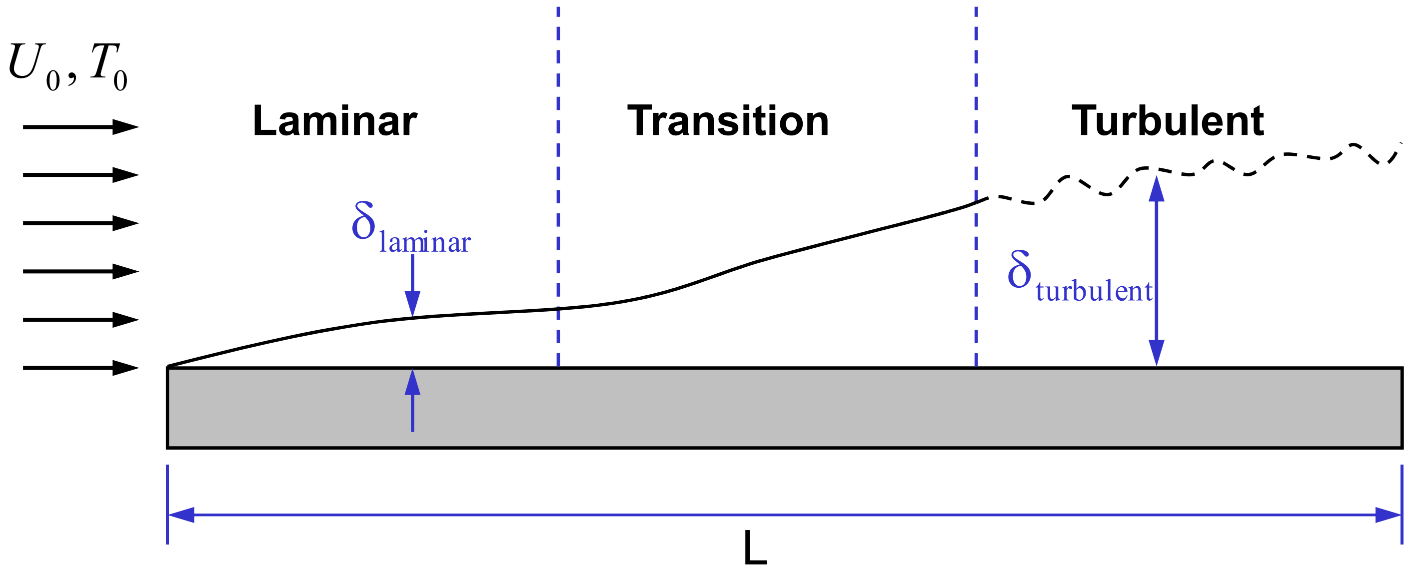
Boundary Layers

- **Important parameters are bulk velocity, bulk temperature, and pressure gradient**



- **Dimensionless variables:**

$$\tilde{u} = \frac{u}{U_0} \qquad \tilde{T} = \frac{T - T_s}{T_0 - T_s}$$

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Boundary Layers

- **Laminar boundary layer**
  - **Mixing is characterized by the ratio of viscous boundary layer thickness to thermal boundary layer thickness.**
- **Turbulent boundary layer**
  - **Mixing is primarily governed by turbulence.**

$$\frac{\delta}{\delta_T} \approx 1$$

- **Heat transfer coefficient – use an available correlation for the friction coefficient, C_f**
  - **Laminar Boundary Layers (exact)**

$$C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}} \qquad \mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

  - **Turbulent Boundary Layers (empirical correlations)**

$$C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}} \qquad \mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$$

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Modeling Turbulent Heat Transfer

- **RANS equations**

$$u = \overline{u} + u' \qquad T = \overline{T} + T'$$

$$\rho\left(\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y}\right) = -\frac{\partial \overline{P}}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial \overline{u}}{\partial y} - \rho\,\overline{u'v'}\right)$$

$$\rho C_p\left(\overline{u}\frac{\partial \overline{T}}{\partial x} + \overline{v}\frac{\partial \overline{T}}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial \overline{T}}{\partial y} - \rho C_p\,\overline{v'T'}\right)$$

- **Boussinesq approximation for Reynolds stresses**
  - **Turbulent viscosity, μ_T, is calculated from some turbulence model:**

$$-\rho\,\overline{u'v'} = \mu_T\frac{\partial \overline{u}}{\partial y}$$

  - **By analogy, Pr_T = 0.85 (from experimental data)**

$$-\rho\,\overline{v'T'} = D_T\frac{\partial \overline{T}}{\partial y} \qquad D_T = \frac{\mu_T}{\mathrm{Pr}_T}$$

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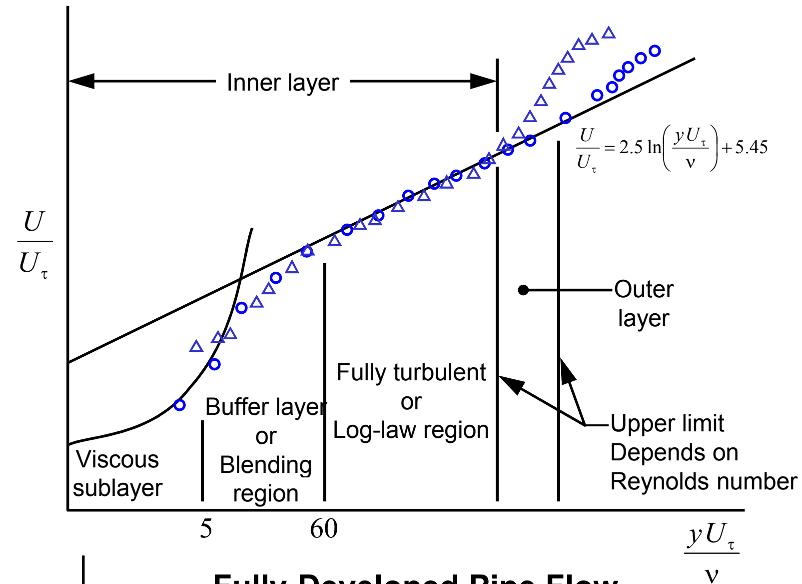
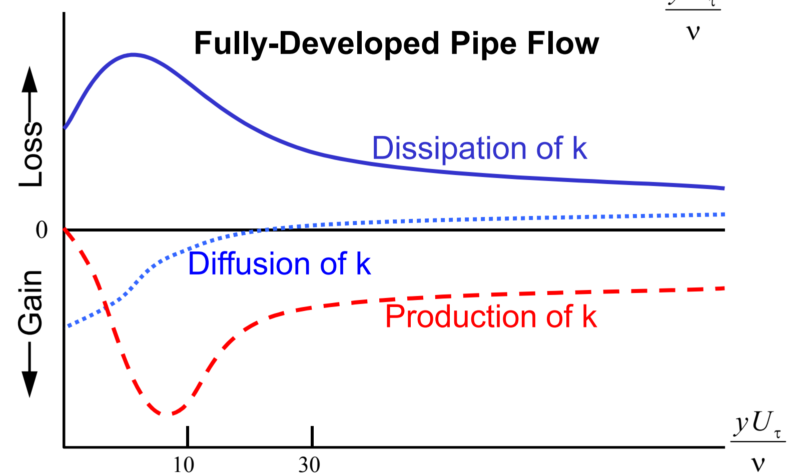
Turbulent Boundary Layer Structure

- **Velocity profile exhibits layered structure identified from dimensional analysis**
  - **Viscous sublayer – Viscous forces dominate, velocity depends on ρ, τ_w, μ, y.**
  - **Outer layer – Depends on mean flow characteristics**
  - **Overlap layer – Log law applies**



- **TKE production and dissipation are nearly equal in the overlap layer (turbulent equilibrium)**

- **Dissipation dominates production in the viscous sublayer region.**



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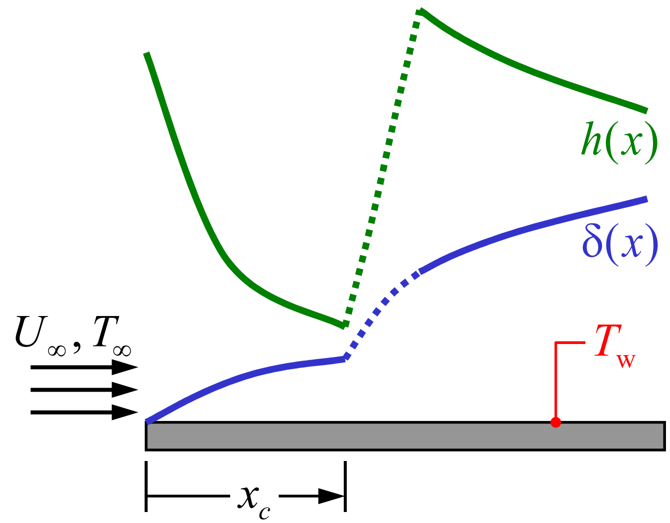
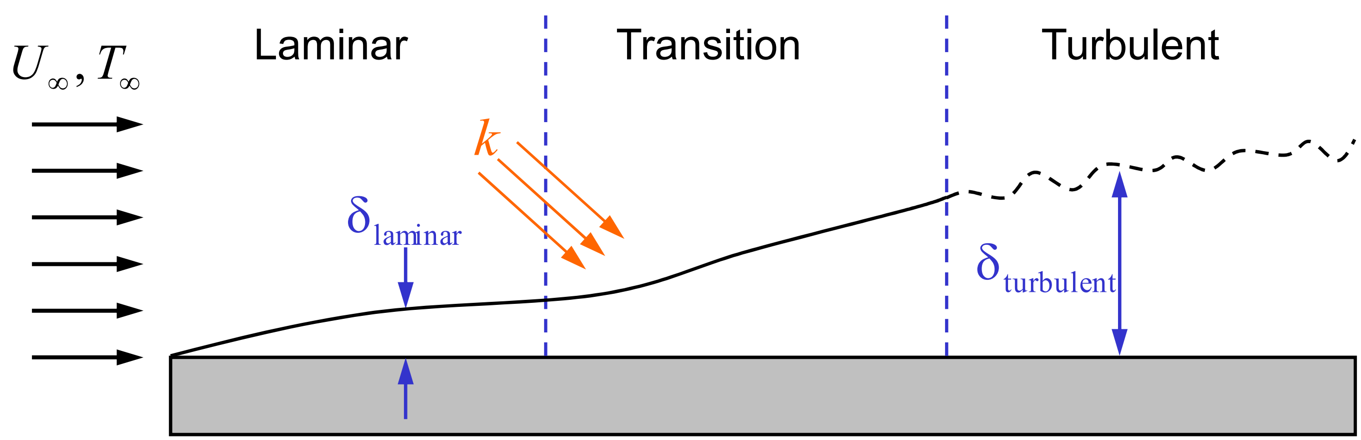
Effects of Transition

- **Spurious jump of C_f and *h* at transition from laminar to turbulent flows (Re_x > 5.105)**
- **Natural transition is a complex phenomenon (for RANS)**
- **RANS: k-k_L-ω and Transition SST models can be used for natural transition, bypass transition, separation induced transition**
  - **Use if extent of laminar flow region is significant**





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Boundary Layer Heat Transfer

- **Impact on numerical modeling**
  - **Use of wall functions for $y^+ >> 1$ (when hypothesis are fulfilled)**
    - **Sensitivity of the results to y^+ (transition, low-Re effect) and Pr**
  - **When hypothesis fails (Non-equilibrium boundary layers, recirculation, stagnation, transition), we need to correctly resolve both the momentum AND thermal viscous sub-layer ($y^+ < 1$)**
    - **This is straightforward for Pr ~ 1 or Pr < 1.**
    - **When Pr is greater than 1, the thermal sublayer is much thinner than the viscous sublayer.**
    - **Small sensitivity to grid resolution (provided that the momentum boundary layer is correctly predicted**
      - **$y^+ \leq 1$ and ~10 cells for $1 < y^+ < 30$**

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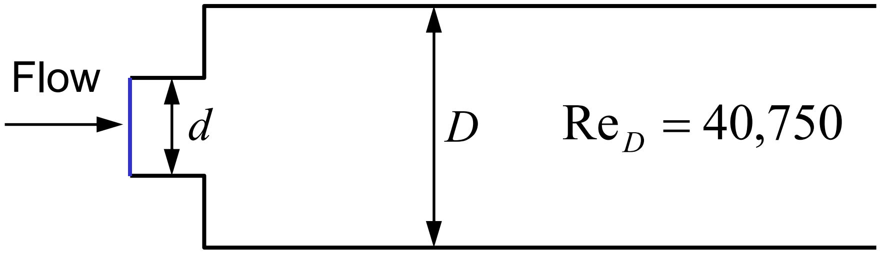
BL Heat Transfer Example – Abrupt Pipe Expansion

- **Abrupt pipe expansion (non-equilibrium boundary layer, recirculation, wall heat transfer)**



- **Mesh: $y^+ \sim 1, 50$**
  - **Inlet: Fully-developed turbulent pipe flow.**
  - **Models: RKE with EWT, SST k–ω**
  - **Enhanced wall treatment (for $y^+ \sim 1$ mesh)**
  - **Standard wall functions (for $y^+ \sim 50$ mesh)**
  - **Both equilibrium and non-equilibrium wall functions were studied.**

J. Baughn, M. Hoffman, R. Takahashi, and B. Launder (1984), "Local Heat Transfer Downstream of an Abrupt Expansion in a Circular Channel with Constant Wall Heat Flux," *ASME J. Heat Transfer*, Vol. 106, No. 4, pp. 789–796.

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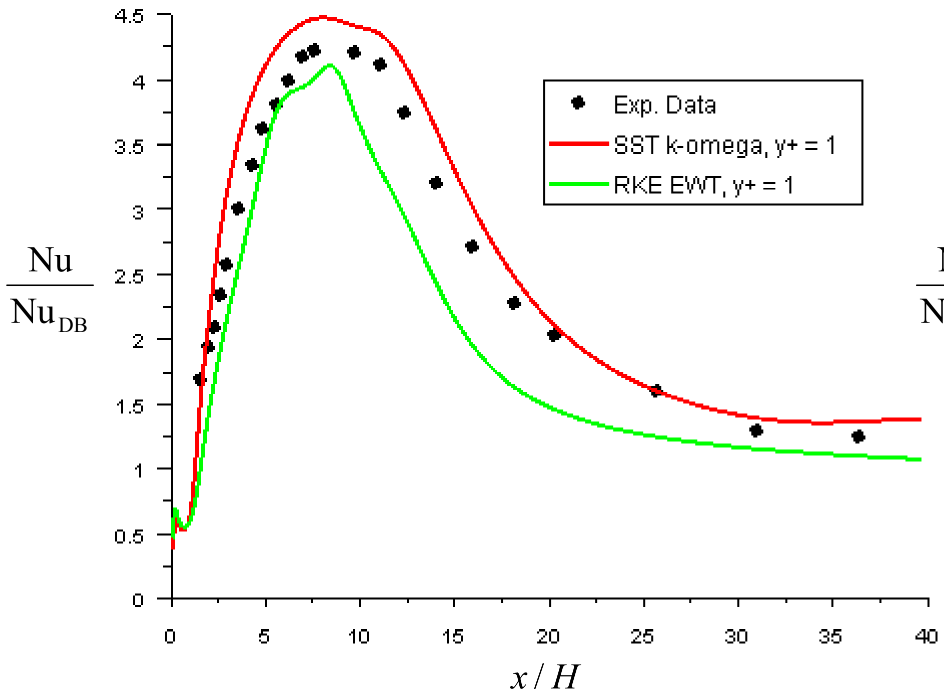
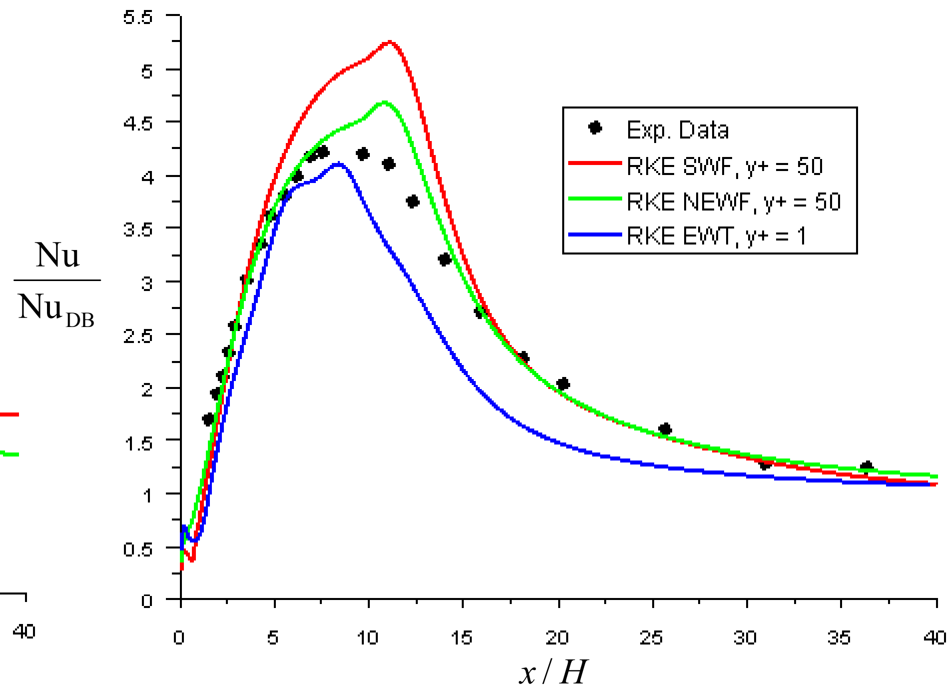
BL Heat Transfer Example –Pipe Expansion

- **Local Nusselt number compared to the Dittus-Boelter correlation (valid for pipe flows).**

$$\mathrm{Nu}_{DB} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$$





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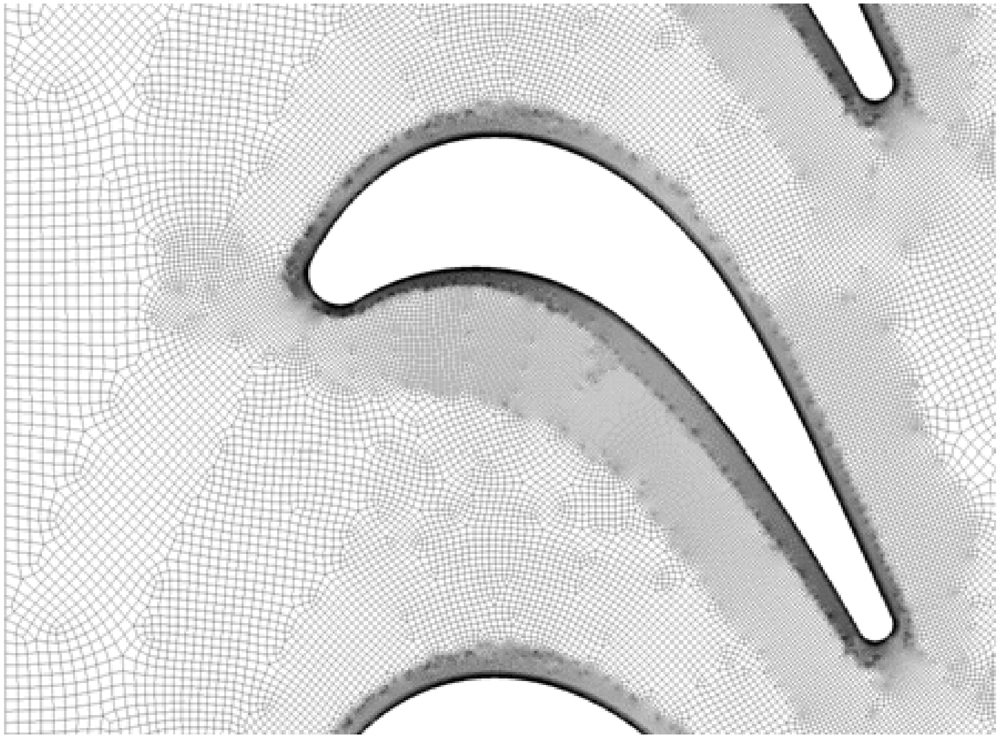
BL Heat Transfer – 2D Turbine Blade

- The suction side BL undergoes a laminar-to-turbulent transition.
- Computed using several near-wall models and low Re models.
  - Two-layer zonal model
  - k–ω models both with and without the “transitional flow” option



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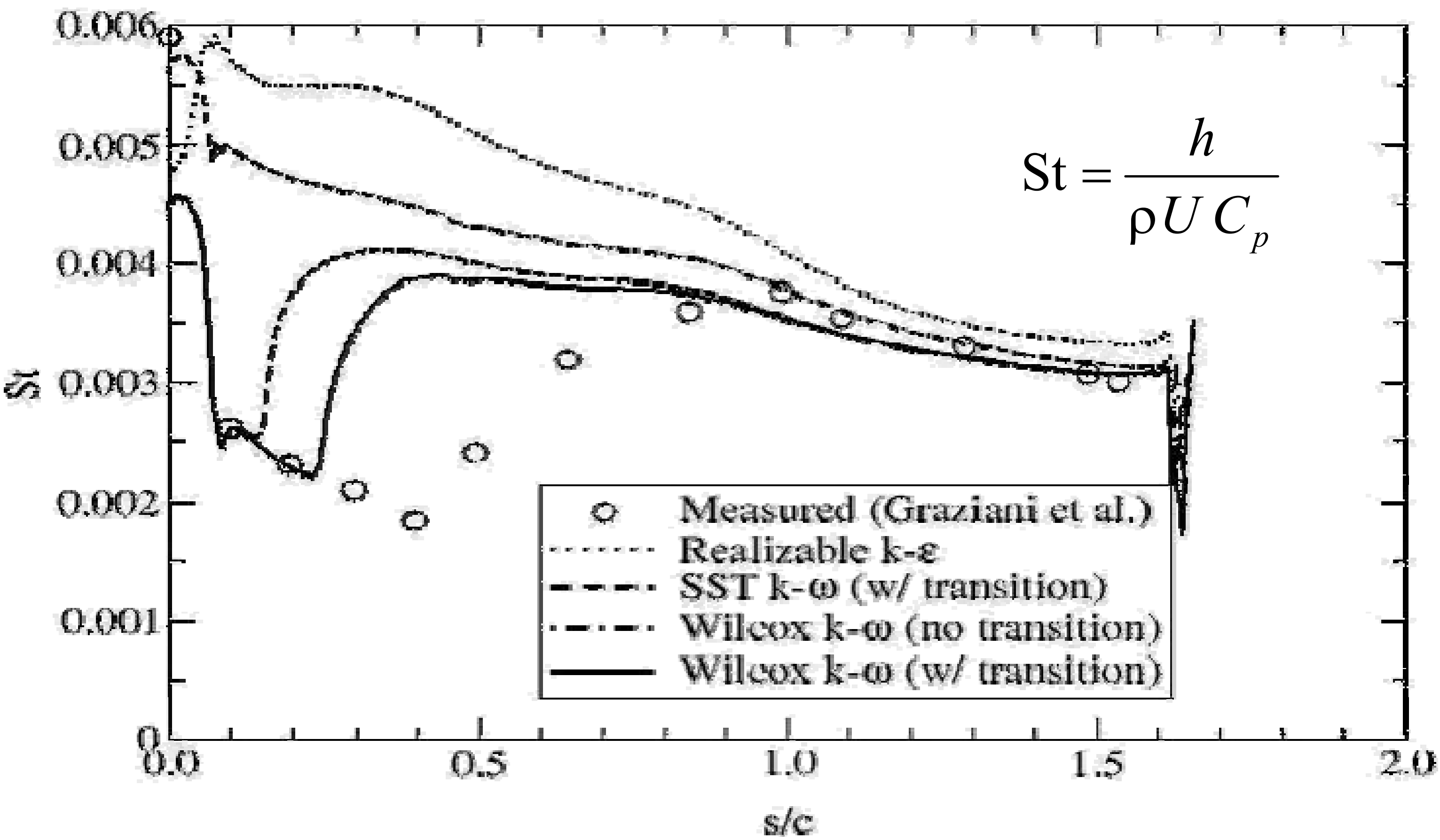
BL Heat Transfer – 2D Turbine Blade

Stanton Number (St)



$$St = \frac{h}{\rho U C_p}$$

Heat Transfer Prediction on the Suction Side

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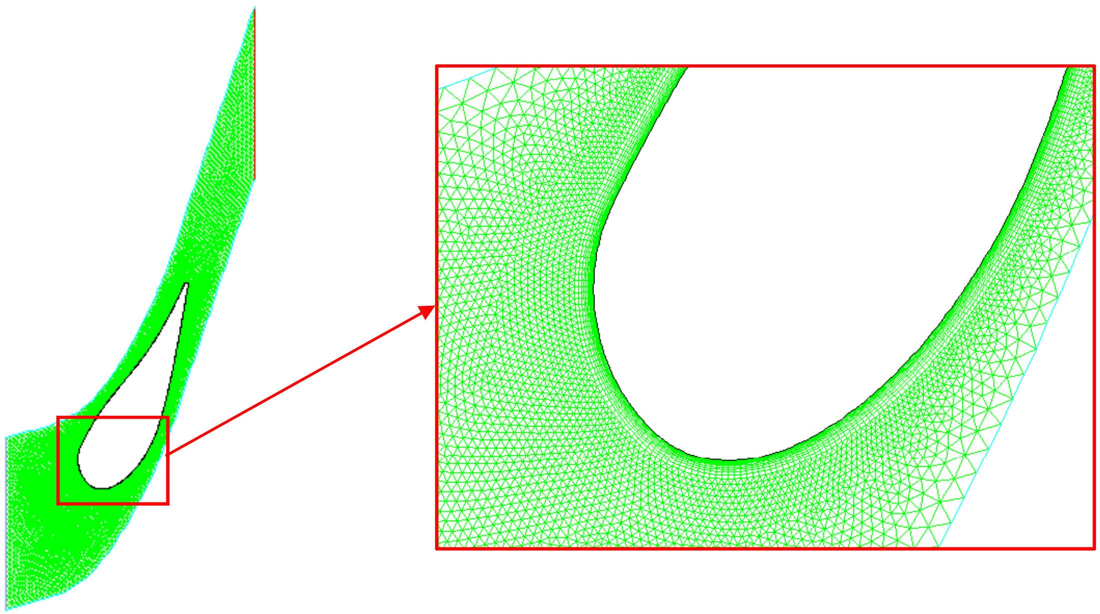
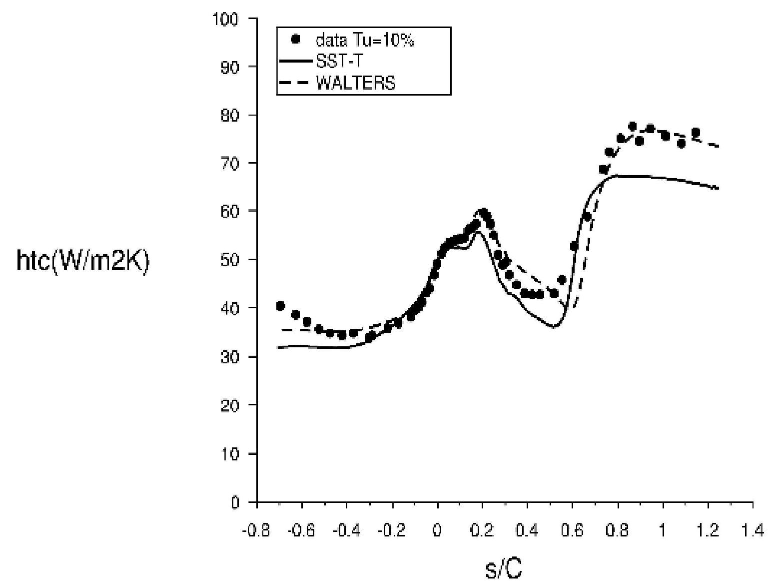
Turbine Blade Heat Transfer with Transition Models

- **VPI Turbine**
  - **Hybrid Mesh: 24,386 cells**
  - **Re = 23,000, U_{in} = 5.85 m/s, Tin = 20 ºC, Chord = 59.4 cm**
  - **Air with constant properties**
  - **Inlet turbulent intensity = 10%**





  - **Both models do a good job of predicting transition point and heat transfer coefficient**

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Example – Impinging Jet

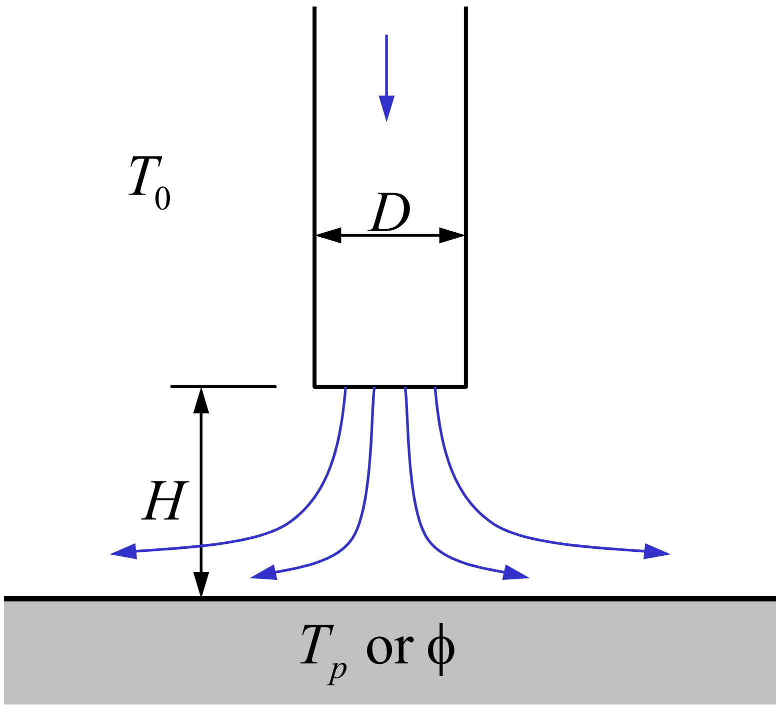
- **Relevant dimensionless parameters**
  - **Height-to-diameter ratio, H/D**
  - **Reynolds number, Re**
  - **Prandtl number, Pr**

- **Quantities analyzed**
  - **Surface heat transfer coefficient**

$$h(x) = \frac{\phi}{T_p - T_0}$$

  - **Nusselt number**

$$\mathrm{Nu}_x = \frac{h(x)\, L}{k_f}$$



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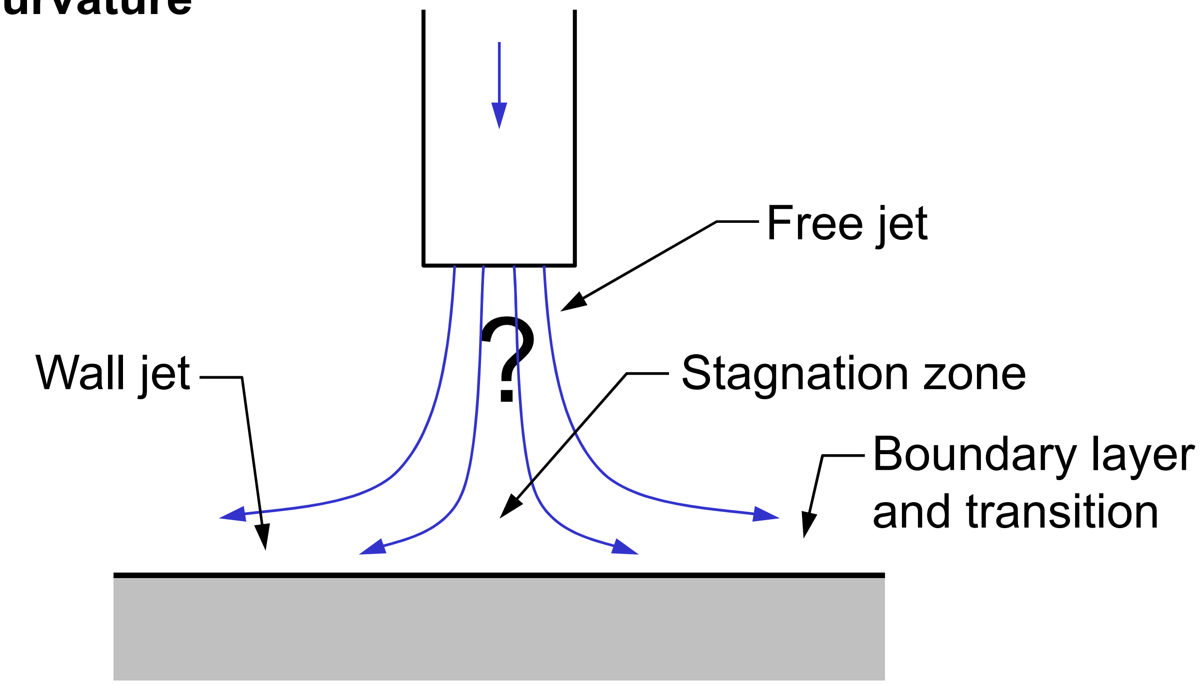
Characteristics of Impinging Jet Flow

- **Modeling challenge – complex flow**
  - **Free jet turbulence**
  - **Stagnation point**
  - **Boundary layer**
  - **Strong streamline curvature**
  - **Transition (?)**



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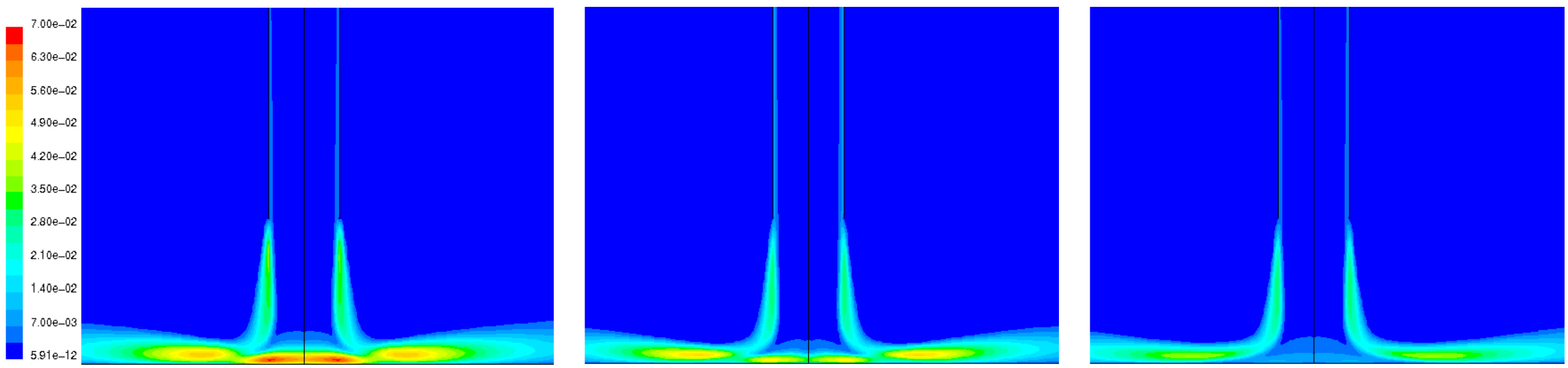
TKE Production at Stagnation Point

- **Physically, decreased production of turbulence is observed at the stagnation point.**
- **Two-equation models tend to overestimate TKE production at the stagnation point**



Standard k–ε **Realizable k–ε** RNG k–ε

Can the production of turbulent kinetic energy be reduced?

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Impinging Jet Example

- **Turbulent kinetic energy transport equation:**

$$\rho \frac{Dk}{Dt} = \underbrace{\frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]}_{\text{Diffusion}} + \underbrace{\mu_T \frac{\partial \overline{u}_i}{\partial x_j}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)}_{\text{Production}} \underbrace{- \rho \varepsilon}_{\text{Dissipation}}$$

- **Modification of production term (Menter, 1992):**

$$P_k = \mu_T \Omega^2$$

- **Text user interface command is**

```
define/models/viscous/turbulent-expert/
```

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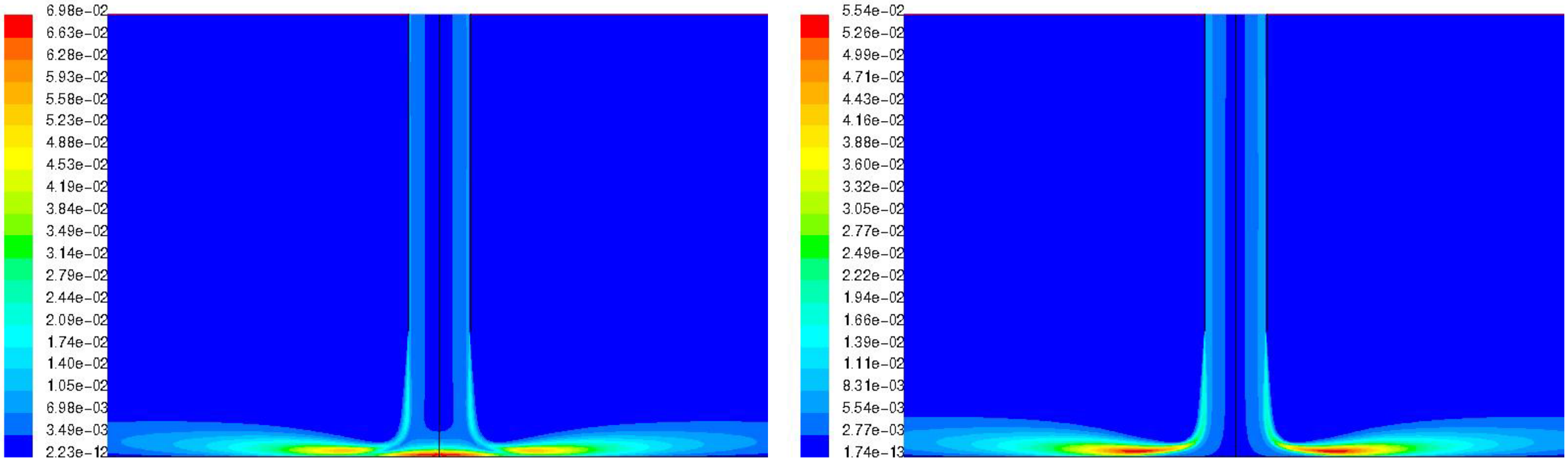
Effect of Modified Production Term



Default Production

Ω-Based Production

k–ω Model

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Flow Calculations ($y^+ = 1$)

- **The following RANS models were evaluated:**
  - **Standard k–ε (SKE)**
  - **RNG k–ε (RKE) – Minimizes TKE at stagnation point.**
  - **Standard k–ω (KW) – Laminar/turbulent transition in boundary layer.**
  - **Modified k–ω (KWW) – Production of TKE based on rotation rate, Ω.**
  - **V^2F model – Accounts for near-wall anisotropy by solving a transport equation for $(v')^2$**

- **Flow characteristics**
  - **Prandtl number: Pr = 0.7**
  - **Reynolds number: Re = 23,000**
  - **Height-to-diameter ratio: H/D = 2.0 and 6.0**

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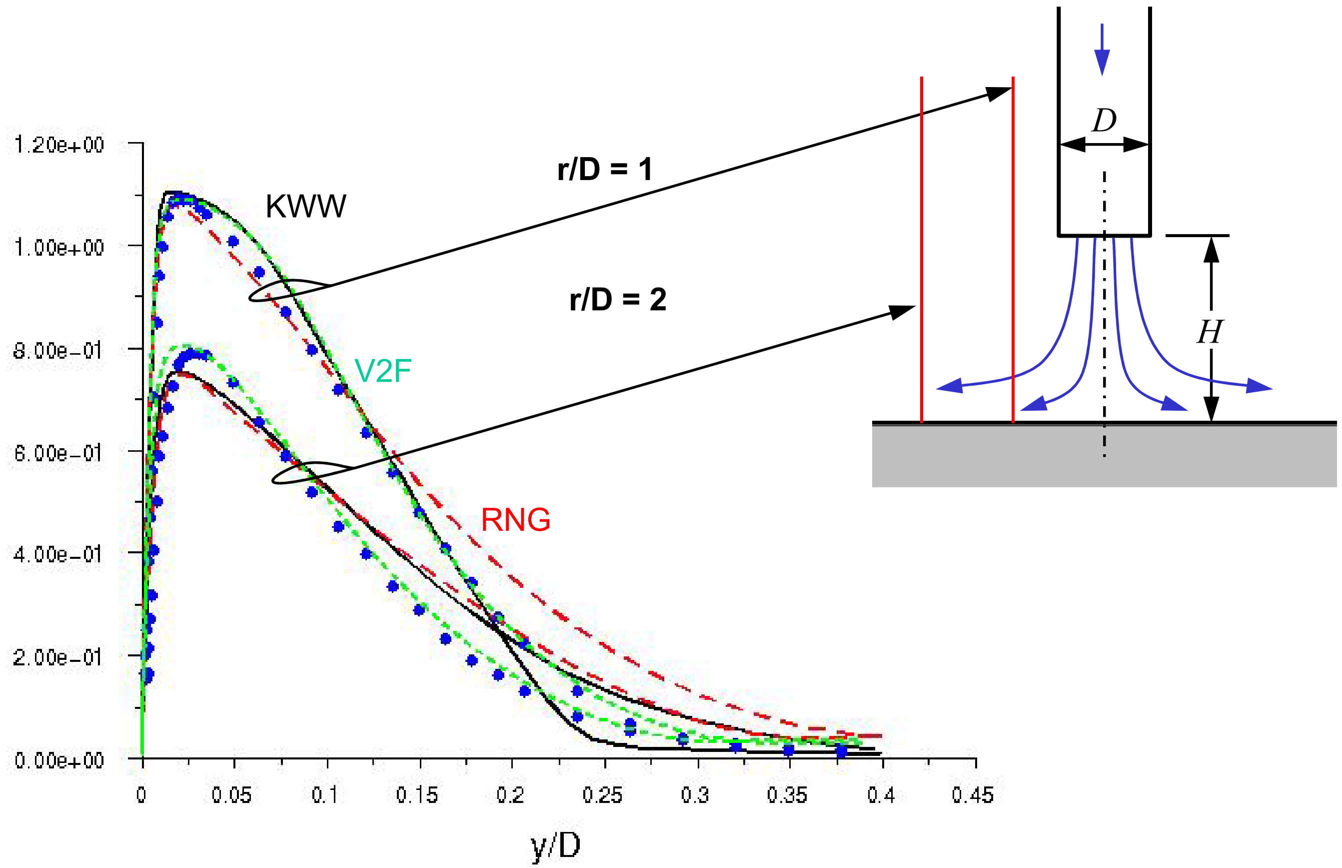
Impinging Jet: Velocity Profiles

- **Results: H/D = 2, Re = 23,000**

Mean velocity profiles



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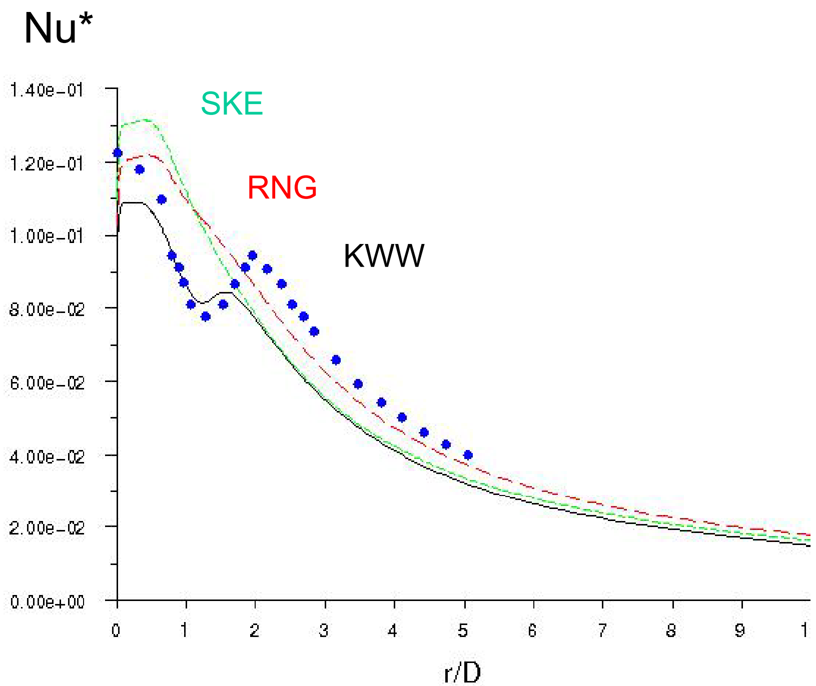
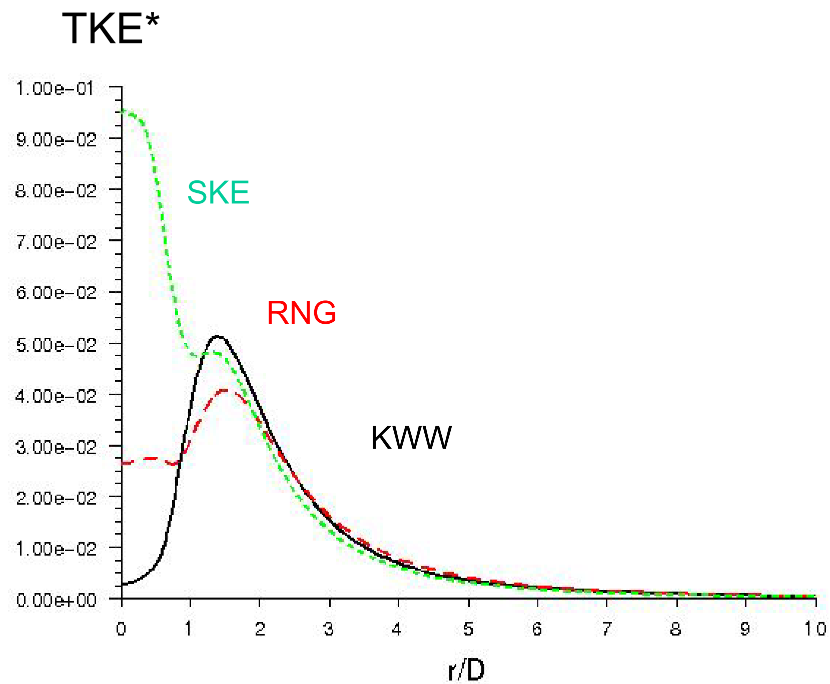
Results from Two-Equation Models

- **Results: H/D = 2, Re = 23,000**



Nusselt Number Re = 23,000



Turbulent Kinetic Energy Re = 23,000

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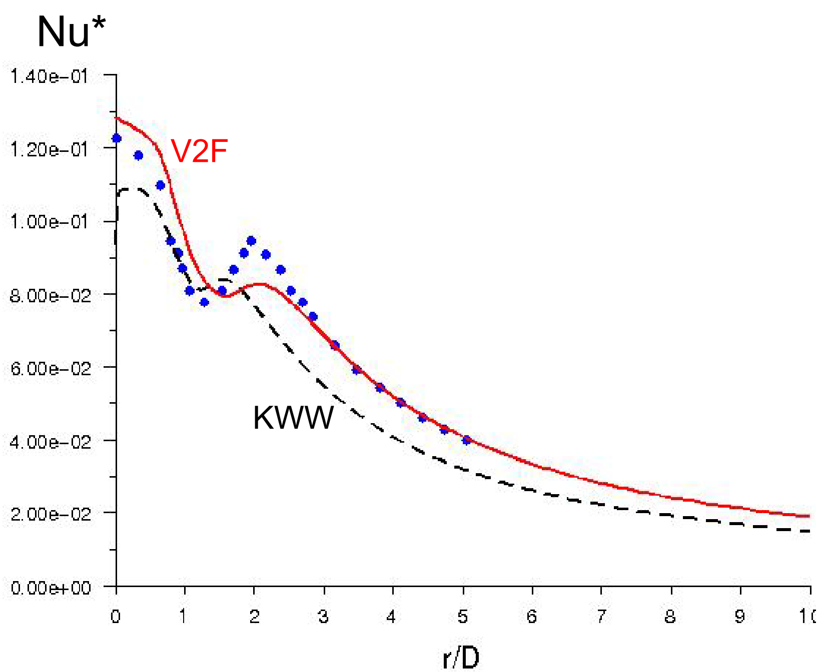
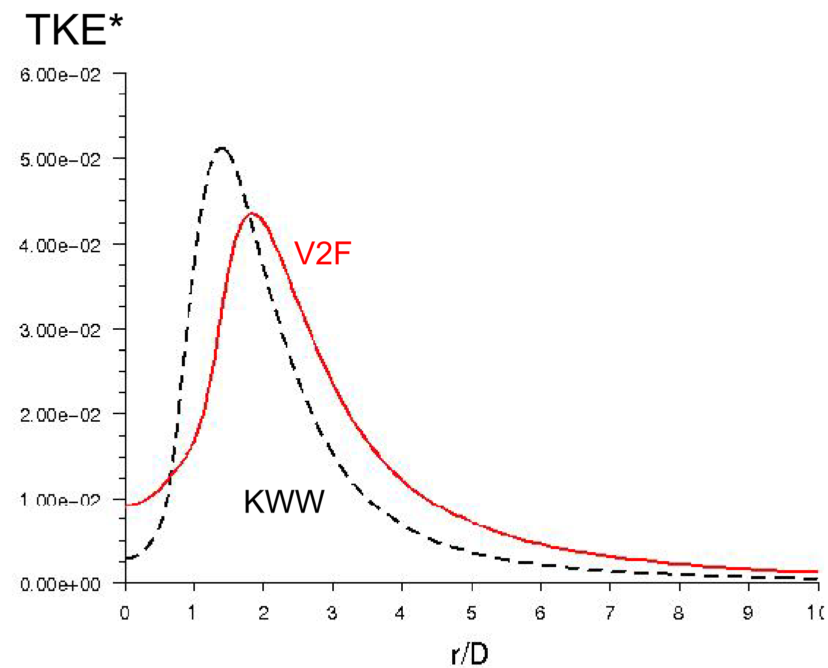
Comparison of k–ω and V2F models

- **Results: H/D = 2, Re = 23,000**



Nusselt Number Re = 23,000



Turbulent Kinetic Energy Re = 23,000

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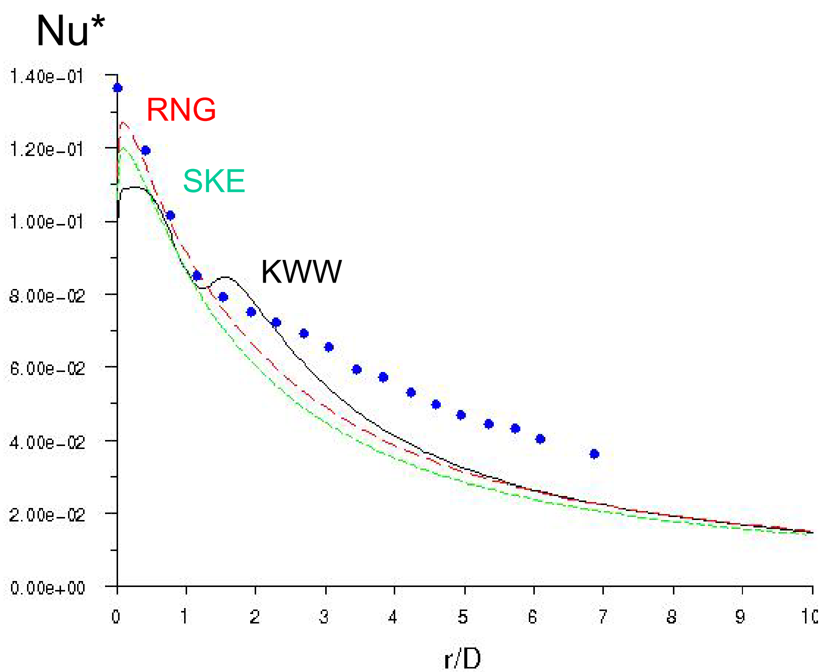
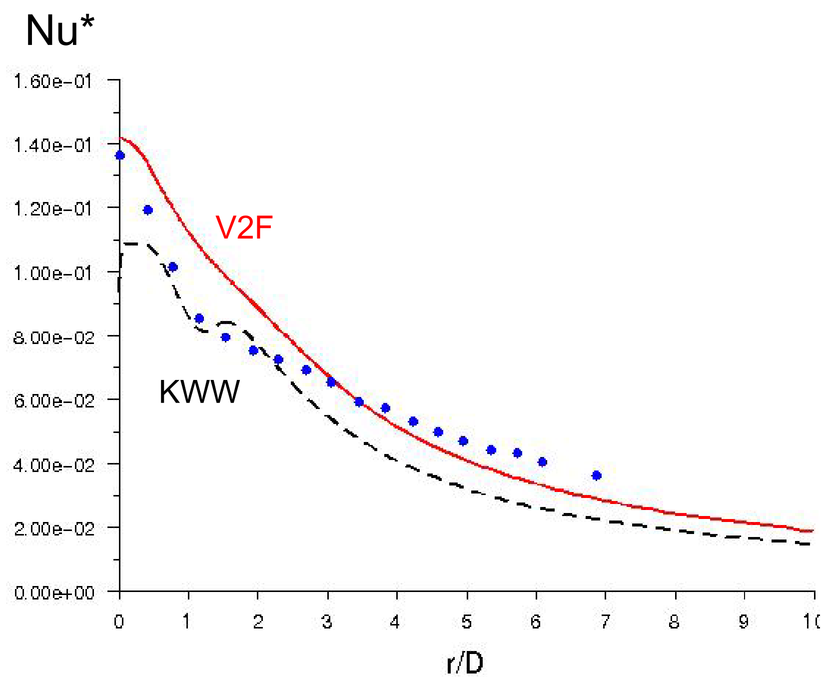
Results from Two-Equation Models

- **Results: H/D = 6, Re = 23.000**



Nusselt Number Re = 23,000



Nusselt Number Re = 23,000

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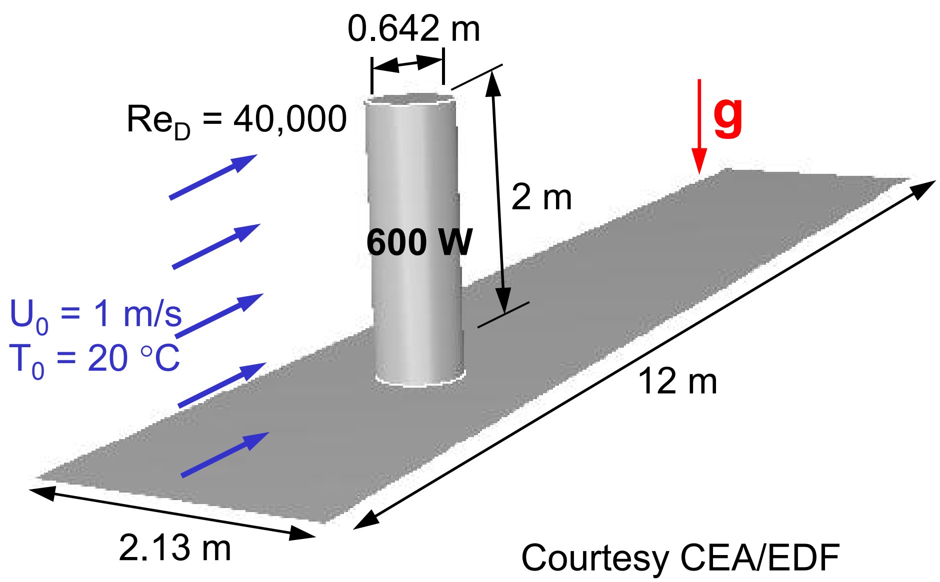
Mixed Convection around a Wall-Mounted Cylinder

- **Re = 40,000 (subcritical flow)**
  - **Laminar BL with turbulent wake**
- **Bluff Body**
  - **Massive separation, vortex shedding**
- **Turbulence model**
  - **SST k–ω with y^+ = 1 to model transition**
  - **LES with dynamic Smagorinsky subgrid model**
- **3 million cell mesh**
- **Mixed convection (buoyancy is important)**
  - **Boussinesq approximation**
- **Cylinder covered by a 5 mm thick steel layer**
- **Fluid/Solid coupled thermal simulation**
  - **D >> d so use of shell conduction is appropriate**



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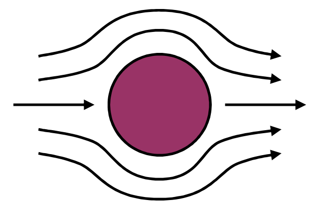
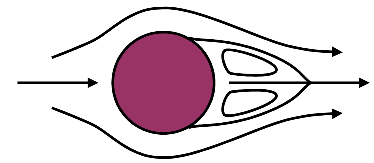
Flow Regimes for Flow Past Cylinders

- **Re < 50**
  - **Laminar wake**

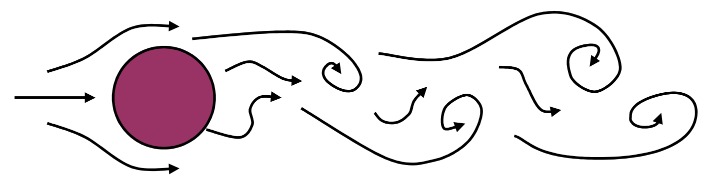


No separation



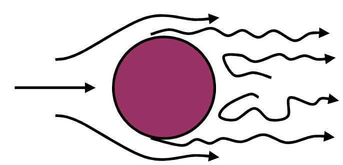
Steady separation bubble

- **50 < Re < 5000 :**
  - **Von-Karman street (laminar BL)**

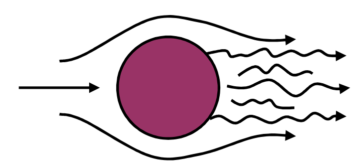


Oscillating Kàrman vortex wake

- **5,000 < Re < 200,000:**
  - **Laminar BL prior to separation (α = 80°). Sub-critical regime**
- **Re > 200,000 “Drag Crisis”**
  - **Turbulent boundary layer prior to separation (α = 120°).**



Laminar boundary layer with wide turbulent wake



Turbulent boundary layer with narrow turbulent wake

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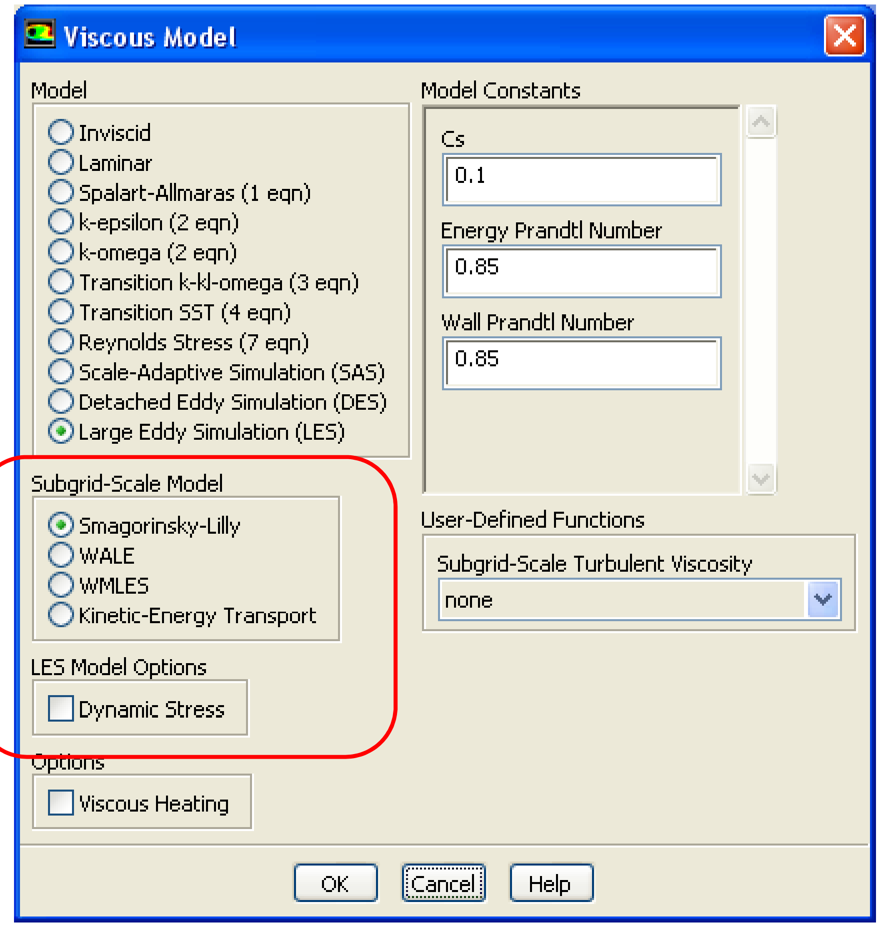
Subgrid Scale Viscosity Models

- **FLUENT 13 offers the following subgrid scale models to be used with LES:**
  - **Smagorinsky model**
  - **WALE model**
  - **Dynamic Smagorinsky model**
  - **Dynamic subgrid kinetic energy transport model**
  - **Wall Modeled LES (WMLES)**
    - **Beta feature in FLUENT 13.0**



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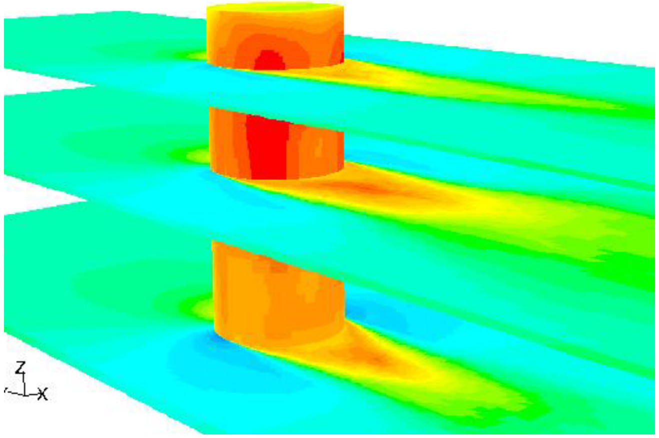
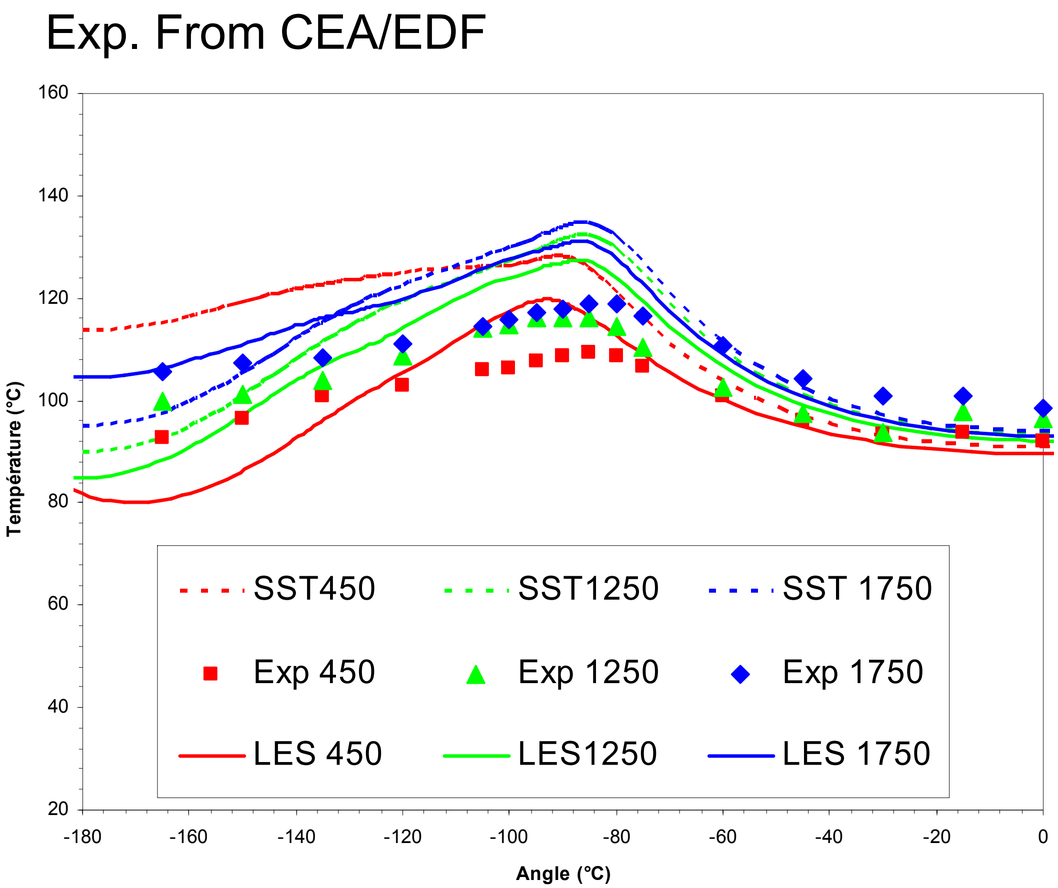
Results – Surface Temperature





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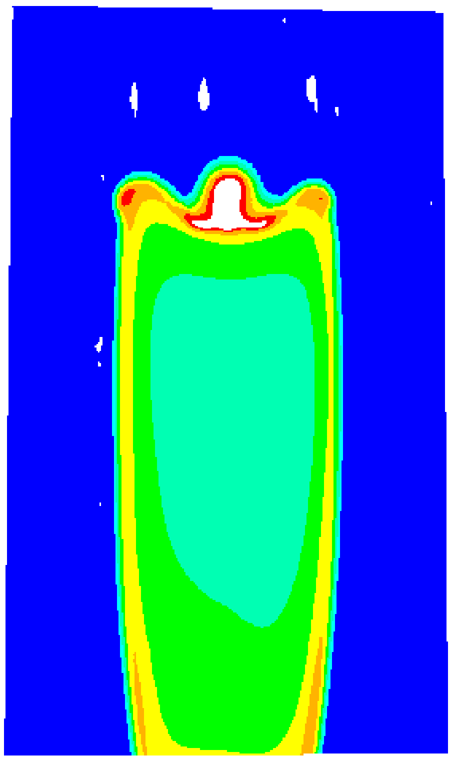
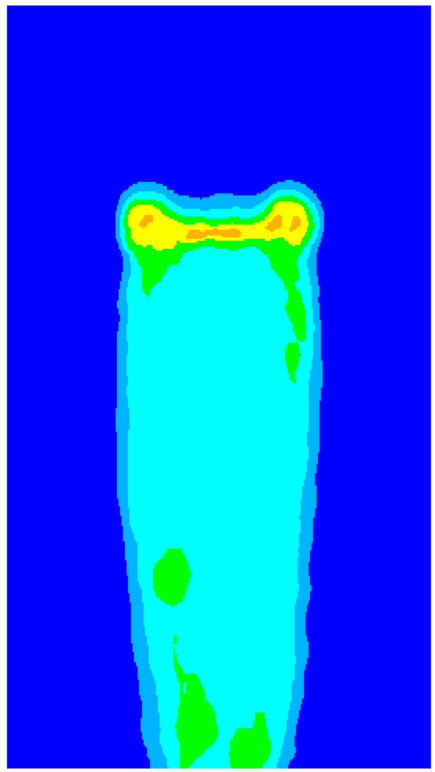
Results – Wake (x = 0.4 m)

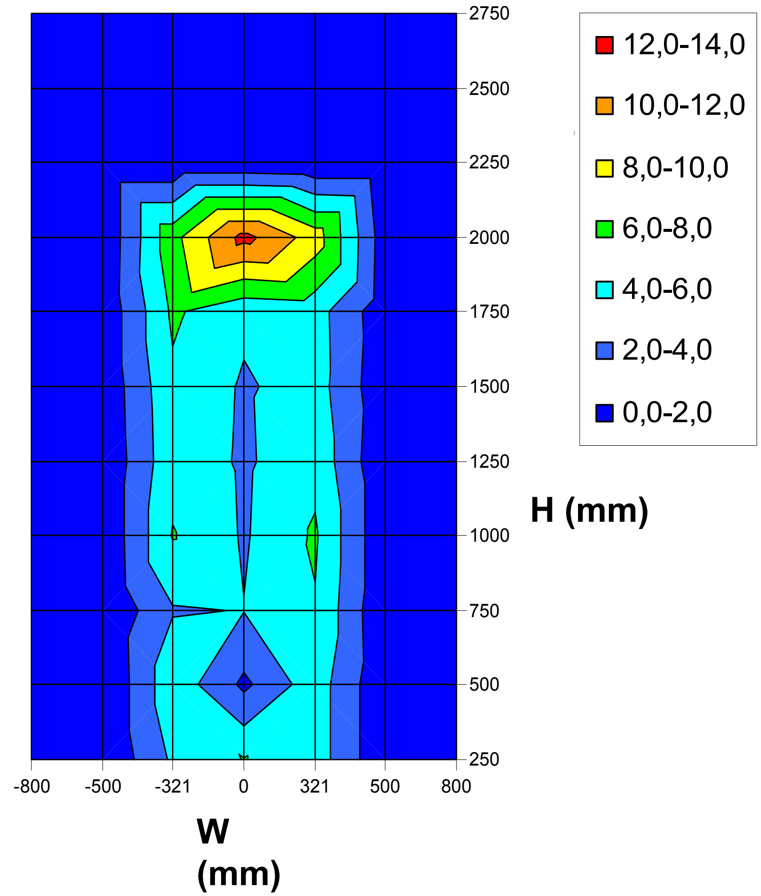
SST



LES



Exp.



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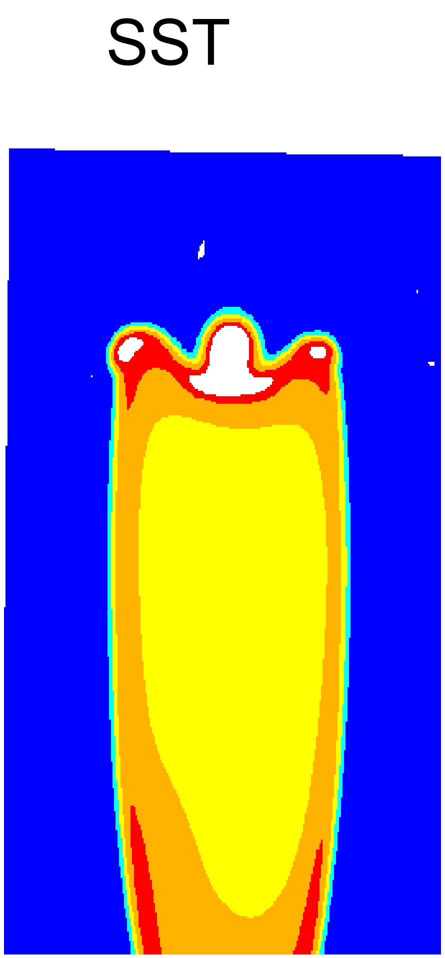
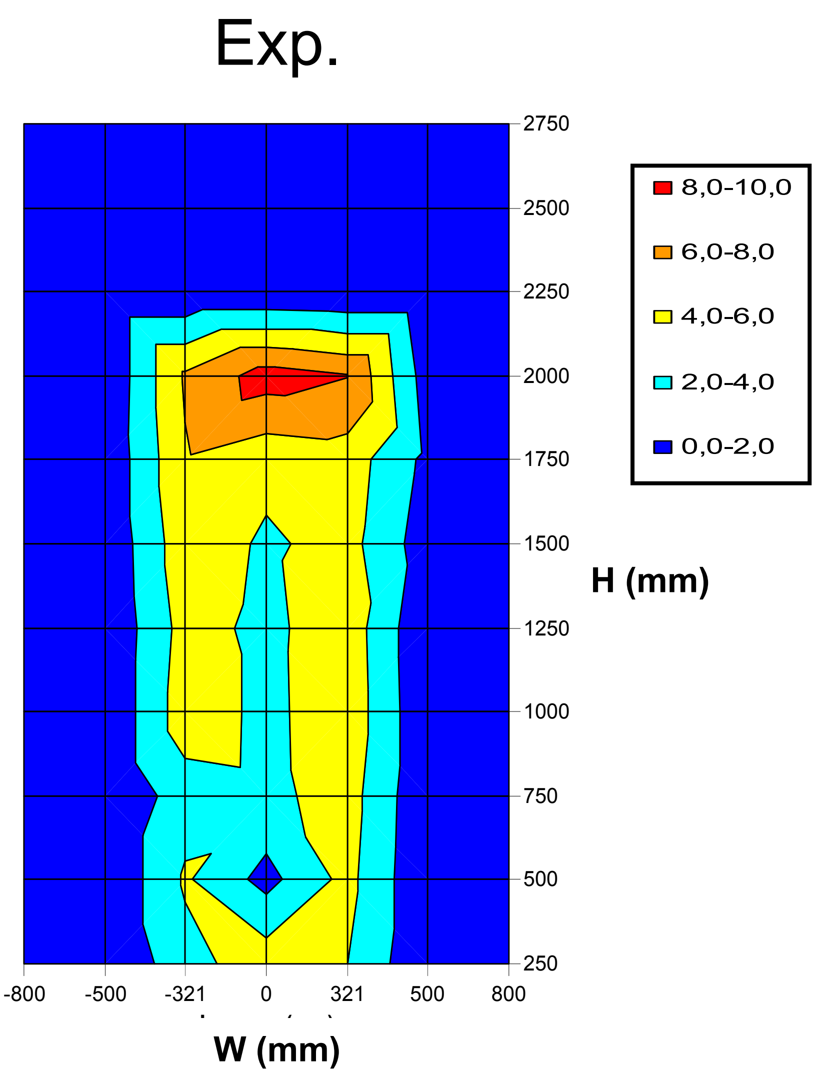
Results – Wake (x = 0.5 m)





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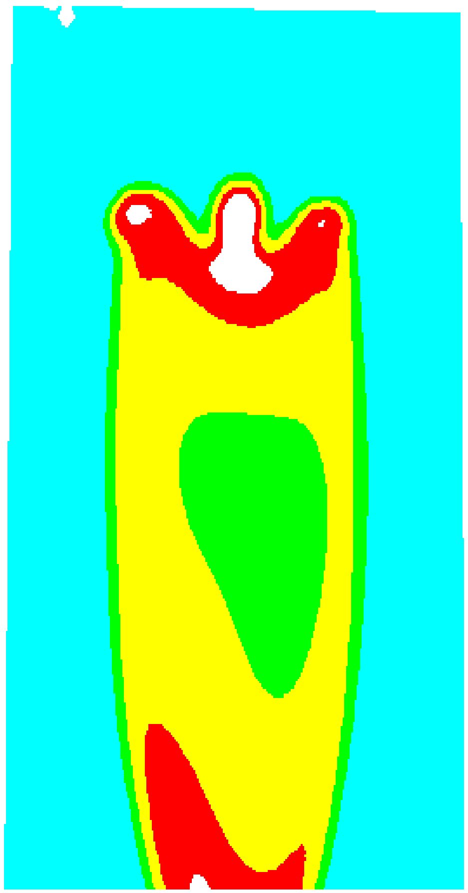
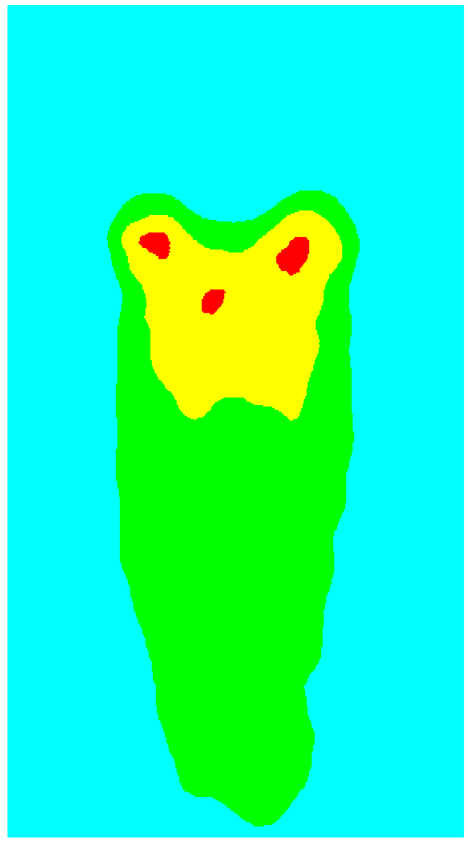
Results – Wake (x = 0.75 m)

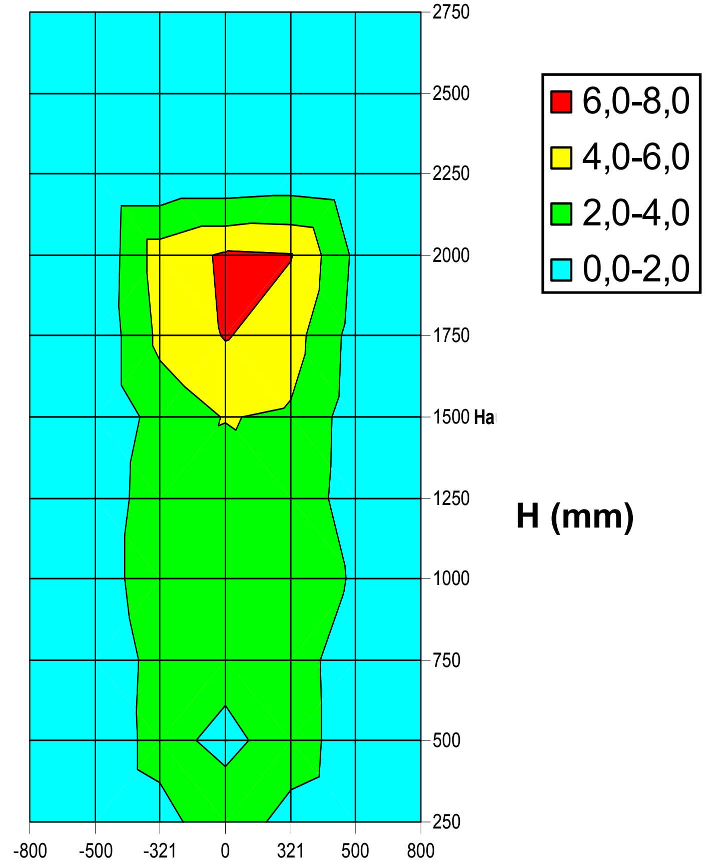
SST



LES



Exp.



W (mm)

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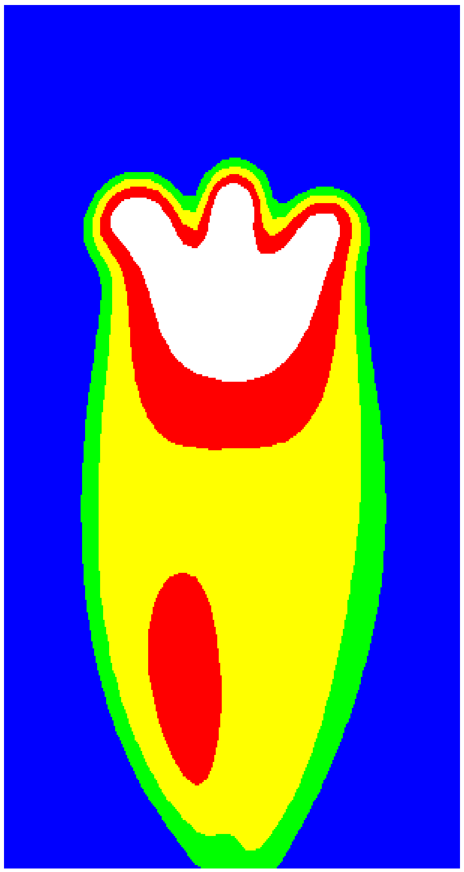
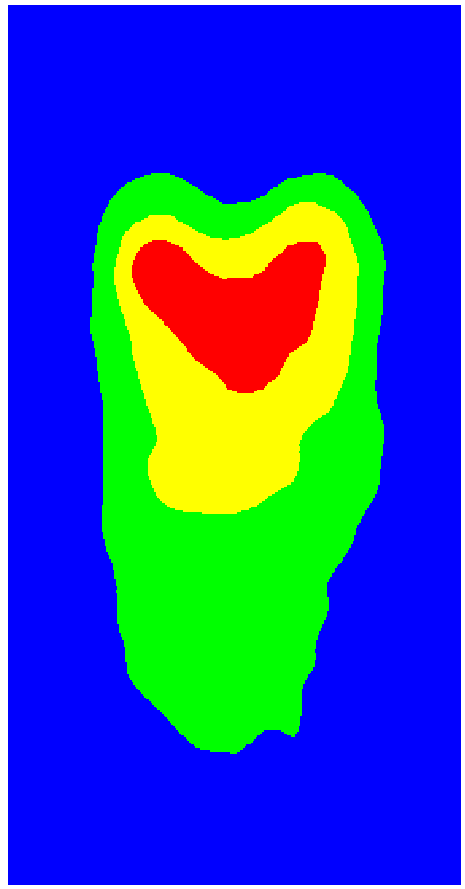
Results – Wake (x = 1.5 m)

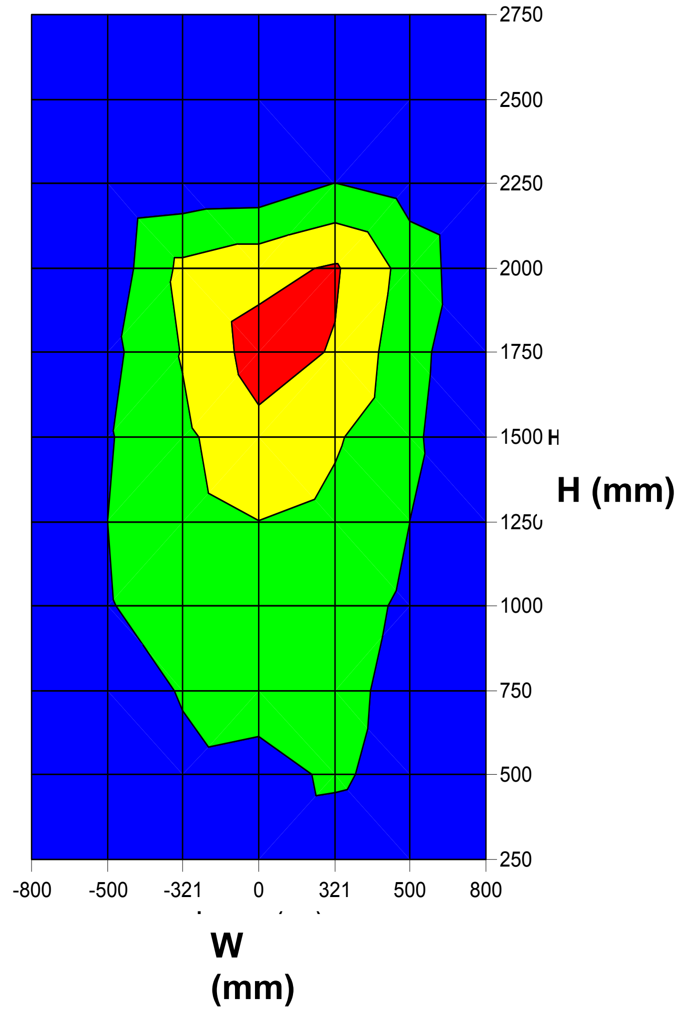
SST



LES



Exp.



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Results – Conclusions

- **Wall temperature comparable between RANS/LES**
- **More accurate wake prediction with LES**
- **CPU time required**
  - **RANS – Days**
  - **LES – Weeks**
- **In this case fluid/solid thermal coupling and large difference between characteristic time scales induce expensive unsteady calculations**

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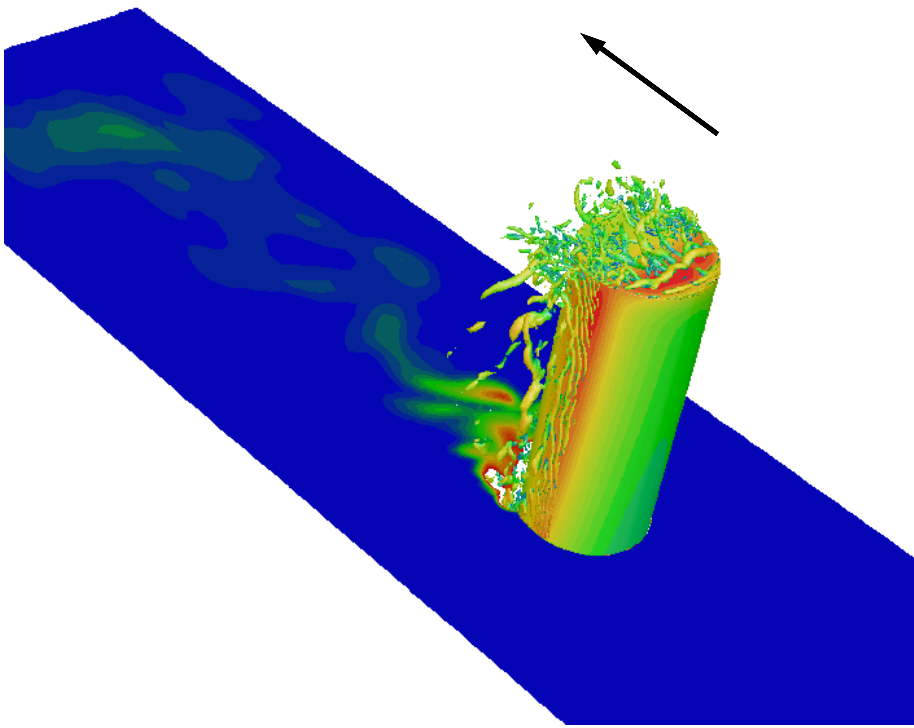
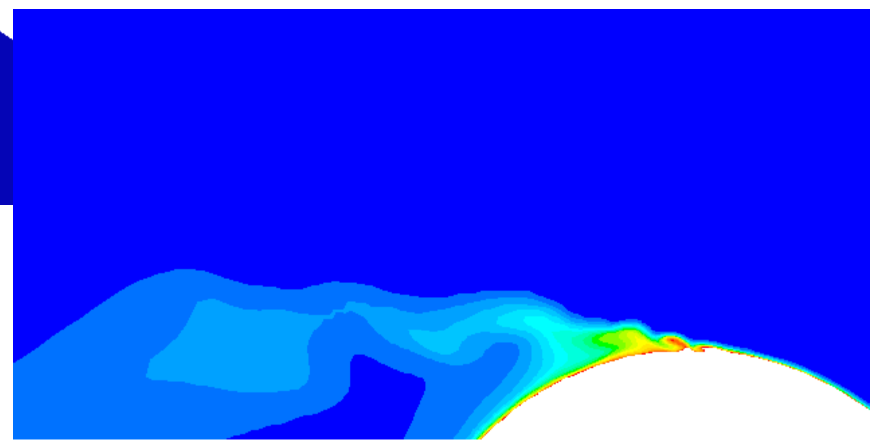
Large Eddy Simulation – Applications



- **Compute unsteady temperature field**
- **Explicit representation of mixing**
- **Accurate min/max fluctuations**
- **Application examples**
  - **Thermal fatigue**
  - **Fluid-structure interaction (FSI)**



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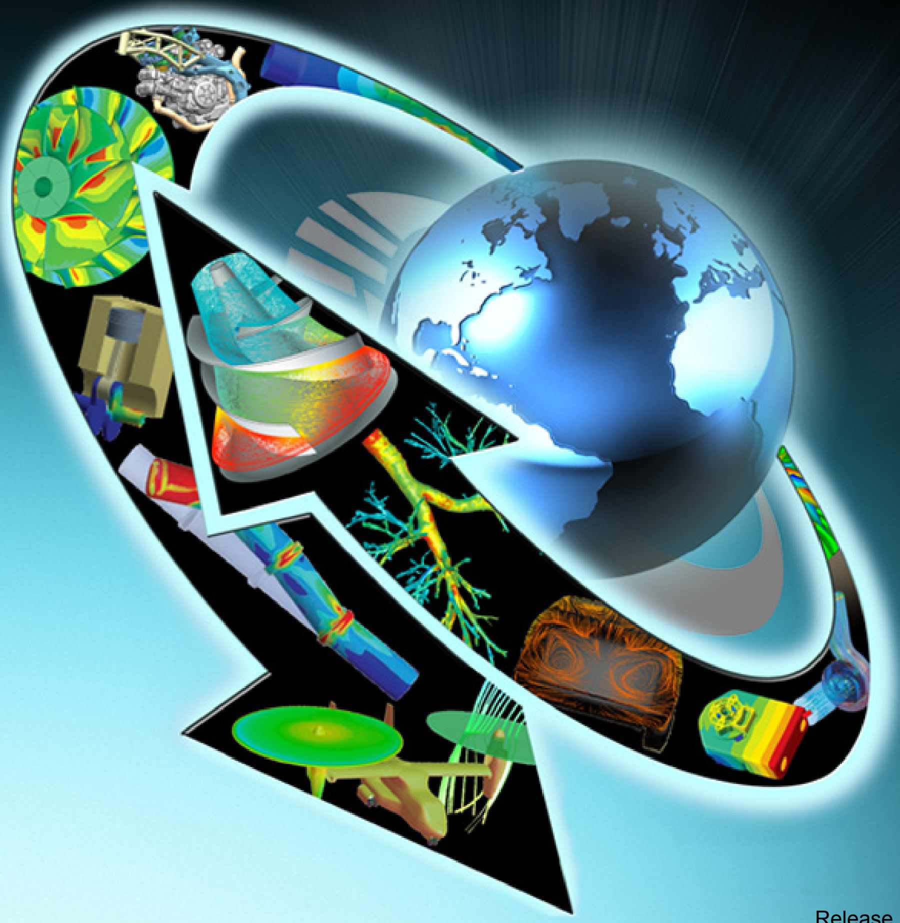
Post-Processing Variables in FLUENT



Heat Transfer Modeling using ANSYS FLUENT

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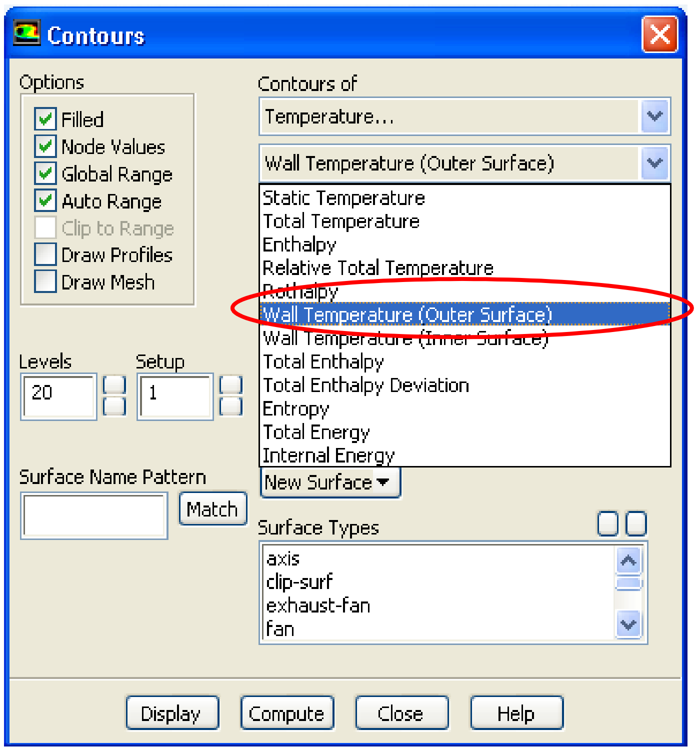
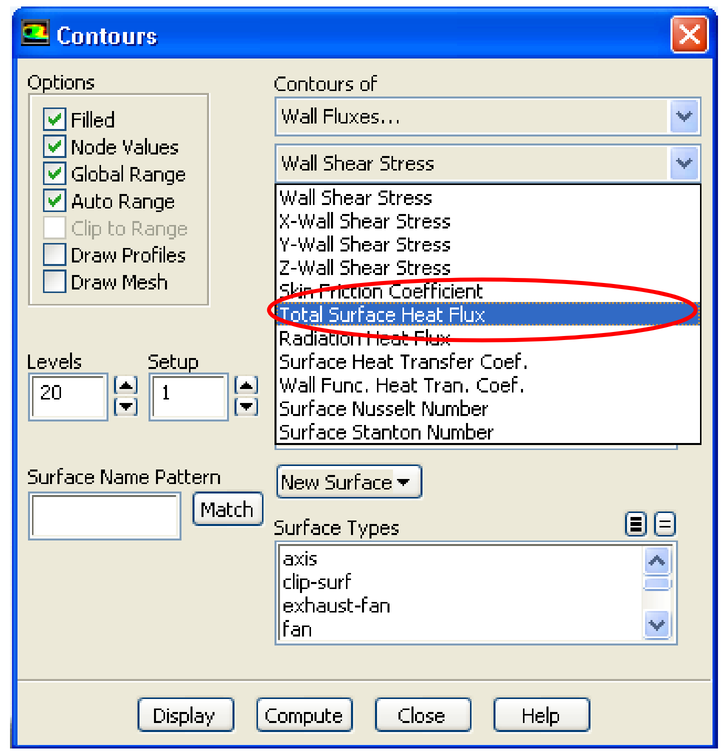
Postprocessing

- **Wall temperature – Use Wall-Temperature (Outer Surface)**
  - **Static Temperature may give a different result.**
- **Convective heat flux – Use Total Surface Heat Flux.**
  - **If radiation is calculated, you need to create a custom field function of Total Surface Heat Flux – Radiation Heat flux**



Wall temperatures are stored at the adjacent cell center



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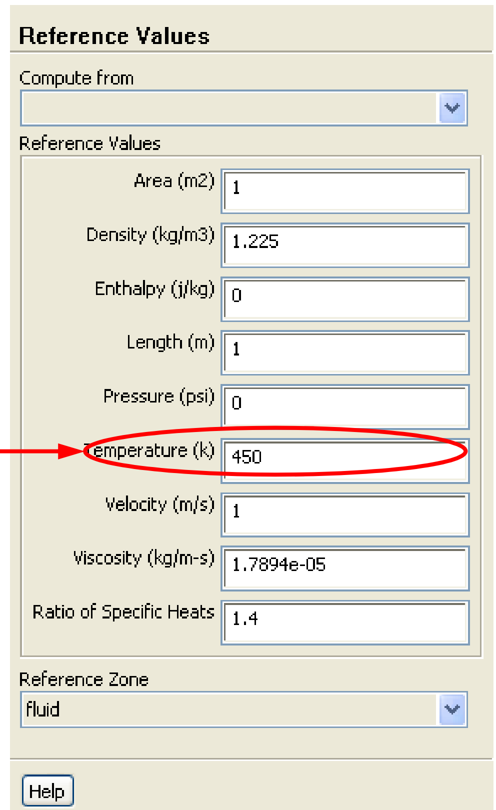
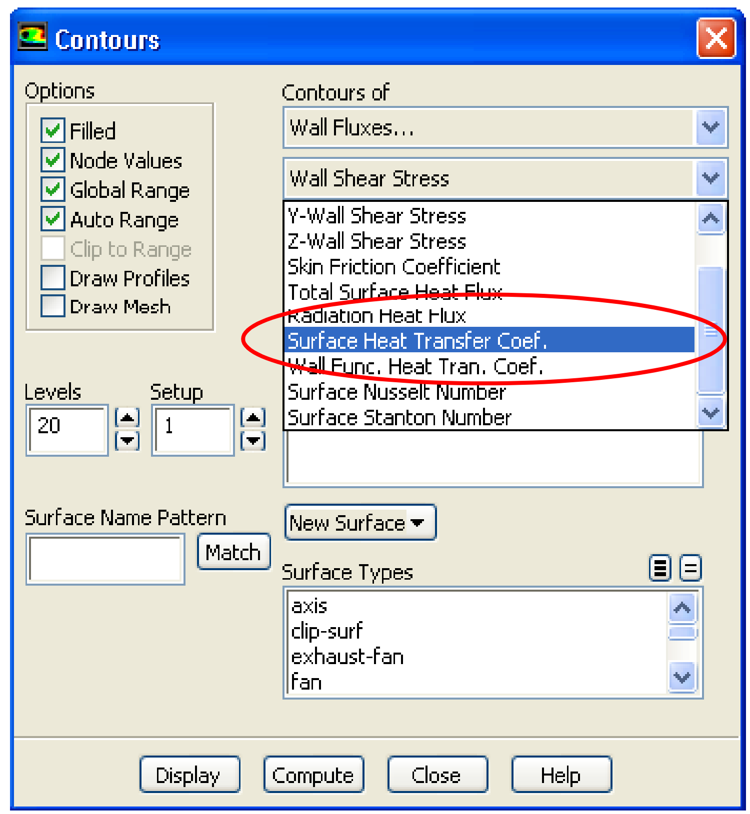
Postprocessing

- **Heat transfer coefficient**
  - **The heat transfer coefficient is computed with a fixed bulk temperature.**
  - **Note that reference temperature must be specified first.**

$$h = \frac{q_{\text{conv+rad}}}{T_w - T_{\text{ref}}}$$





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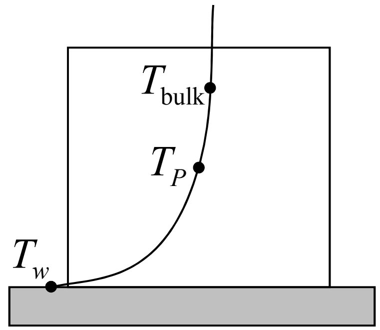
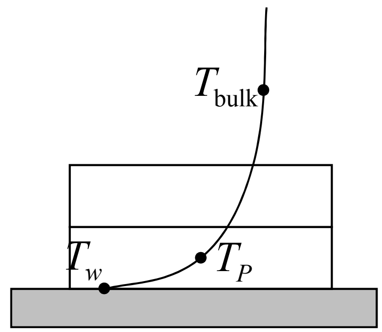
Postprocessing

- **For complex flows, calculating the bulk temperature can be difficult.**
- **If you need a local reference temperature, the problem is even more complex.**
- **In some cases, running an adiabatic case first can give you a local reference temperature profile on the wall.**
  - **For example, solve hot impinging jet in a cold environment onto a cold plate will yield an acceptable reference temperature profile.**





- **Wall function heat transfer coefficient (depends on value of y⁺)**

$$h = \frac{q_{\text{conv}}}{T_w - T_p}$$

Temperature at cell center

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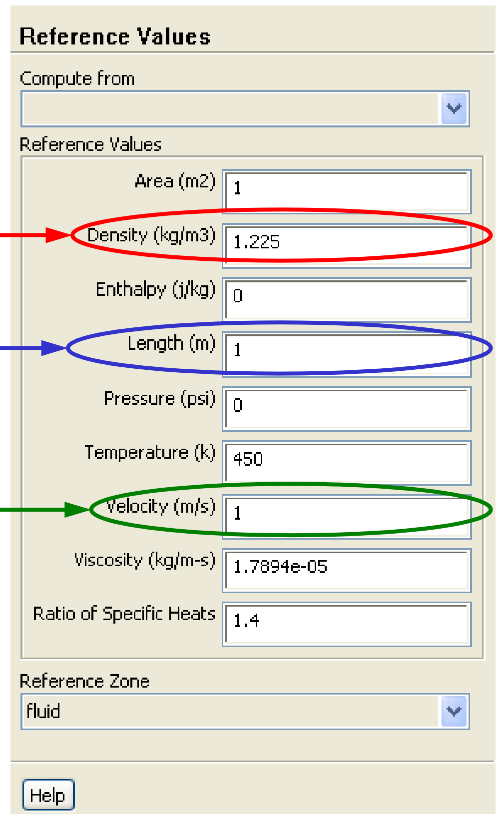
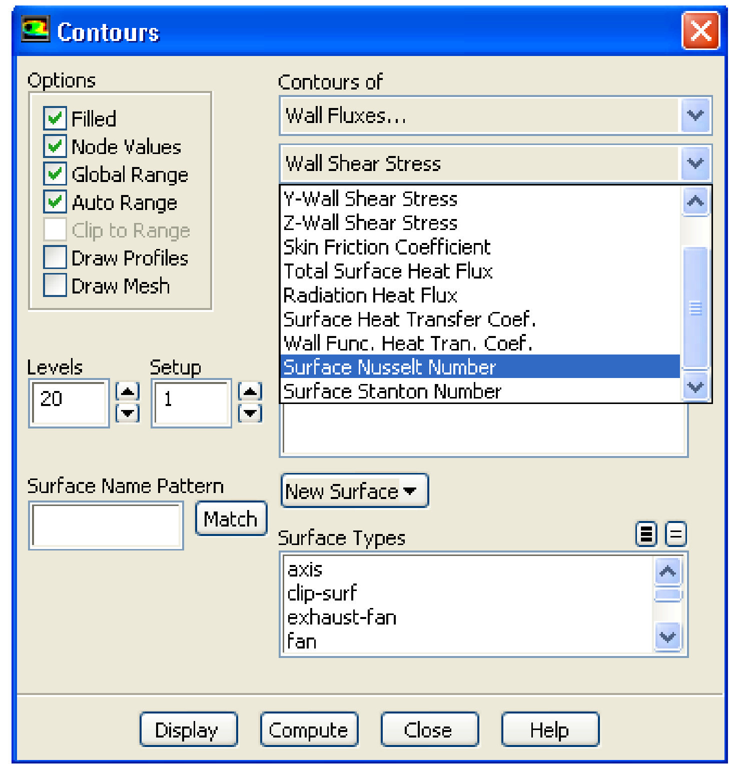
- **Nusselt number**

$$\mathrm{Nu} = \frac{h L_{\mathrm{ref}}}{k}$$

- **Stanton Number**

$$\mathrm{St} = \frac{h}{\rho_{\mathrm{ref}} C_p V_{\mathrm{ref}}}$$





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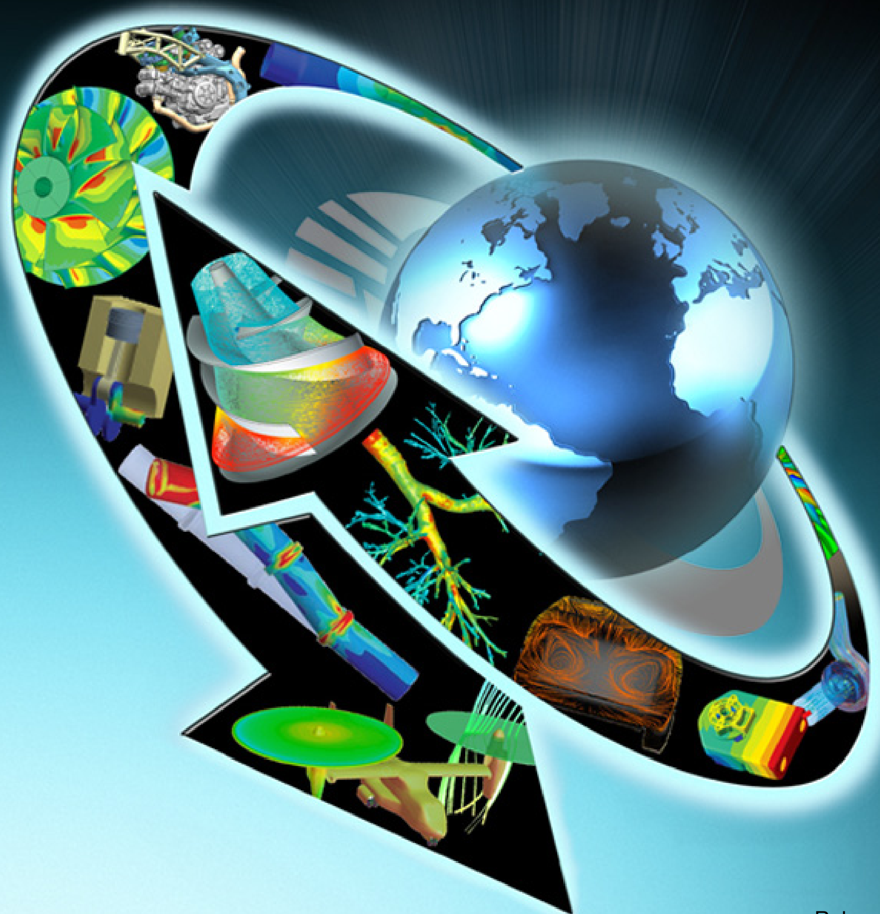
Customer Training Material

Appendix



Lecture 3

Forced Convection

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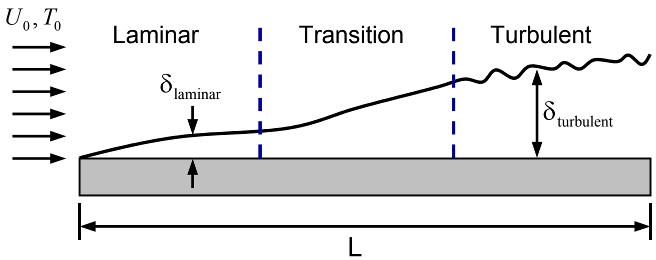
Reynolds Analogy

- **Boundary Layer Equations:**



$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + v\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{P}}{\partial \tilde{x}} + \frac{1}{\text{Re}}\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}$$

$$\tilde{u}(\tilde{x},0) = \tilde{v}(\tilde{x},0) = 0$$

$$\tilde{u}(\tilde{x},\infty) = u_\infty / U_0$$

$$\tilde{u}\frac{\partial \tilde{T}}{\partial \tilde{x}} + v\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{\text{Re}\,\text{Pr}}\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2}$$

$$\tilde{T}(\tilde{x},0) = 0 \quad \tilde{T}(\tilde{x},\infty) = 1$$

- **Wall Fluxes:**

$$C_f = \frac{2}{\text{Re}_L}\left.\frac{\partial \tilde{u}}{\partial \tilde{y}}\right|_{\tilde{y}=0} \qquad \text{Nu} = \left.\frac{\partial \tilde{T}}{\partial \tilde{y}}\right|_{\tilde{y}=0}$$

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Boundary Layers

- **Reynolds analogy**
  - **If dP/dx ~ 0, Pr ~ 1 (constant properties)**

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}}\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} \qquad \begin{array}{l} \tilde{u}(\tilde{x},0) = \tilde{v}(\tilde{x},0) = 0 \\ \tilde{u}(\tilde{x},\infty) = 1 \end{array}$$

$$\tilde{u}\frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} \qquad \begin{array}{l} \tilde{T}(\tilde{x},0) = 0 \\ \tilde{T}(\tilde{x},\infty) = 1 \end{array}$$

- **In dimensionless form, equations are of the same form. Thus, the solutions for dimensionless velocity and dimensionless temperature should be equivalent.**

$$C_f \frac{\mathrm{Re}_L}{2} = \mathrm{Nu} \qquad\qquad \frac{C_f}{2} = \mathrm{St}$$

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Some Definitions

$$\tau_w = \mu \left. \frac{\partial \bar{u}}{\partial y} \right|_{y=0} \qquad U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

$$q_w = k \left. \frac{\partial \bar{T}}{\partial y} \right|_{y=0} \qquad T_\tau = \frac{q_w}{\rho C_p U_\tau}$$

$$\mu_T = l_{mix}^2 \left| \frac{\partial \bar{u}}{\partial y} \right| \qquad l_{mix} = \kappa y$$

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Wall Functions

- **BL Momentum RANS equations**

$$\cancelto{0}{\bar{u}\frac{\partial \bar{u}}{\partial x}} + \cancelto{0}{\bar{v}\frac{\partial \bar{u}}{\partial y}} = \cancelto{0}{-\frac{\partial \bar{P}}{\partial x}} + \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right)$$

$$\underbrace{\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}}_{\tau_{\text{total}}} = \text{constant}$$

- **BC at the wall (y = 0):** $$\tau_{\text{total}}\Big|_{y=0} = \mu\frac{\partial \bar{u}}{\partial y}\bigg|_{y=0} = \rho U_\tau^2$$

- **Mixing length model** $$-\rho\overline{u'v'} = \mu_T\frac{\partial \bar{u}}{\partial y} = \kappa\, y^2\left|\frac{\partial \bar{u}}{\partial y}\right|^2$$

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Boundary Layers

- **Viscous sublayer**

$$\underbrace{\mu\frac{\partial \bar{u}}{\partial y} - \rho\,\cancelto{0}{\overline{u'v'}}}_{\tau_{\text{total}}} = \rho U_\tau^2 \quad \Rightarrow \quad \boxed{u^+ = y^+}$$

- **Turbulent region**

$$\underbrace{\cancelto{0}{\mu\frac{\partial \bar{u}}{\partial y}} - \rho\,\overline{u'v'}}_{\tau_{\text{total}}} = \rho U_\tau^2 \quad \Rightarrow \quad \boxed{u^+ = \frac{1}{\kappa}\ln\left(y^+\right) + C}$$

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Boundary Layers

- **Boundary layer energy equation**

$$\rho C_p \left(\cancelto{0}{\bar{u} \frac{\partial \bar{T}}{\partial x}} + \cancelto{0}{\bar{v} \frac{\partial \bar{T}}{\partial y}} \right) = \frac{\partial}{\partial y} \left(k \frac{\partial \bar{T}}{\partial y} - \rho C_p \overline{v' T'} \right)$$

$$\underbrace{k \frac{\partial \bar{T}}{\partial y} - \rho C_p \overline{v' T'}}_{q_{\text{total}}} = \text{constant}$$

- **BC at the wall (y = 0):**

$$q_{\text{total}}\Big|_{y=0} = k \frac{\partial \bar{T}}{\partial y}\Big|_{y=0} = \rho C_p T_\tau U_\tau$$

- **Reynolds analogy:**

$$-\rho \overline{v' T'} = \frac{\mu_T}{\mathrm{Pr}_T} \frac{\partial \bar{T}}{\partial y} = \frac{\kappa y^2}{\mathrm{Pr}_T} \left| \frac{\partial \bar{u}}{\partial y} \right| \frac{\partial \bar{T}}{\partial y}$$

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Boundary Layers

- **Viscous sublayer**

$$\underbrace{k\frac{\partial \overline{T}}{\partial y} - \rho C_p \cancel{\overline{v'T'}}^{\,0}}_{q_{\text{total}}} = \rho C_p T_\tau U_\tau \Rightarrow \boxed{T^+ = \Pr y^+}$$

- **Turbulent region**

$$\underbrace{\cancel{k\frac{\partial \overline{T}}{\partial y}}^{\,0} - \rho C_p \overline{v'T'}}_{q_{\text{total}}} = \rho C_p T_\tau U_\tau \Rightarrow \boxed{T^+ = \frac{\Pr_T}{\kappa}\ln\left(y^+\right) + f\left(\Pr\right)}$$

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Wall Functions in FLUENT

- **Non-equilibrium effect and pressure gradient effect**

$$\mu_T = \rho \kappa C_\mu^{1/4} k^{1/2} y$$

  - **Use Prandtl-Komolgorov eddy-viscosity model**

$$y^* \equiv \frac{\rho C_\mu^{1/4} k_P^{1/2} y_P}{\mu}$$

$$U^* = \frac{1}{\kappa} \ln\left(E y^*\right)$$

$$U^* \equiv \frac{U_P C_\mu^{1/4} k_P^{1/2}}{\tau_w / \rho}$$

  - **Keep pressure gradient in boundary layer equations (partially cancel the inertial terms)**

$$\frac{\widetilde{U} C_\mu^{1/4} k^{1/2}}{U_\tau^2} = \frac{1}{\kappa} \ln\left(E \frac{\rho C_\mu^{1/4} k^{1/2} y}{\mu} \right)$$

$$y_v \equiv \frac{\mu y_v^*}{\rho C_\mu^{1/4} k_P^{1/2}}$$

$$\widetilde{U} = U - \frac{1}{2} \frac{dP}{dx} \left[\frac{y_v}{\rho \kappa \sqrt{k}} \ln\left(\frac{y}{y_v}\right) + \frac{y - y_v}{\rho \kappa \sqrt{k}} + \frac{y_v^2}{\mu} \right]$$

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Wall Functions in FLUENT

- **Jayatilleke: Wide range of Prandtl number**

$$T^* \equiv \frac{(T_w - T_p)\rho C_P C_\mu^{1/4} k_P^{1/2}}{\dot{q}}$$

$$= \begin{cases} \Pr y^* + \dfrac{\rho \Pr U_p^2 C_\mu^{1/4} k_P^{1/2}}{2\dot{q}} & \text{for } y^* < y_T^* \\ \Pr_t \left[\dfrac{1}{k} \ln(E y^*) + P \right] + \dfrac{\rho C_\mu^{1/4} k_P^{1/2}}{2\dot{q}} \left[\Pr_t U_P^2 + (\Pr - \Pr_t) U_c^2 \right] & \text{for } y^* > y_T^* \end{cases}$$

$$P = 9.24 \left[\left(\frac{\Pr}{\Pr_t} \right)^{3/4} - 1 \right] \left[1 + 0.28 \exp\left(-0.007 \frac{\Pr}{\Pr_t} \right) \right]$$

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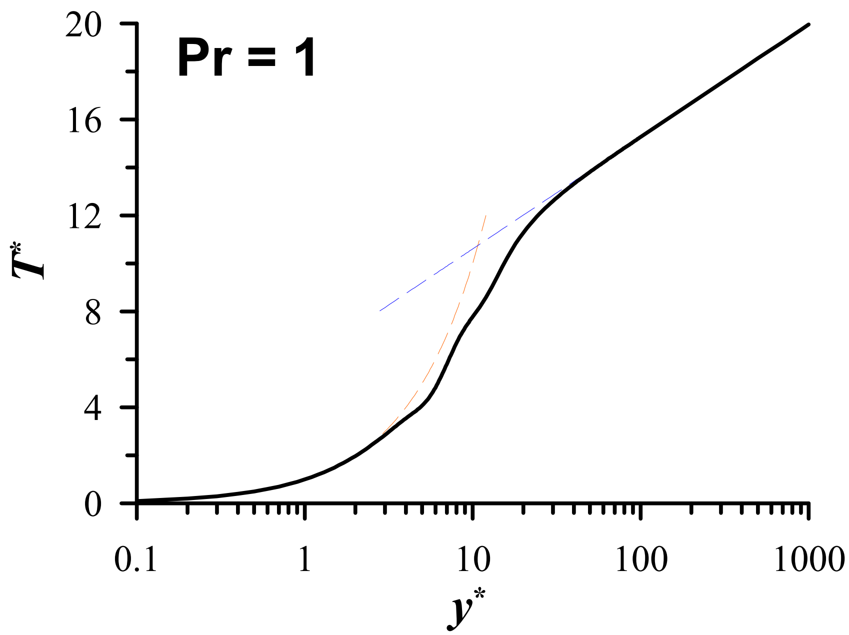
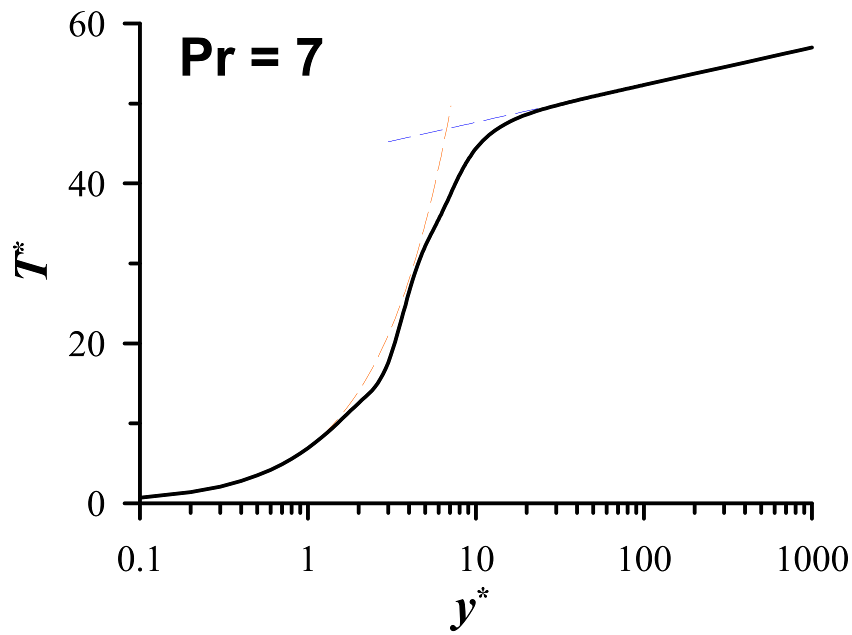
Turbulent Thermal Boundary Layers

- **The wall laws are also functions of Prandtl number.**
- **Viscous sublayer thickness defined as the intersection between viscous and logarithmic law.**

y^+ ~ 10 for Momentum and for Pr = 1

$$y^+ = y_T^+ = f(\Pr, \Pr_T)$$





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