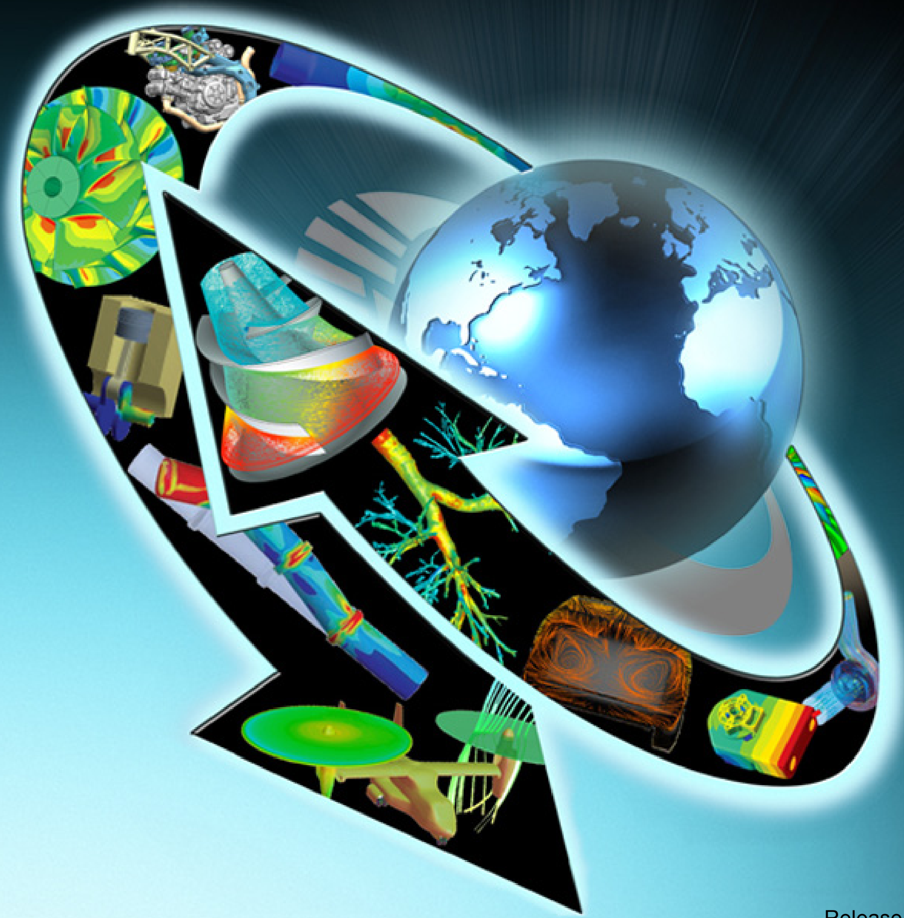
Customer Training Material

Lecture 8

Heat Transfer in Porous Media

Heat Transfer Modeling using ANSYS FLUENT



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Agenda

- **Introduction**
- **Porous Media Characterization**
- **The Representative Elementary Volume (REV) Concept**
- **Theory**
- **The Standard Approach Used in FLUENT**
- **Non-Equilibrium Heat Transfer – Two-Equation Model**
- **Conclusions**

Introduction

- Industrial examples
  - Fuel cells
  - Catalytic converters
  - Filters
  - Food products
  - Tobacco products
- Like in all multiphase or heterogeneous systems, transport phenomena are important.
- The focus of this presentation is mainly heat transfer by convection
  - Introduction to the elementary representative volume concept
  - Governing equations
  - Closure model
  - Two models for predicting porous media heat transfer
    - One-equation model (local thermal equilibrium model)
    - Two-equation model (non-equilibrium model)

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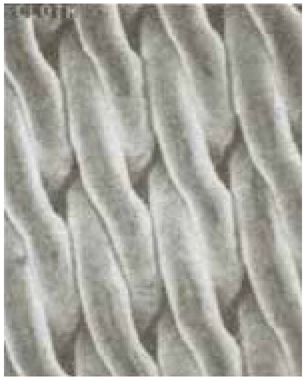
Porous Media Characterization

- **Definition**
  - **Porous media is a solid with many complex pores.**
  - **Shape and connectivity of the pores are important.**
- **Porous media exhibits large diversity in**
  - **Structure (shape)**
  - **Nature**
  - **Spatial scales**



Steel Filter
(Ordered Structure)

Steel Fiber Porous Match
(Random Structure)

Sand Pack
(Random Structure)

Porous Media Characterization

- **Transfer phenomena in porous media strongly depend on solid matrix geometry.**

- **Characteristic variables for porous media**
  - **Porosity (defined as the ratio of void volume to total volume)**
    - **Global porosity – percentage of pore volume or void space, or that volume within the porous region that can contain fluids**

$$\gamma = 1 - \frac{\text{Global Volume}}{\text{Solid Matrix Volume}}$$

    - **Effective porosity – the interconnected pore volume or void space in a rock that contributes to fluid flow. Effective porosity excludes isolated pores and is therefore typically less than the global porosity.**

$$\gamma_{\text{eff}} = \frac{\text{Volume of Active Pore Spaces}}{\text{Sample Volume}}$$

- **Specific surface area – Ratio of interfacial area to specific volume**

$$A_S = \frac{\text{Fluid - Solid Interfacial Area}}{\text{Sample Volume}}$$

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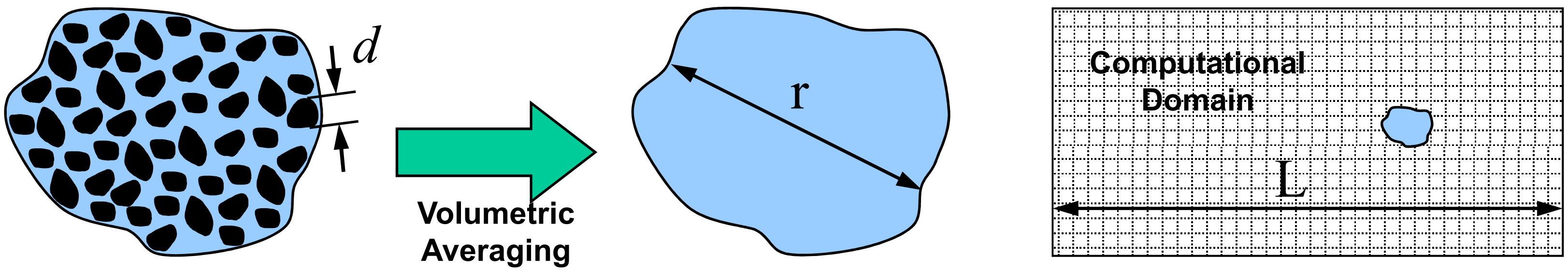
Representative Elementary Volume

- **Geometric problem**
  - **Two largely different length scales**
    - **Pore or grain scale, d**
    - **Porous media length scale, L >> d**
  - **In general, solving the flow field at the pore scale is impractical**

- **Can we describe the flow field at a larger, more practical scale?**
  - **Concept of upscaling**
  - **Method of volumetric averaging**

Representative Elementary Volume

- Representative elementary volume
  - Large enough to characterize the material
  - Small enough to maintain spatial description



Pore scale　　REV scale　　Local Discretization

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Assumptions and Limitations

- **The porous media model in FLUENT introduces flow resistance parameters that can be obtained either analytically or empirically.**
  - **Porous zones remain fluid type.**
  - **Sink terms are included in the momentum equations which account for the resistance forces of solid materials onto the fluid.**

- **The porous media treatment is subject to the following assumptions and limitations:**
  - **The volume blockage that is physically present is not modeled. Instead, a superficial velocity is calculated which represents the fluid velocity through the porous zone.**
  - **Interaction between porous media and turbulence is approximated.**

- **Other limitations apply. Refer to Chapter 7 of the FLUENT 6.3 User Guide.**

Theory

- **In porous zones, the continuity equation remains unchanged, except that it is formulated in terms of superficial velocity.**

- **Momentum equation contains an additional body force term, F**

$$\frac{\partial}{\partial t}(\varepsilon \rho_f) + \nabla \cdot (\rho_f \mathbf{U}) = 0$$

- **For homogenous porous media:**

$$\frac{\partial(\rho_f \gamma \mathbf{U})}{\partial t} + \nabla \cdot (\rho_f \mathbf{U}\mathbf{U}) = -\nabla P + \nabla \cdot \bar{\bar{\boldsymbol{\tau}}} + \rho \mathbf{g} + \mathbf{F}$$

$$F_i = \sum_j -D_{ij} \mu U_j + \sum_j C_{ij} \frac{\rho \|\mathbf{U}\| U}{2}$$

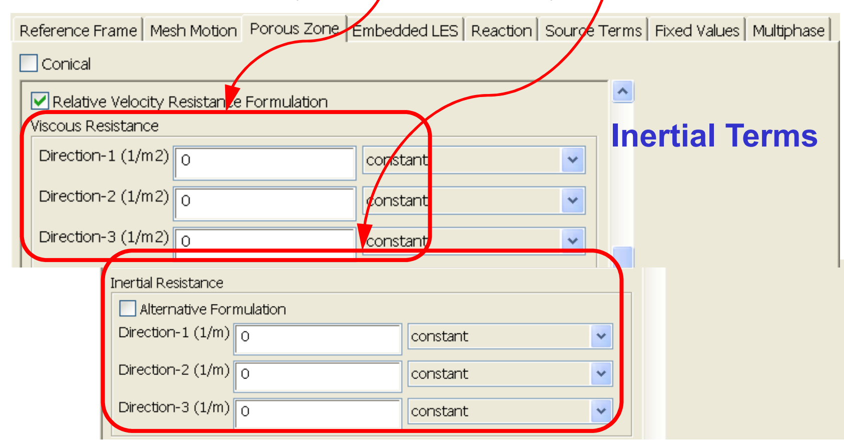
Viscous Terms

Inertial Terms

$$\mathbf{F} = -\left(\frac{\mu \mathbf{U}}{\alpha} + C_2 \frac{\rho \|\mathbf{U}\| \mathbf{U}}{2} \right)$$

Permeability

Inertial resistance factor



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Standard Approach Used in FLUENT

- **FLUENT uses the local equilibrium (one-equation) approach. This roughly means that the local fluid temperature is approximately equal to the porous matrix temperature.**

- **Range of application:**
  - **Situations where there is local heat balance in porous media**
  - **Limited to partial anisotropy**

- **Energy equation**

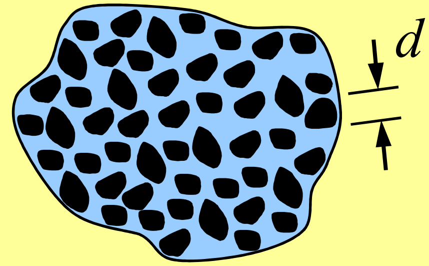
$$\underbrace{(\rho C_p)_{\mathrm{pm}}}_{(\rho C_p)_s (1-\gamma) + (\rho C_p)_f \gamma} \left(\frac{\partial T_{\mathrm{pm}}}{\partial t} + \mathbf{U} \cdot \nabla T_{\mathrm{pm}} \right) = \nabla \cdot \left(k_{\mathrm{eff}} \nabla T_{\mathrm{pm}} \right)$$

- **How do we determine the effective conductivity?**

Local Equilibrium (One Equation Model)

Fluid part: $$\left(\rho C_p\right)_f \left(\frac{\partial T_f}{\partial t} + \mathbf{U} \cdot \nabla T_f\right) = \nabla \cdot \left(k_f \, \nabla T_f\right)$$

Solid part: $$\left(\rho C_p\right)_s \frac{\partial T_s}{\partial t} = \nabla \cdot \left(k_s \, \nabla T_s\right)$$

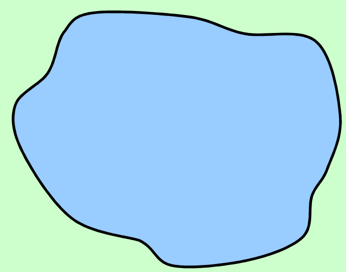


Pore Scale, d

Upscaling (Phase Averaging)

$$\underbrace{\left(\rho C_p\right)_{pm}}_{\left(\rho C_p\right)_s (1-\gamma) + \left(\rho C_p\right)_f \gamma} \left(\frac{\partial T_{pm}}{\partial t} + \mathbf{U} \cdot \nabla T_{pm}\right) = \nabla \cdot \left[\left(\underbrace{k_{pm} + k_d}_{}\right) \nabla T_{pm}\right]$$

Closure model
Real conductivity matrices



REV or Domain Scale

$$k_{pm} = f(k_f, k_s, \gamma, \text{structure})$$

$$k_d = f(\text{microscopic velocity field heterogeneity})$$

Closure Model

- Volumetric averaging introduces effective conductivity matrices.
  - Matrices depend on
    - Thermal conductivity of each void,
    - Material porosity,
    - Solid matrix structures,
    - Thermal dispersion – Consequence of microscopic velocity field heterogeneity
  - Several methods exist to characterize these matrices
    - Experimental
    - Geometric
    - Empirical
    - Local numerical simulation

- Characterization is a complex problem and will not be discussed in this presentation.

Effective Thermal Conductivity

- Isotropic porous media
  - For isotropic porous media, the effective conductivity can be estimated using a porosity-weighted average of fluid and solid parts.

$$k_{\text{eff}} = k_f \, \gamma + k_s \,(1-\gamma)$$

  - UDF is needed in order to simulate spatial dependence.

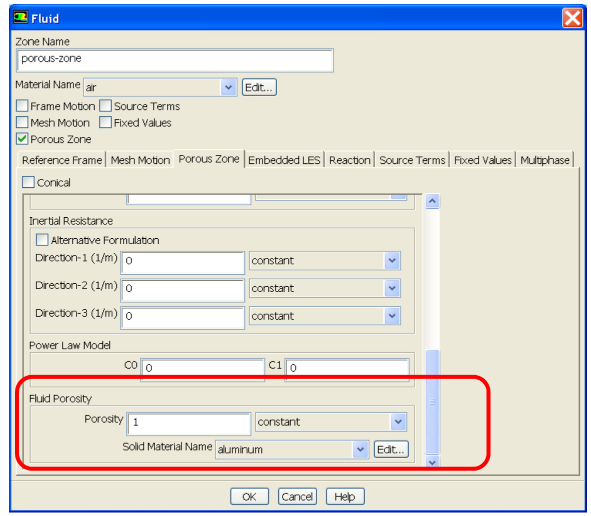
- Anisotropic porous media
  - For anisotropic porous media, effective conductivity can be estimated using porosity-weighted average of fluid scalar conductivity and solid conductivity matrix.
  - Spatial dependence is possible via UDF.

$$[k_{\text{eff}}] = \gamma \underbrace{\begin{bmatrix} k_f & 0 & 0 \\ 0 & k_f & 0 \\ 0 & 0 & k_f \end{bmatrix}}_{\text{Fluid contribution}} + \underbrace{(1-\gamma)\begin{bmatrix} (k_{11})_s & (k_{12})_s & (k_{13})_s \\ (k_{21})_s & (k_{22})_s & (k_{23})_s \\ (k_{31})_s & (k_{32})_s & (k_{33})_s \end{bmatrix}}_{\text{Solid contribution}}$$

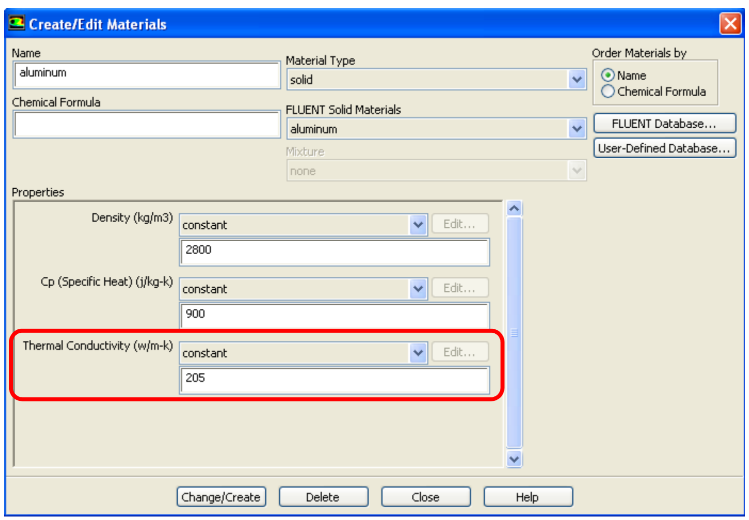
Standard Approach Used in FLUENT

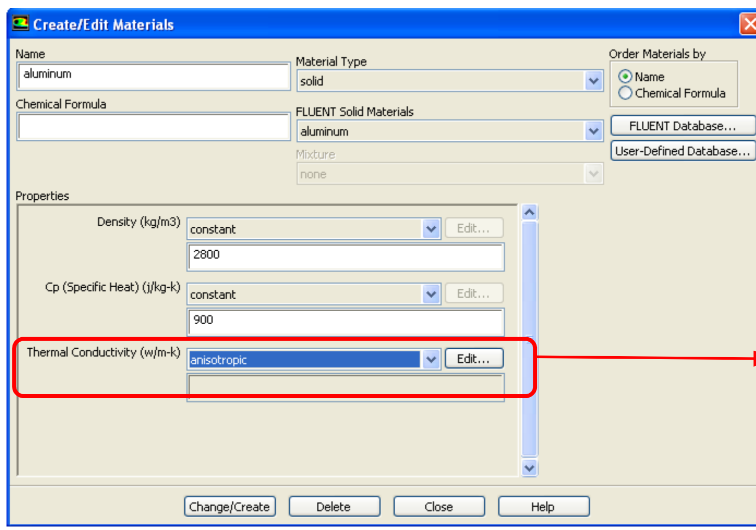
- **User-defined effective conductivity**
  - **Fluids – Scalar value**
  - **Solids – Scalar or matrix value**

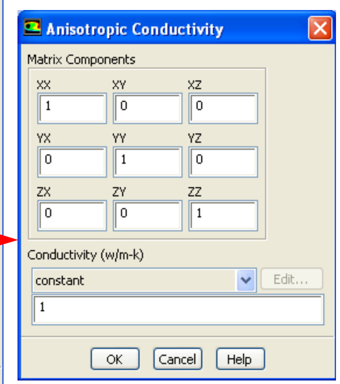
Cell Zone Conditions → Edit...



Materials → Solid → Create/Edit...







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Non-Equilibrium Heat Transfer

- **Range of validity**
  - **Thermal equilibrium is not assumed**
  - **Isotropic heat transfer**

- **Two separate but coupled energy equations are solved.**
  - **Energy equation for the fluid region**
  - **Energy equation for the solid matrix region (requires a user-defined scalar).**

- **In the non-equilibrium case, the system of governing equations require additional closure relationships.**
  - **Fluid thermal dispersion (isotropic)**
  - **Solid thermal diffusion (isotropic)**
  - **Exchange coefficient at fluid-solid interface**

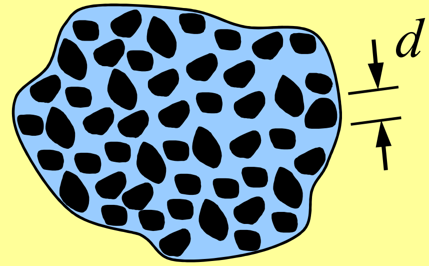
Local Imbalance (Two-Equation Model)

Fluid part:

$$\left(\rho C_p\right)_f\left(\frac{\partial T_f}{\partial t}+\mathbf{U}\cdot\nabla T_f\right)=\nabla\cdot\left(k_f\,\nabla T_f\right)$$

Solid part:

$$\left(\rho C_p\right)_s\frac{\partial T_s}{\partial t}=\nabla\cdot\left(k_s\,\nabla T_s\right)$$



Pore Scale, d

Upscaling (Phase Averaging)

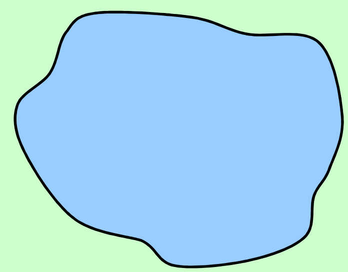
$$\left(\rho C_p\right)_f\left(\frac{\partial T_f}{\partial t}+\mathbf{U}\cdot\nabla T_f\right)=\nabla\cdot\left(k_f\cdot\nabla T_f\right)+\underbrace{h\left(T_s-T_f\right)}_{\text{Coupling}}$$

Closure model

Effective conductivity matrices

Coupling

$$\left(\rho C_p\right)_s\left(1-\gamma\right)\frac{\partial T_s}{\partial t}=\nabla\cdot\left(k_s\cdot\nabla T_s\right)+h\left(T_f-T_s\right)$$

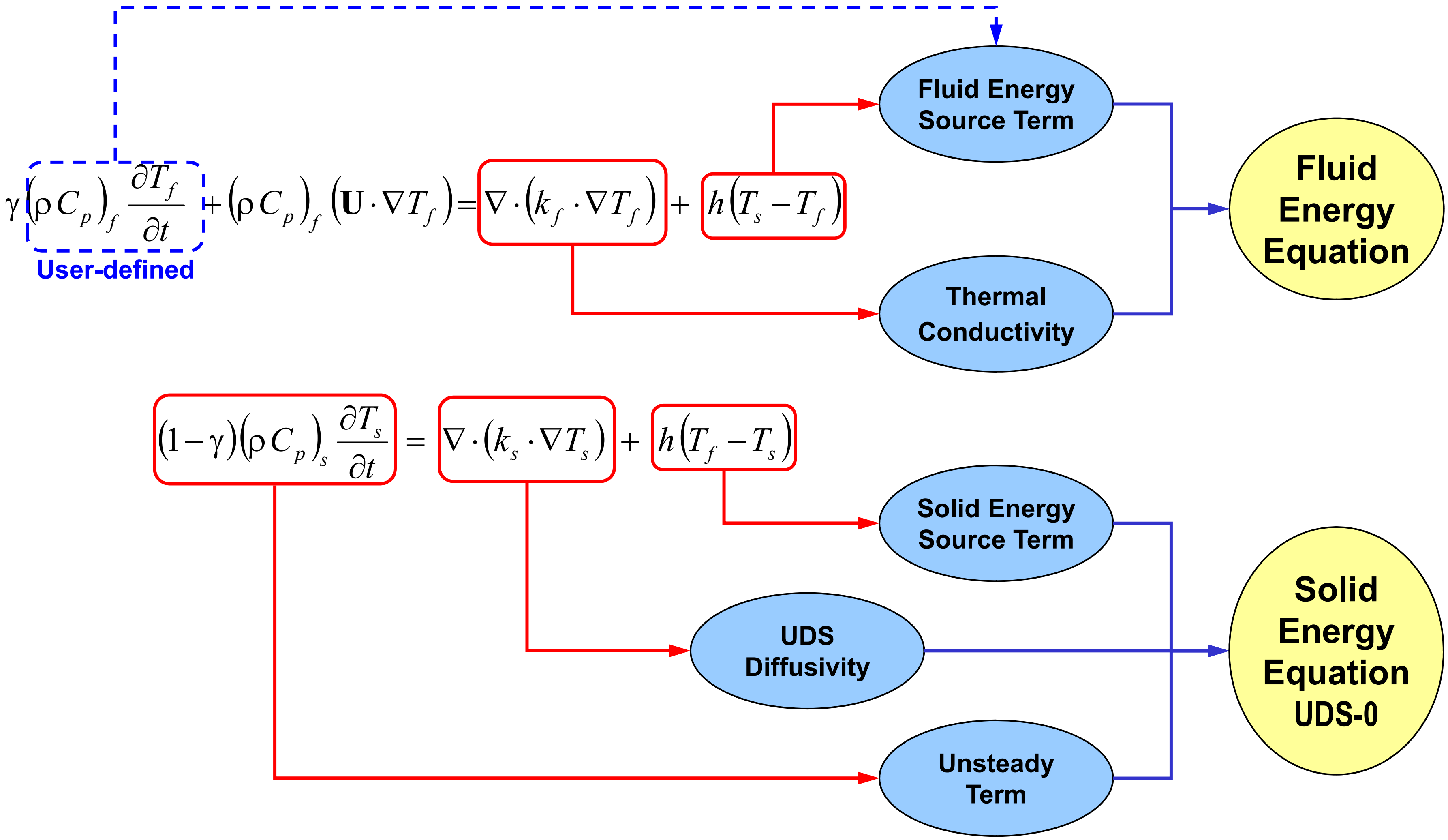


REV or Domain Scale

Non-Equilibrium Heat Transfer

$$\gamma\left(\rho C_p\right)_f \frac{\partial T_f}{\partial t} + \left(\rho C_p\right)_f \left(\mathbf{U}\cdot\nabla T_f\right) = \boxed{\nabla\cdot\left(k_f\cdot\nabla T_f\right)} + \boxed{h\left(T_s - T_f\right)}$$

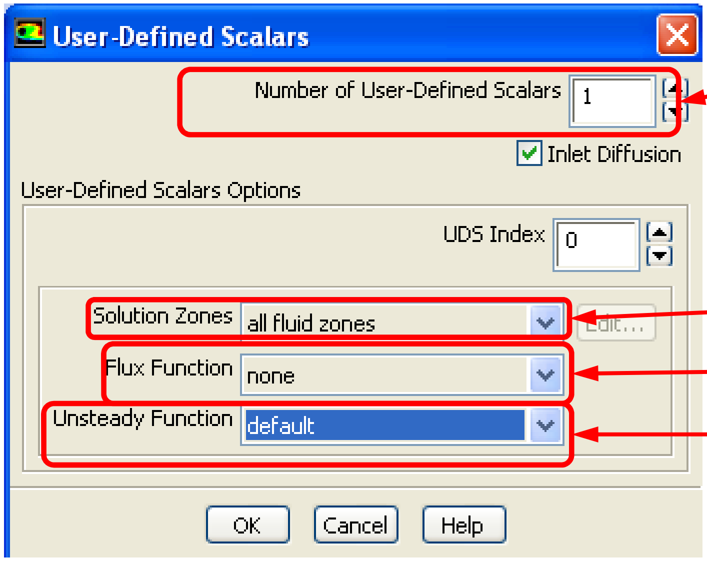
$$\boxed{\left(1-\gamma\right)\left(\rho C_p\right)_s \frac{\partial T_s}{\partial t}} = \boxed{\nabla\cdot\left(k_s\cdot\nabla T_s\right)} + \boxed{h\left(T_f - T_s\right)}$$



Non-Equilibrium Heat Transfer

- **Declaration of User-Defined Scalars (UDS)**

Define → User-Defined → Scalars...



Number of User-Defined Scalars = 1
UDS-0 = Energy equation for solid region

Select only porous fluid zones.

Convective term is neglected.

Standard unsteady term or user-defined unsteady term (option available with unsteady solver only).

Non-Equilibrium Heat Transfer

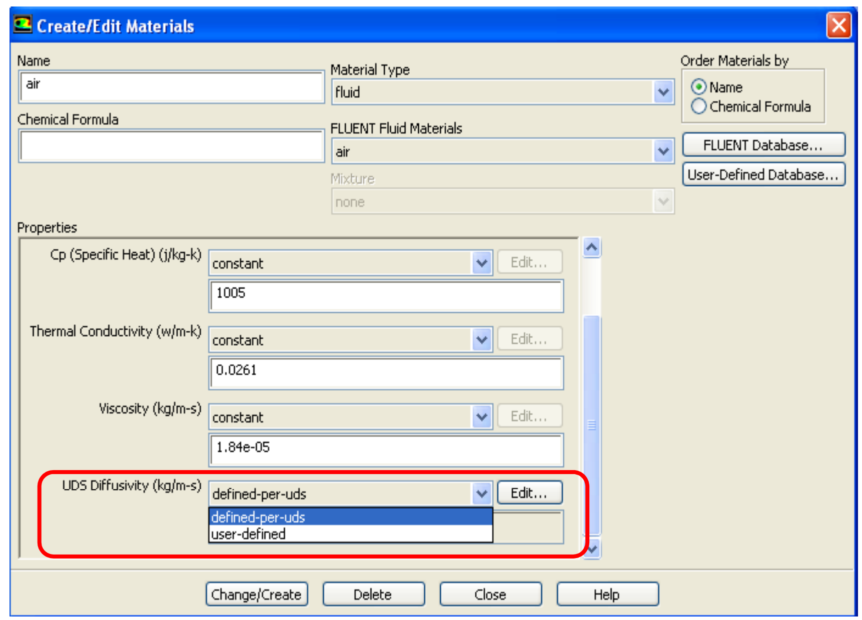
- **Treatment of unsteady term in solid energy equation**
  - **The default unsteady term cannot be used. We must use a user-defined unsteady term. If the scalar we define is**
  - **Then the following temporal discretization and linearization schemes are possible:**
    - **First-order implicit:** $$\phi = (1-\gamma)(\rho C_p)_s T_s$$
    - **Second-order implicit** $$\frac{\partial \phi}{\partial t} = \frac{\phi_n - \phi_{n-1}}{\Delta t}$$
    - **Linearization:** $$\frac{\partial \phi}{\partial t} = \frac{3\,\phi_n - 4\,\phi_{n-1} - \phi_{n-2}}{2\,\Delta t}$$

$$\frac{\partial \phi}{\partial t} = A + B\,\phi$$

Non-Equilibrium Heat Transfer

- **Diffusivity treatment for the solid energy equation**
  - **Scalar equation is solved in all zones.**
  - **The diffusion term characterizes thermal dispersion in porous media**
    - **Must be physical inside the porous media**
    - **Must equal zero outside the porous media**
  - **User-defined diffusivity via UDF**

Materials → Fluid → Create/Edit...



Non-Equilibrium Heat Transfer

- **Treatment of source term characterizing convective exchanges for energy solid equation**
  - **Source term depends on volumetric heat transfer coefficient.**

$$h_V = A_s\, h_s$$

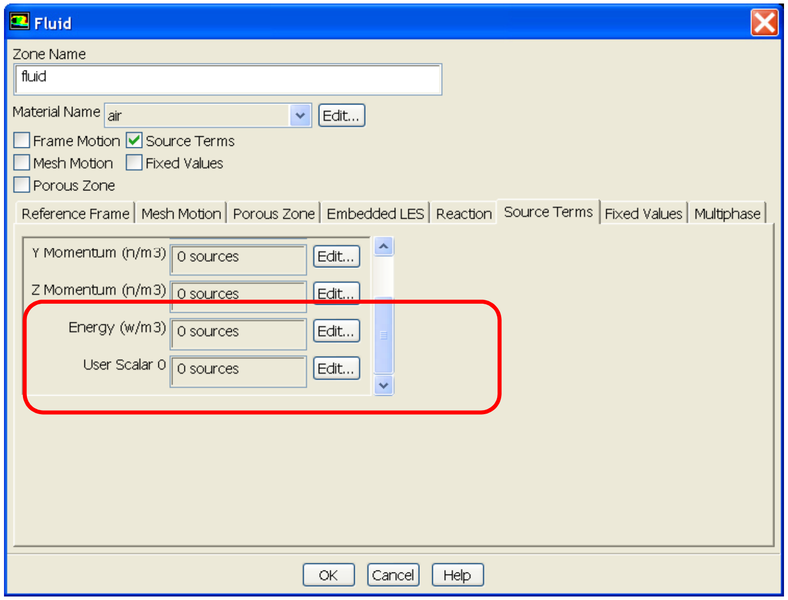
$$A_s = \frac{\text{Interstitial surface area}}{\text{Porous elementary volume}}$$

  - **Source term depends of local solid-fluid temperature differential**

$$S_\phi = h_V\left(T_s - T_f\right)$$

  - **Using source term in porous fluid zone**

Cell Zone Condition → Edit…



Non-Equilibrium Heat Transfer

- Calculation of source term in the fluid energy equation
  - Energy equation solved by FLUENT (default)

$$\left(\rho C_p\right)_f \frac{\partial T_f}{\partial t} + \left(\rho C_p\right)_f \mathbf{U} \cdot \nabla T_f = \nabla \cdot \left(k_f \cdot \nabla T_f\right) + S$$

  - Equation to solve :

$$\gamma \left(\rho C_p\right)_f \frac{\partial T_f}{\partial t} + \left(\rho C_p\right)_f \mathbf{U} \cdot \nabla T_f = \nabla \cdot \left(k_f \cdot \nabla T_f\right) + h_V \left(T_s - T_f\right)$$

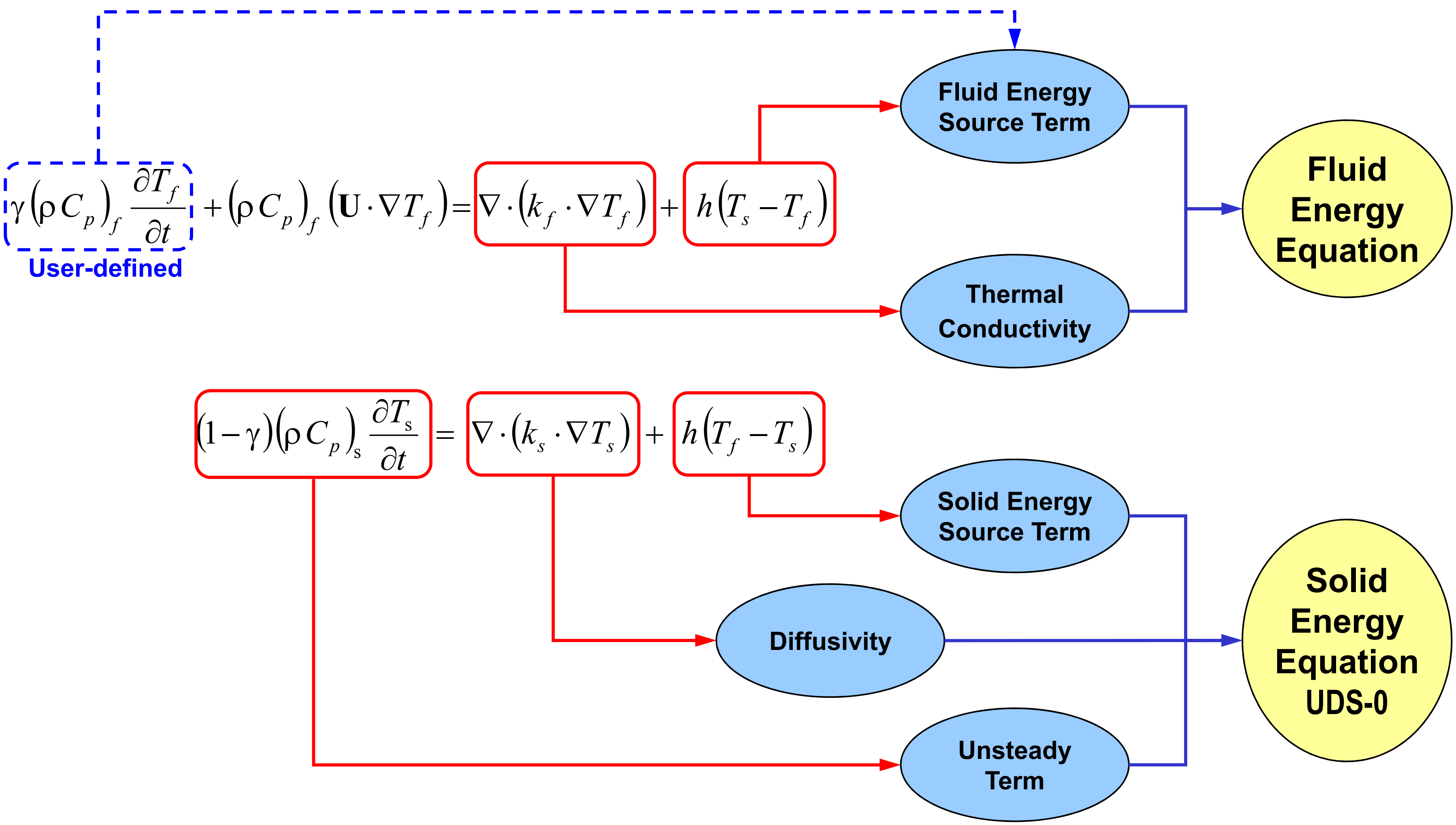
  - Define new equation :

$$\underbrace{\left(\rho C_p\right)_f \frac{\partial T_f}{\partial t}}_{\text{Unsteady}} + \underbrace{\left(\rho C_p\right)_f \mathbf{U} \cdot \nabla T_f}_{\text{Convection}} = \underbrace{\nabla \cdot \left(k_f \cdot \nabla T_f\right)}_{\text{Diffusion}} + \underbrace{h_V \left(T_s - T_f\right) + \left(1 - \gamma\right)\left(\rho C_p\right)_f \frac{\partial T_f}{\partial t}}_{\text{User-Defined Source}}$$

  - The source term is therefore

$$S = h_V \left(T_s - T_f\right) + \left(1 - \gamma\right)\left(\rho C_p\right)_f \frac{\partial T_f}{\partial t}$$

Non-Equilibrium Heat Transfer



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Conclusions

- Convective heat transfer in porous media can be approached in FLUENT using two different methods.
  - One-Equation Model – The standard approach
  - Two-Equation Model – UDF approach

- Closure model remains a complex problem.
  - Effective conductivity characterization
  - Specific heat transfer coefficient in porous media.

- It is possible to introduce a diffusivity matrix in the scalar equations.
  - Presented approach can be generalized either directly or via UDF.
    - Anisotropic thermal conductivity in solid zone (matrix structure effect).
    - Thermal dispersion matrix in fluid zone (velocity field heterogeneity effect)

References

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