Eulerian Model

Advanced Multiphase
Modeling Course

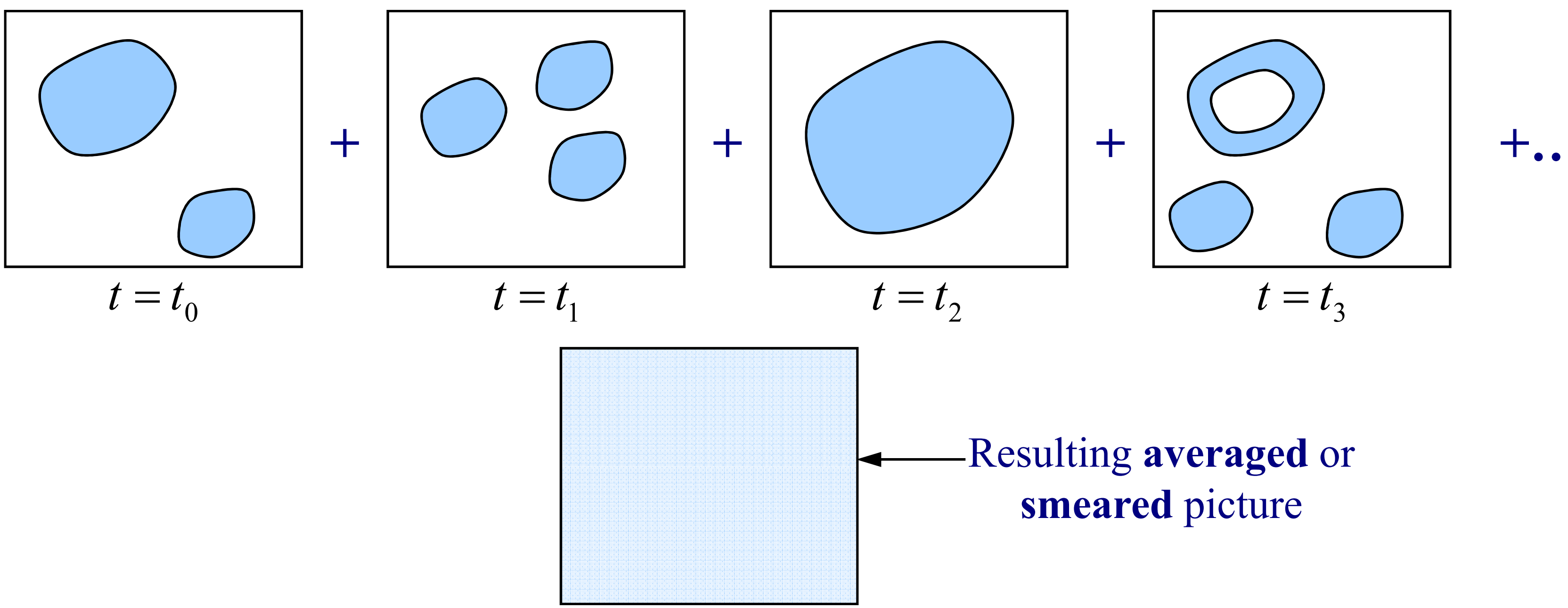
Contents

Eulerian Model – Applicability

- Used to model dispersed phase (solid particles, bubbles, droplets) in continuous (primary) phase (liquid or gas).
- Every phase has its own mass, momentum and energy conservation.
- Conservation equations of different phases are coupled via interfacial terms – these terms are modeled. Closeness of these terms to reality determines accuracy of the model.
- Most of interfacial terms depend on particle size (diameter) and most dispersed flows are not monodispersed so knowledge of representative particle diameter is essential.
- Most interfacial terms are non-linear so convergence of Euler model may be slow.

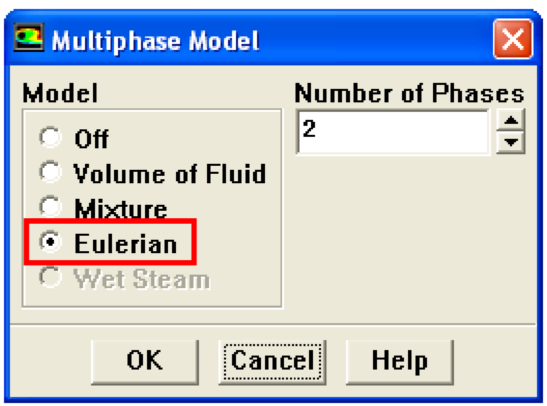
Eulerian Model – Origin

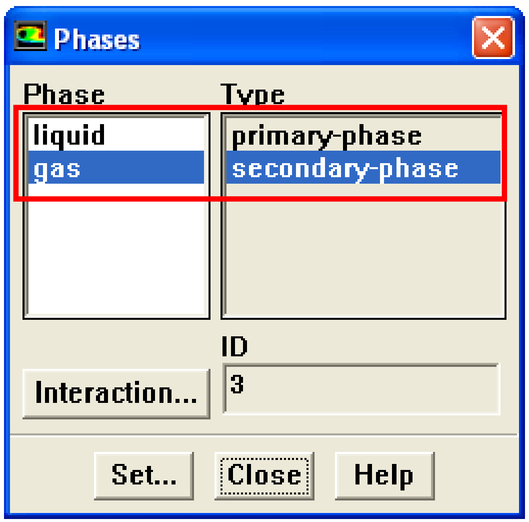
- Results from averaging – exact flow equations near the interface are smeared out.
- This averaging creates a closure problem involving interfacial terms.

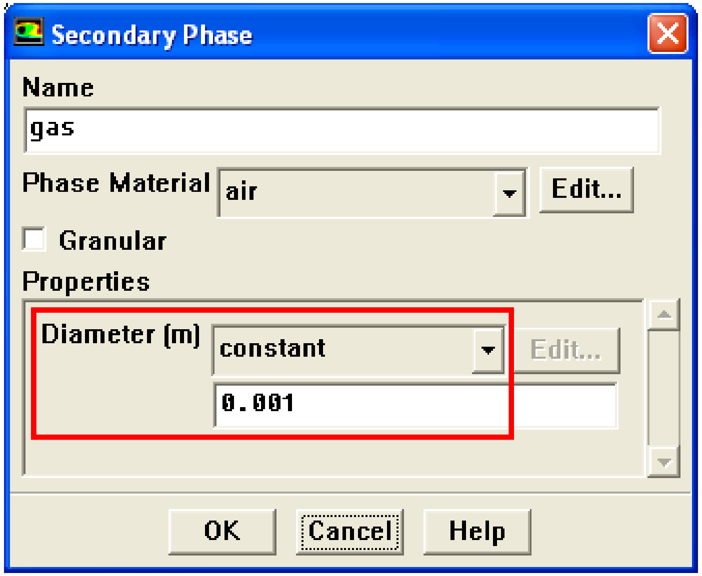


Eulerian Model Inputs

- First phase in the phase list is always primary – this is continuous phase which carries dispersed phases.
- There can be more than one dispersed phase.
- For dispersed phase an input for particle diameter is required – it will be used in calculation of interfacial terms.

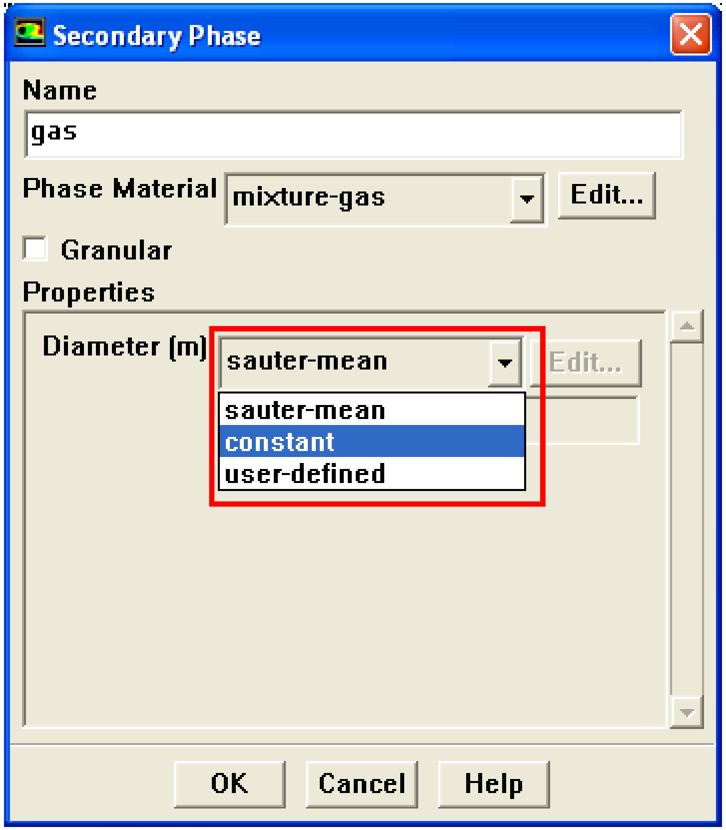






Eulerian Model Inputs

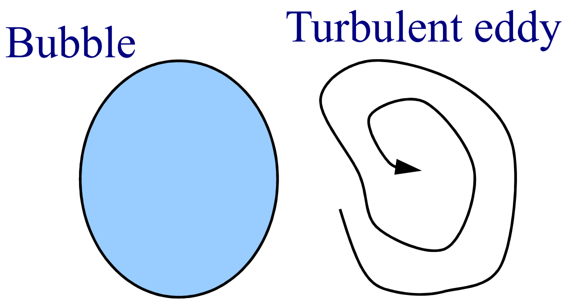
- There are three ways to calculate dispersed phase diameter
  - Constant – if you know representative size of dispersed phase describing your size distribution
  - UDF – if you know some correlation for your particle size as function of local flow parameters (velocity, temperature, pressure)
  - Using Population Balance model (Sauter-mean)



Eulerian Model Inputs

- Example of equilibrium bubble or droplet size in turbulent flow
  - Weber number – ratio of surface tension force and turbulent eddy shear stress:

$$\tau_t = 2\rho_c(\varepsilon d)^{2/3} + \Delta p = \frac{\sigma}{d} \longrightarrow \mathrm{We}_{\mathrm{crit}} = \frac{\tau_t}{\Delta p}$$



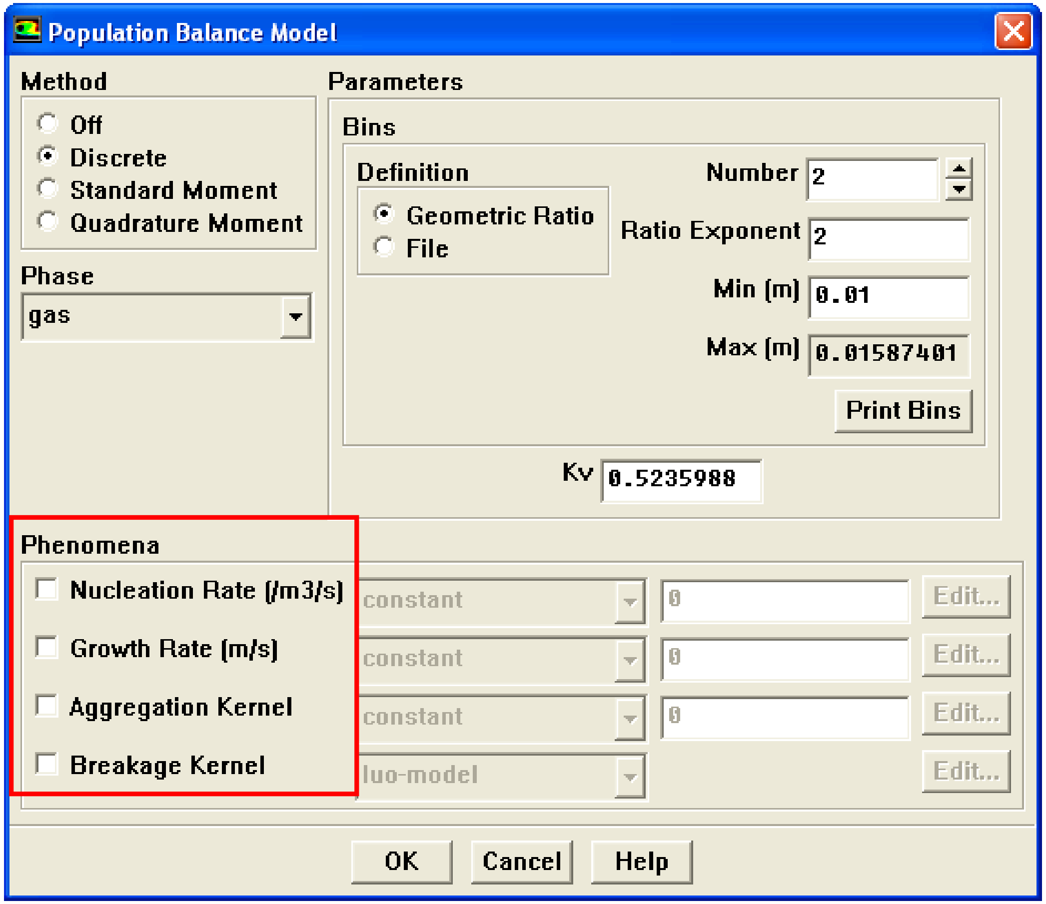
  - Critical Weber number serves as criteria for equilibrium between coalescence and breakup and allows to find equilibrium particle diameter:

$$d = \left(\frac{\sigma\, \mathrm{We}_{\mathrm{crit}}}{\rho_L\, \varepsilon^{2/3}}\right)^{3/5}$$

$\mathrm{We}_{\mathrm{crit}} = 2.48$ for bubbles in liquid
$\mathrm{We}_{\mathrm{crit}} = 1.17$ for droplets in liquid

Eulerian Model Inputs

- Population Balance Model
  – Add-on module to FLUENT. The PBM is the most comprehensive model to simulate particle size distribution.
- Includes solution of additional equations which describe particle size evolution due to coalescence, breakup, nucleation and mass change phenomena.



Eulerian Model Inputs

- Mass conservation equation for phases 1 and 2:

$$\frac{\partial(\rho_1\alpha_1)}{\partial t} + \nabla\cdot(\rho_1\alpha_1\mathbf{v}_1) = S_{\alpha_1} + \dot{m}_{21}$$

$$\frac{\partial(\rho_2\alpha_2)}{\partial t} + \nabla\cdot(\rho_2\,\alpha_2\,\mathbf{v}_2) = S_{\alpha_2} - \dot{m}_{21}$$

- Difference between VOF mass conservation:
  - Eulerian is smoothly varying, while in VOF model VOF has a jump.
  - In Eulerian, each phase has its own velocity field, while in VOF model the velocity field is shared by all phases.

Eulerian Model Inputs

- Momentum conservation equation for phases 1 and 2:

$$\boxed{\frac{\partial(\rho_1\alpha_1\mathbf{v}_1)}{\partial t}+\nabla\cdot(\rho_1\alpha_1\mathbf{v}_1\mathbf{v}_1)}=\boxed{-\alpha_1\nabla p+\alpha_1\rho_1\mathbf{g}}+\nabla\cdot(\alpha_1\,\overline{\overline{\boldsymbol{\tau}}})+\boxed{K_{12}(\mathbf{v}_2-\mathbf{v}_1)}+\mathbf{F}_{12}$$

Single phase terms – known exactly

Drag forces – unknown exactly

$$\boxed{\frac{\partial(\rho_2\alpha_2\mathbf{v}_2)}{\partial t}+\nabla\cdot(\rho_2\alpha_2\mathbf{v}_2\mathbf{v}_2)}=\boxed{-\alpha_2\nabla p+\alpha_2\rho_2\mathbf{g}}+\boxed{\nabla\cdot(\alpha_2\overline{\overline{\boldsymbol{\tau}}})}+\boxed{K_{12}(\mathbf{v}_1-\mathbf{v}_2)}+\boxed{\mathbf{F}_{12}}$$

Turbulent stress and other interfacial forces – unknown exactly

Eulerian Model Inputs

- Drag force is the most important interfacial force
  - Written as exactly known drag on sphere in irrotational flow (Stokes law) times correction to account for real drag

$$K_{12} = \frac{\alpha_1 \alpha_2 \rho_1 f}{\tau_{12}}$$

f (correction)

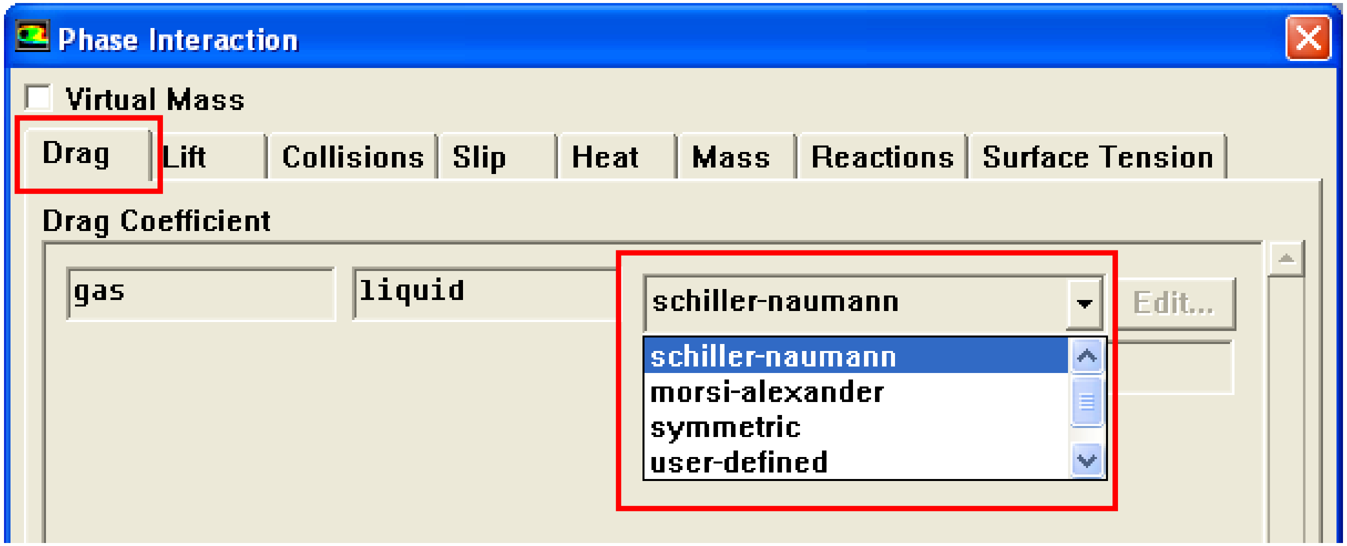
$$\tau_{12} = \frac{\rho_1 d_2^2}{18 \mu_1} = \text{Stokes time scale}$$

  - Correction is a function of particle Reynolds number for spherical particles:

$$\mathrm{Re}_p = \frac{\rho_1 d_2 \|\mathbf{v}_1 - \mathbf{v}_2\|}{\mu_1}$$

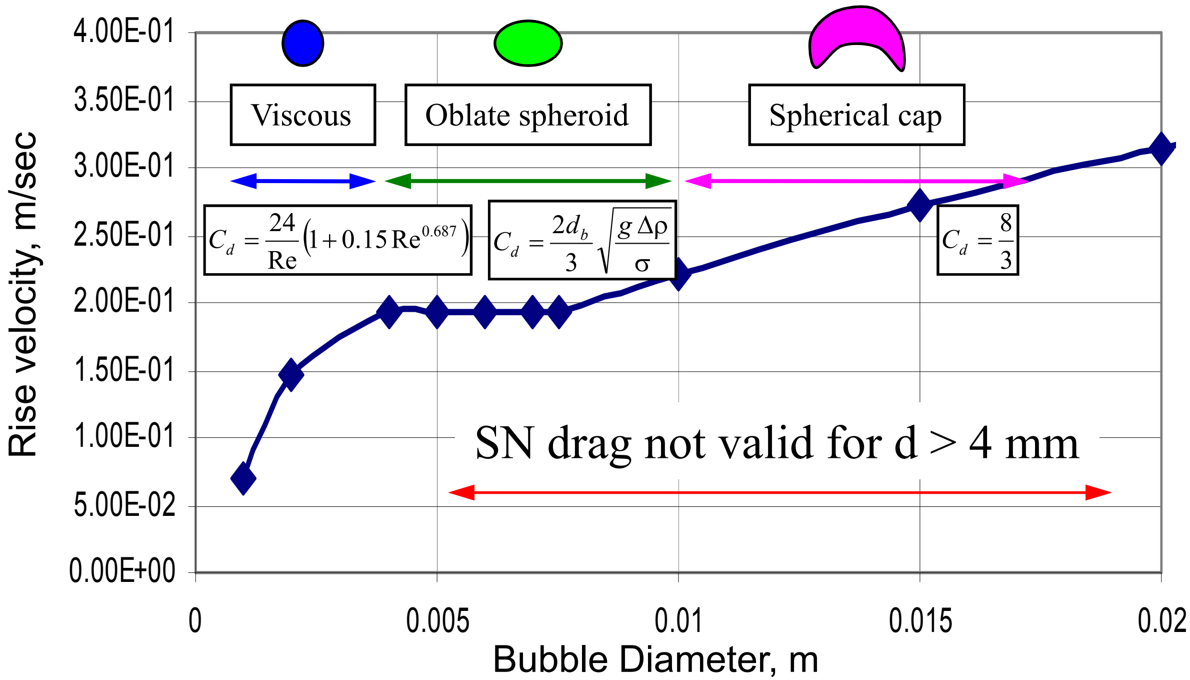
Eulerian Model Inputs

- Drag corrections available in FLUENT
  - **Schiller-Naumann** – valid for all spherical particles
  - **Morsi-Alexander** – Practically close to Schiller-Naumann (not often recommended).
  - **Symmetric** – Approximate extension of Schiller-Naumann for situations where the secondary phase volume fraction can approach a value of 1, i.e., it will become a primary phase in reality



Eulerian Model Inputs

- Cases where the standard FLUENT drag laws are not applicable
  - If particle shape substantially deviates from spherical (large bubbles)
  - If particle concentration becomes high (VOF > 30%)
  - Example of rise velocity of bubbles in liquid and various drag laws is below. These drag laws can be implemented via UDF

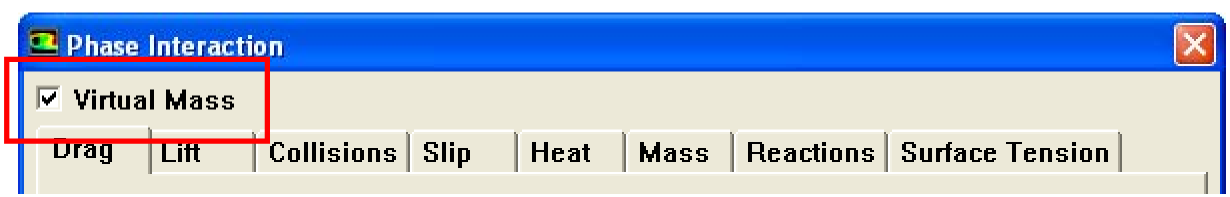


Eulerian Model Inputs

- Virtual mass force
  - Caused by relative acceleration between phases

$$\mathbf{F}_{\mathrm{vm},\,12} = C_{\mathrm{vm}}\,\alpha_2\,\rho_1\left[\left(\frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_1 \cdot \nabla \mathbf{u}_1\right) - \left(\frac{\partial \mathbf{u}_2}{\partial t} + \mathbf{u}_2 \cdot \nabla \mathbf{u}_2\right)\right]$$

  - Practically significant only mesh size is less than length scale of acceleration
  - This situation is very rare – one example might be gas injector in liquid as distance very close to injector, but in reality we rarely need to resolve in Euler model such scales

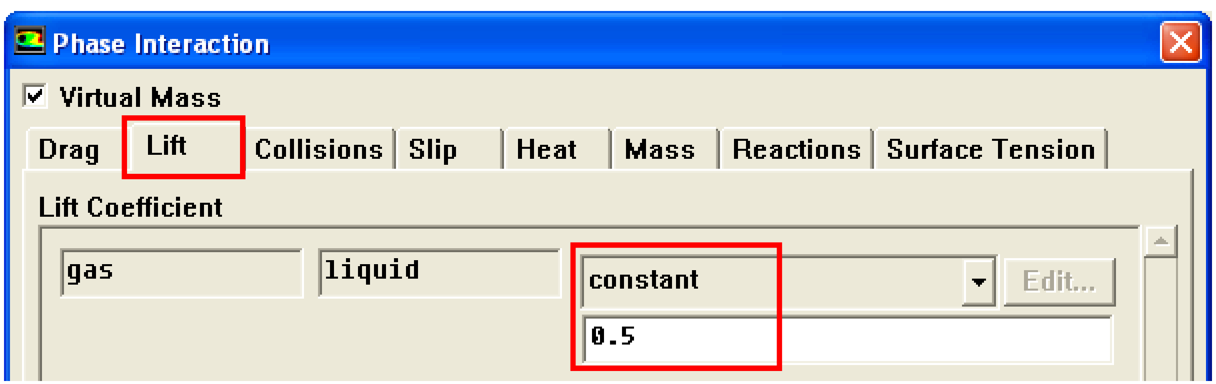


Eulerian Model Inputs

- Lift force
  - Caused by the shearing effect of the fluid onto the particle

$$\mathbf{F}_{12} = C_L\, \alpha_2\, \rho_1 (\mathbf{u}_2 - \mathbf{u}_1) \times \nabla \times \mathbf{u}_1$$

  - Significant only for turbulent bubbly flows in channels – acts near the wall
  - For small bubbles lift coefficient is positive, for large bubbles it may become negative – UDF can be used for general case



Eulerian Model Inputs

- Energy conservation equation for phase 1 and 2:

$$\boxed{\frac{\partial(\rho_1\alpha_1 h_1)}{\partial t} + \nabla\cdot(\rho_1\alpha_1 h_1 \mathbf{v}_1)} = \boxed{-\alpha_1\frac{\partial p}{\partial t} + \nabla\cdot(\alpha_1 \overline{\overline{\boldsymbol{\tau}}}\nabla\mathbf{v}_1) - \nabla\cdot\mathbf{q}_1}$$
$$\boxed{+Q_{12}(T_2 - T_1)} - \boxed{\dot{m}_{12}h_1}$$

Single phase terms – known exactly

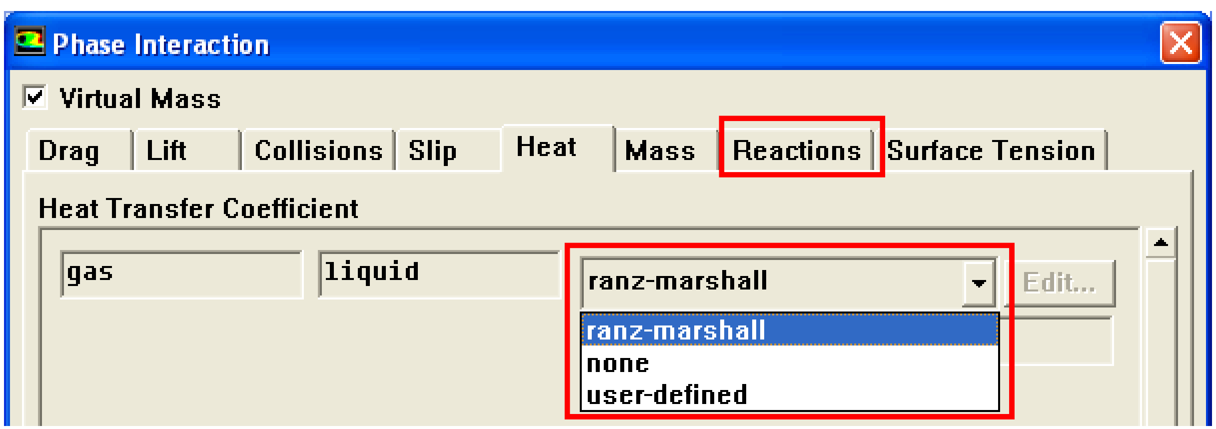
Thermal exchange – unknown exactly

$$\boxed{\frac{\partial(\rho_1\alpha_1 h_1)}{\partial t} + \nabla\cdot(\rho_1\alpha_1 h_1 \mathbf{v}_1)} = \boxed{-\alpha_1\frac{\partial p}{\partial t} + \nabla\cdot(\alpha_1 \overline{\overline{\boldsymbol{\tau}}}\nabla\mathbf{v}_1) - \nabla\cdot\mathbf{q}_1}$$
$$\boxed{+Q_{12}(T_2 - T_1)} + \boxed{\dot{m}_{12}\left(h_1 + h_1^f - h_2^f\right)}$$

Latent heat release – known exactly

Eulerian Model Inputs

- Interfacial mass exchange term
  - Ranz-Marshall correlation is valid for heat exchange between spherical particle and continuous phase in contact with this particle.
  - Not valid for heat transfer between secondary phases – interparticle heat transfer.
  - Ranz-Marshall is functionally similar to Schiller-Naumann.



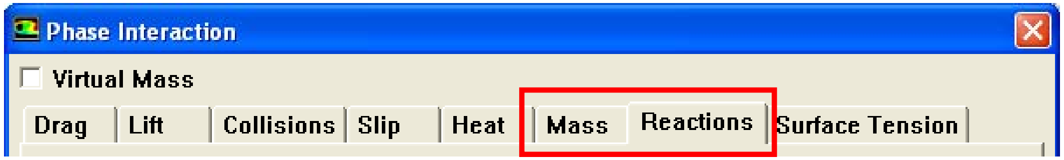
$$Q_{12} = h_{12}\left(T_1 - T_2\right)$$

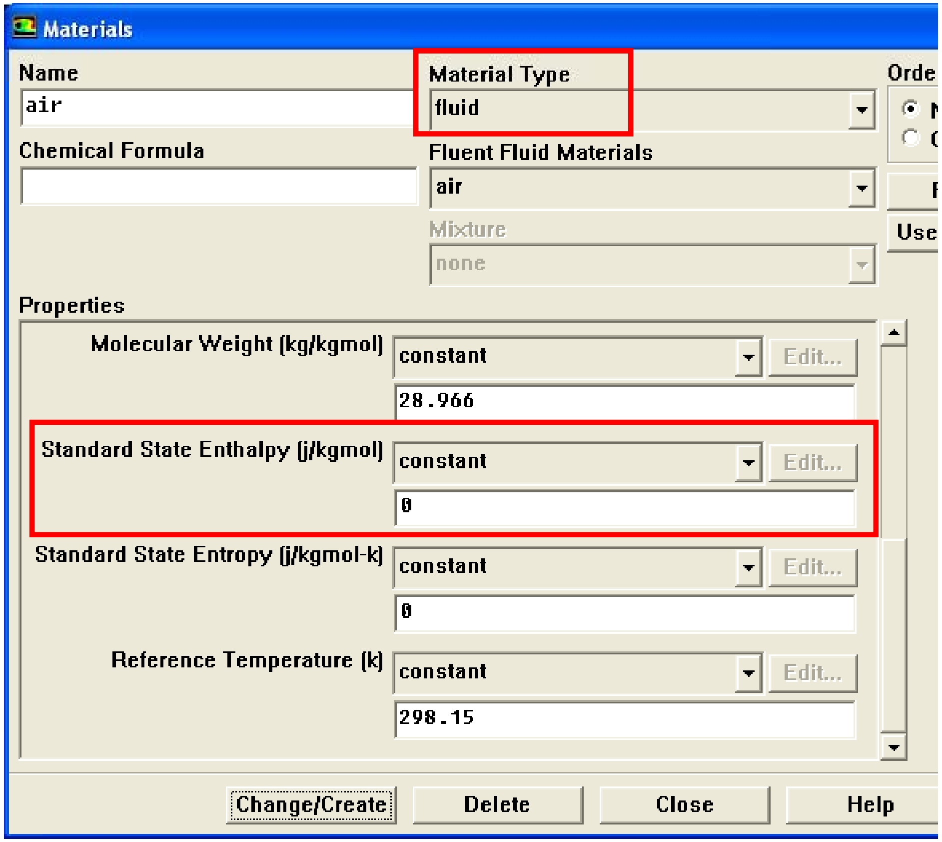
$$h_{12} = \frac{6\,\kappa_1 \alpha_1 \alpha_2 \,\mathrm{Nu}_p}{d_2^2}$$

$$\mathrm{Nu}_p = 2.0 + 0.6\,\mathrm{Re}_p^{1/2}\,\mathrm{Pr}_{\mathrm{prim}}^{1/3}$$

Eulerian Model Inputs

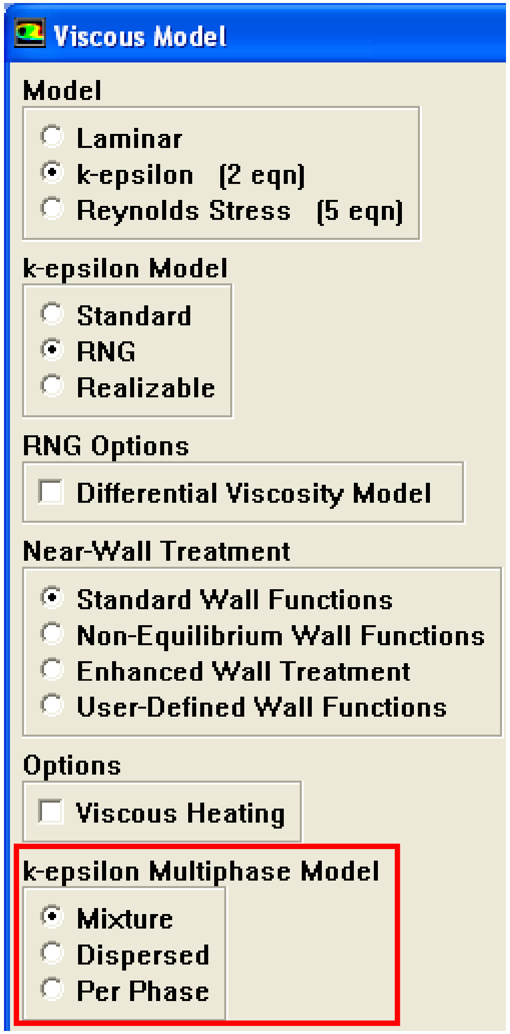
- Latent heat is accounted for when mass transfer is prescribed through standard means.
- Latent heat is calculated from standard state enthalpy of species/phase participating in mass exchange.
- Material type must be fluid
- Be aware of values of standard state enthalpy – only enthalpy difference matters. For example, vapor enthalpy must be larger than liquid enthalpy.





Eulerian Model Inputs

- The majority of multiphase flows are turbulent.
- Common approach – turbulence in primary phase can be modeled by single phase model (k–ε, RSM, etc.)
- More advanced – turbulence in primary phase can be modified by particles
  - Three incarnations of single phase turbulence models with Eulerian multiphase:
    - Mixture – Assume that all phases flow like a single fluid (mixture) and turbulence is shared.
    - Dispersed – Turbulence is in primary phase only.
    - Per phase – Never used (for development only)



Eulerian Model Inputs

- Mixture flow field and Mixture material properties are constructed from phase flow fields and phase material properties
- Turbulence equations are solved for Mixture fluid just like in single phase – although primitive, this approach yields reasonable agreement with experiments.
- Dispersed model equations have possibility to include influence of particles (text command, see manual) – fluctuating part of the drag force.

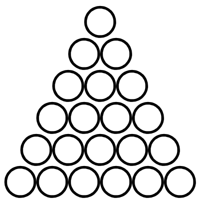
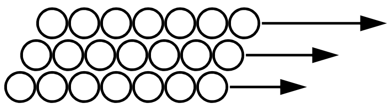
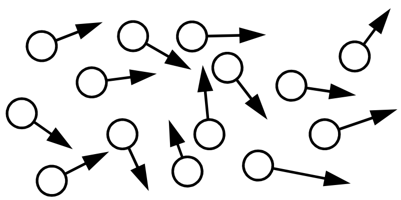
$$\frac{\partial(\alpha_k \rho_k k_k)}{\partial t_k} + \nabla \cdot (\alpha_k \rho_k \vec{u}_k k_k) = \nabla \cdot \left(\frac{\mu_k^t}{\sigma_k} \nabla k_k \right) + G_k - \alpha_k \rho_k \varepsilon_k + \Pi_{k_k}$$

$$\frac{\partial(\alpha_k \rho_k \varepsilon_k)}{\partial t_k} + \nabla \cdot (\alpha_k \rho_k \vec{u}_k \varepsilon_k) = \nabla \cdot \left(\frac{\mu_k^t}{\sigma_\varepsilon} \nabla \varepsilon_k \right) + \frac{\varepsilon_k}{k_k} \left(C_{\varepsilon_1} G_k - C_{\varepsilon_2} \alpha_k \rho_k \varepsilon_k \right) + \Pi_{\varepsilon_k}$$

$$\Pi_k = \alpha_k \rho_k \frac{f_{\text{drag}}}{\tau_{ki}} (k_{ik} - 2k_k) \qquad \Pi_{\varepsilon_k} = c_{\varepsilon 3} \frac{\varepsilon_k}{k_k} \Pi_{k_k}$$

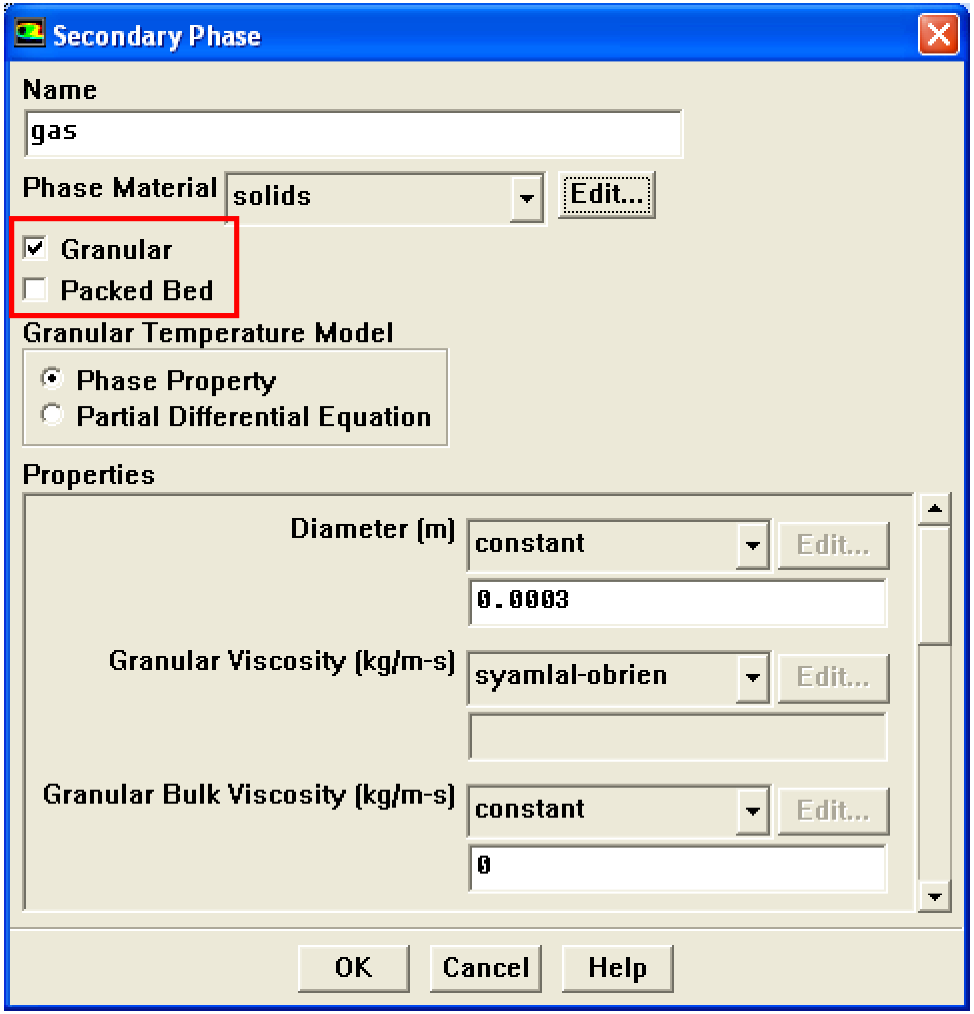
Eulerian Model Inputs

- Granular model – based on kinetic theory, cannot model solid mechanics, can model packed beds and transition between fluidized and packed bed

Elastic Regime	Plastic Regime	Viscous Regime
Stagnant	Slow flow	Rapid flow
Stress is strain dependent	Stress is strain independent	Stress is strain dependent
Elasticity	Solid mechanics	Kinetic theory
		

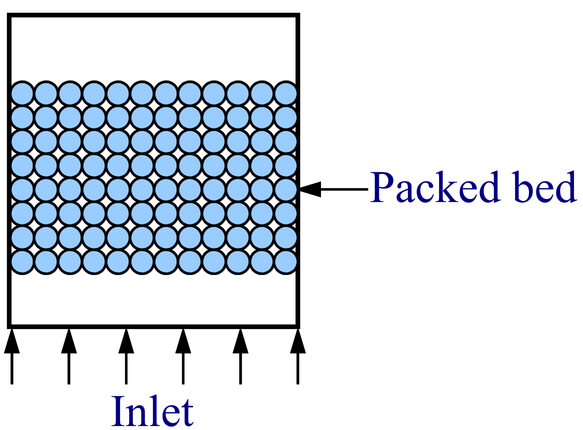
Eulerian Model Inputs

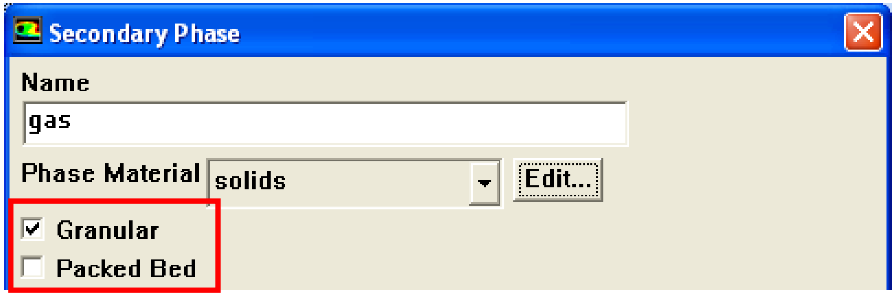
- Granular model is a result of non-ideal gas theory.
  - Colliding molecules are replaced by colliding particles.
  - After averaging, additional solid stress in momentum equation models particle collisions.
- Several options may be available for inputs – we need to understand the difference.
- Convergence issues may arise if the user does not understand what the inputs mean.

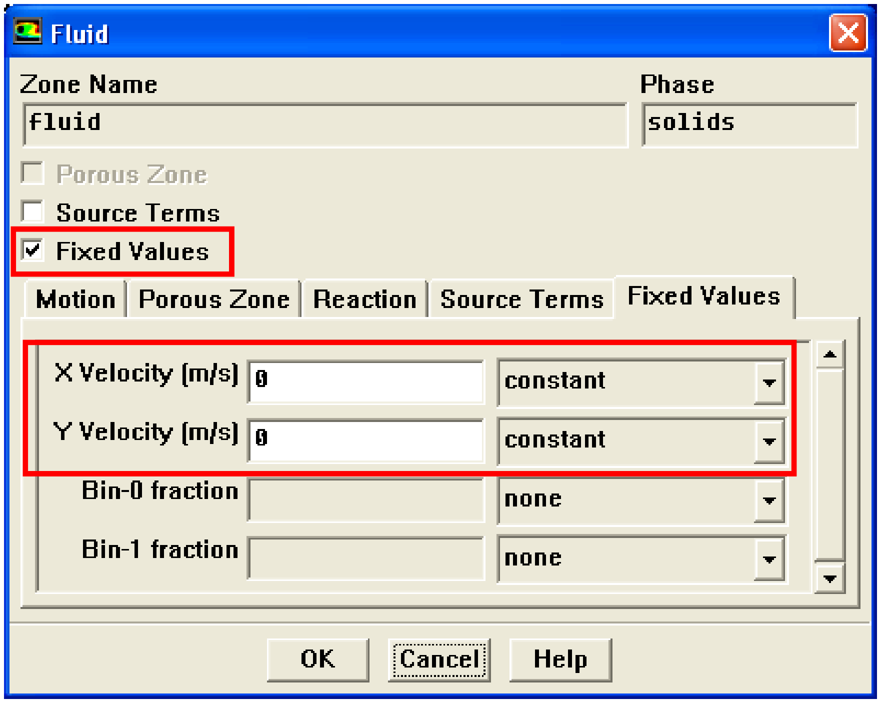


Eulerian Model Inputs

- Packed Bed – assumes that particles are tightly packed and do not move, i.e., their velocity is zero
  - Do not forget to fix the secondary phase velocity to zero in the entire domain!
  - This is the best PHYSICAL representation of packed beds (better than porous media).







Eulerian Model Inputs

- Solid stress inputs – pressure and viscosities in the solid solid stress
- Solid stress – result of non-ideal gas theory applied to fluidized beds

$$\frac{\partial(\alpha_s \rho_s \mathbf{u}_s)}{\partial t} + \nabla \cdot (\alpha_s \rho_s \mathbf{u}_s \mathbf{u}_s) = -\alpha_s \nabla p_f + \boxed{\nabla \cdot \overline{\boldsymbol{\tau}}_s} + \sum_{s=1}^{n} \left(\mathbf{R}_{fs} + \dot{m}_{fs} \mathbf{u}_{fs} \right) + \mathbf{F}_s$$

$$\overline{\boldsymbol{\tau}}_s = -P_s \mathbf{I} + 2\alpha_s \mu_s \mathbf{S} + \alpha_s \left(\lambda_s - \frac{2}{3}\mu_s \right) \nabla \cdot \mathbf{u}_s \mathbf{I}$$

$$\mathbf{S} = \tfrac{1}{2}\left[\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T \right] = \text{Strain rate}$$

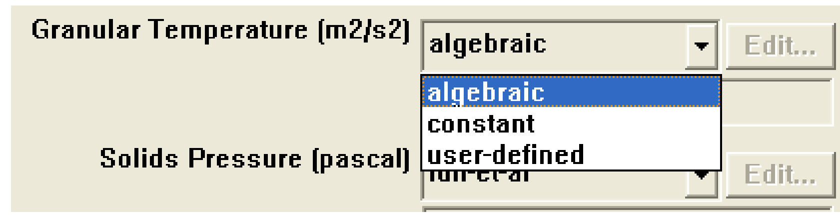
$$P_s = \text{Solids Pressure}$$

$$g_o = \text{Radial distribution function}$$

$$\lambda_s, \mu_s = \text{Solids bulk and shear viscosity}$$

Eulerian Model Inputs

- Granular temperature (GT) model
  - GT – RMS of particle velocity fluctuations
  - Separate transport equation is needed to calculate GT.
  - All components of solid stress include GT
  - Phase Property means that the differential equation will be either reduced to an algebraic equation or made constant by dropping convection and diffusion terms (this may overpredict GT).
  - Partial Differential Equation – full equation solved.



$$\frac{3}{2}\left[\frac{\partial(\alpha_s\rho_s\theta_s)}{\partial t}+\nabla\cdot(\alpha_s\rho_s\mathbf{u}_s\theta_s)\right]=\overbrace{\boxed{\bar{\bar{\boldsymbol{\tau}}}_s:\nabla\mathbf{u}_s}}^{\text{Production}}$$

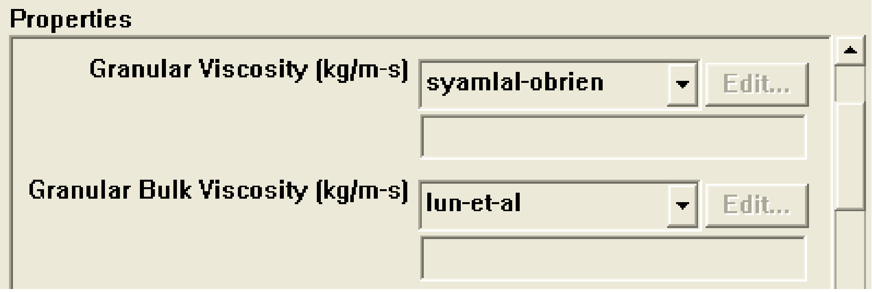
$$+\underbrace{\boxed{\nabla\cdot(\kappa_{\theta s}\nabla\theta_s)}}_{\text{Diffusion}}-\underbrace{\boxed{\gamma_s}}_{\text{Dissipation}}+\overbrace{\boxed{\phi_{lm}+\phi_{fs}}}^{\text{Exchange}}$$

Eulerian Model Inputs

- Solid Stress inputs
  - Granular Viscosity – Kinetic part of total granular viscosity. Represents contribution from free flying particles. The default is Syamlal-O'Brien

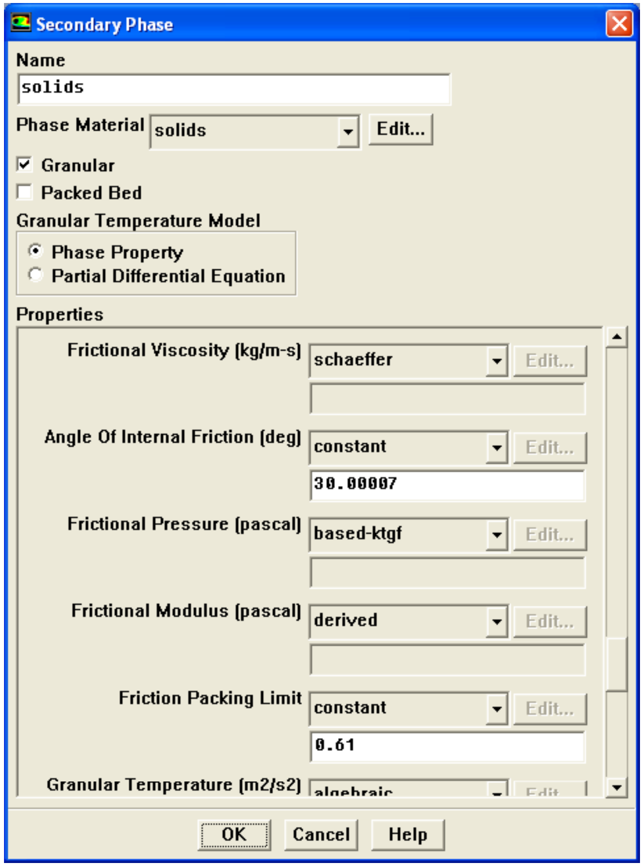
$$\mu_s = \mu_{s\,(\text{collision})} + \boxed{\mu_{s\,(\text{kinetic})}} + \mu_{s\,(\text{friction})}$$

  - Gidaspow option should be used for non-dilute flows only. It has incorrect limiting behavior when granular volume fraction approaches zero.
  - Granular Bulk Viscosity – bulk viscosity representing resistance of particles to compression/expansion. Default is zero which is incorrect. Lun et al. formulation should be used instead.

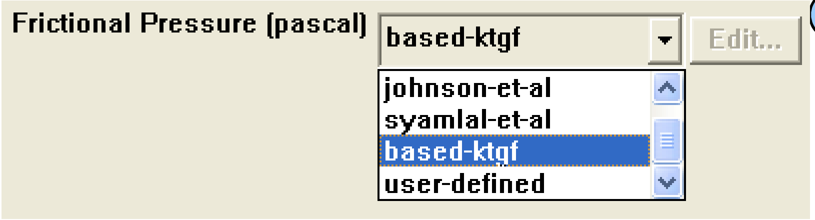


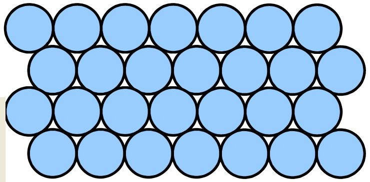
Eulerian Model Inputs

- Solid Stress inputs
  - Frictional Viscosity – becomes important when volume fraction of granular phase approaches the packing limit. In this case, particles do not collide much but rather rub against each other. Packing limit $\alpha_{s,\,max}$ is defined as the maximum achievable volume fraction of the granular phase. For packed spheres it is 0.63. Schaeffer's expression is used which requires frictional angle definition.
  - Frictional Pressure – near packing limit frictional pressure is important. ktfg – derived from kinetic theory, Johnson et al. and Syamlal et al.



$$p_s = p_{\text{kinetic}} + \boxed{p_{\text{friction}}}$$

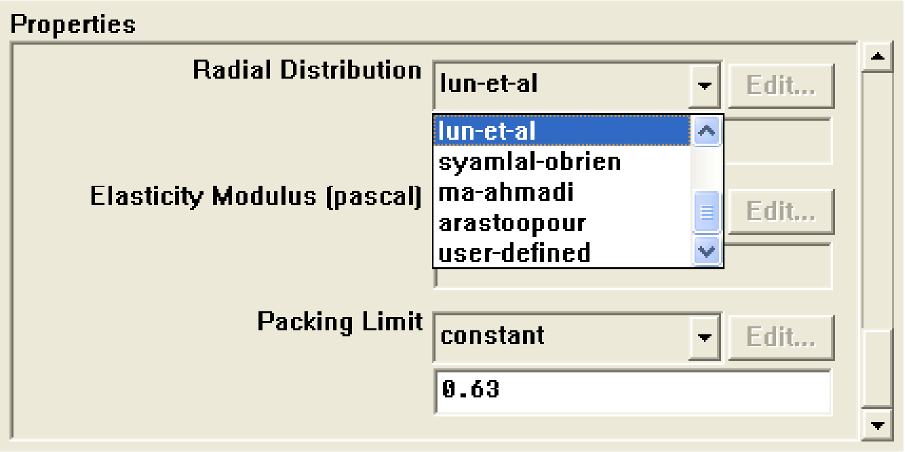




$$\alpha_{s,\max} = 0.63$$

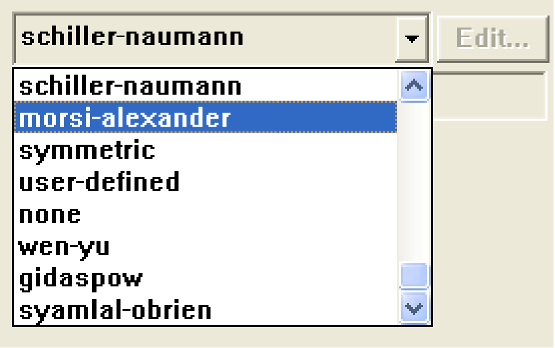
Eulerian Model Inputs

- Solid Stress inputs
  - Frictional Modulus – derivative, $G = \partial p_{fric} / \partial \alpha_{fric} > 0$ determines dynamics of pressure near the packing limit. Important for convergence. When VOF of granular phase approaches the packing limit, solid stresses become very large. This happens because solid stress is proportional to the radial distribution function which determines the probability of collision. This function becomes very large at packing limit, so the solid pressure becomes large too and it reduces solid velocity to near zero value – thus physically correct transition between fluidized and packed bed happens.
  - No exact equation is available for radial function. Several options are available from different authors (see 23.5.5 of Manual)



Eulerian Model Inputs

- Liquid-particle drag force
  - Granular flows are dense so drag coefficients are based on single particle drag + concentration effect
  - Syamlal and O'Brien (1989)
    - Based on correlation for minimum fluidization velocity
  - Wen and Yu (1966)
    - Extension of Schiller-Nauman drag law on high particle concentration case
  - Gidaspow (1992)
    - Uses Wen and Yu for dilute flows, turns into Ergun law for fixed bed for concentration close to packing limit



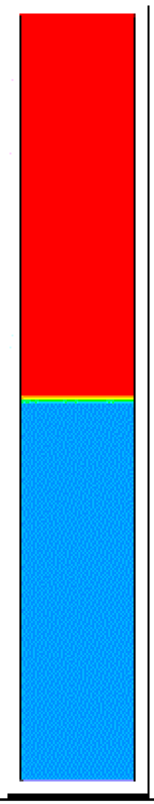
- Use the one that correctly predicts the terminal velocity in dilute flow
- In bubbling beds ensure that the minimum fluidized velocity is correct
- It depends strongly on the particle diameter: correct diameter for non-spherical particles and/or to include clustering effects.

Eulerian Model Inputs

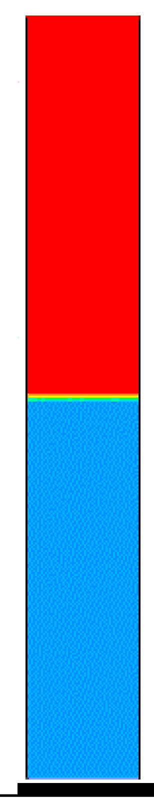
$$v_{r,s} = 0.5\left(A - 0.06\,\mathrm{Re}_s + \sqrt{(0.06\,\mathrm{Re}_s)^2 + 0.12\,\mathrm{Re}_s(2B - A) + A^2} \right)$$

Settling velocity

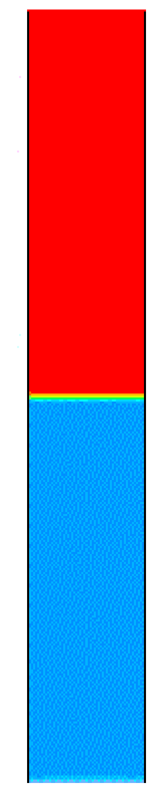
A and B – adjustable constants



Standard Syamlal w/inlet at 21 cm/s



Syamlal tuned at w/ inlet at 8 cm/s



Syamlal Tuned w/ inlet 21cm/s

Eulerian Model Inputs

- Each phase can be a a mixture of species
  - Species conservation equation of species i in phase q
  - Two ways to exchange mass between phases:
    - heterogeneous reaction between species of different phases
    - Mass exchange between species of one phase and bulk of another (non-mixture) phase

Heterogeneous reaction between species *i* of phase *q* and bulk phase *p*

$$\frac{\partial\left(\alpha_q \rho_q Y_q^i\right)}{\partial t} + \nabla \cdot \left(\alpha_q \rho_q \mathbf{v}_q Y_q^i\right) = -\nabla \cdot \alpha_q \mathbf{J}_q^i + \alpha_q R_q^i + \alpha_q S_q^i + \sum_{p=1}^{n} \left(\dot{m}_{pq^i} - \dot{m}_{q^i p}\right) + R$$

Mass transfer between species *i* of phase *q* and bulk phase *p*

Eulerian Model Inputs

- Connection between phase species and total mass conservation of the phase – some of the species conservation equations must be equivalent to phase conservation equations

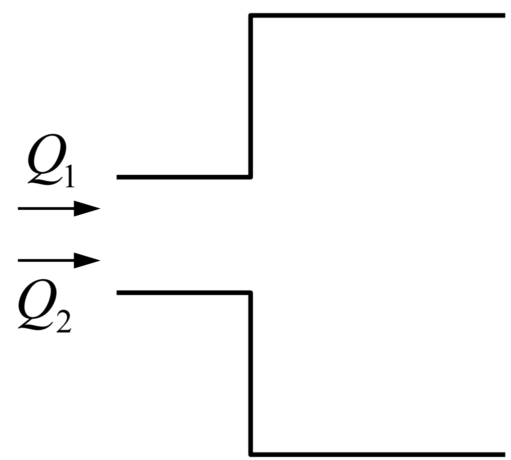
$$\underbrace{\frac{\partial}{\partial t}\left(\alpha_q\rho_q\sum_{i=0}^{I}Y_q^i\right)}_{\frac{\partial(\rho_q\alpha_q)}{\partial t}}+\underbrace{\nabla\left(\alpha_q\rho_q\mathbf{v}_q\sum_i Y_q^i\right)}_{\nabla\cdot(\rho_q\alpha_q\mathbf{v}_q)}=\underbrace{-\nabla\cdot\alpha_q\sum_i\mathbf{J}_q^i}_{0}$$

$$+\underbrace{\alpha_q\sum_i R_{qR}^i}_{0}+\underbrace{\alpha_q\sum_i S_q^i}_{S_{\alpha_q}}+\underbrace{\sum_p\left[\sum_i\left(\dot m_{pq^i}-\dot m_{q^ip}\right)\right]+\sum_i R^i}_{\sum_i Y_q^i}$$

Eulerian Model Inputs

- Boundary conditions
  - Velocity inlet requires specification of velocity for all phases and VOF for all secondary phases
  - Usually only volume or mass flow rates at inlet are known
    - Assume that physical velocities of all phases are equal to *U*. In such case for two phases we have:

$$Q_1 = A\alpha_1 U,\ \ \mathrm{m^3/sec}$$
$$Q_2 = A\alpha_2 U,\ \ \mathrm{m^3/sec}$$
$$\alpha_1 + \alpha_2 = 1$$

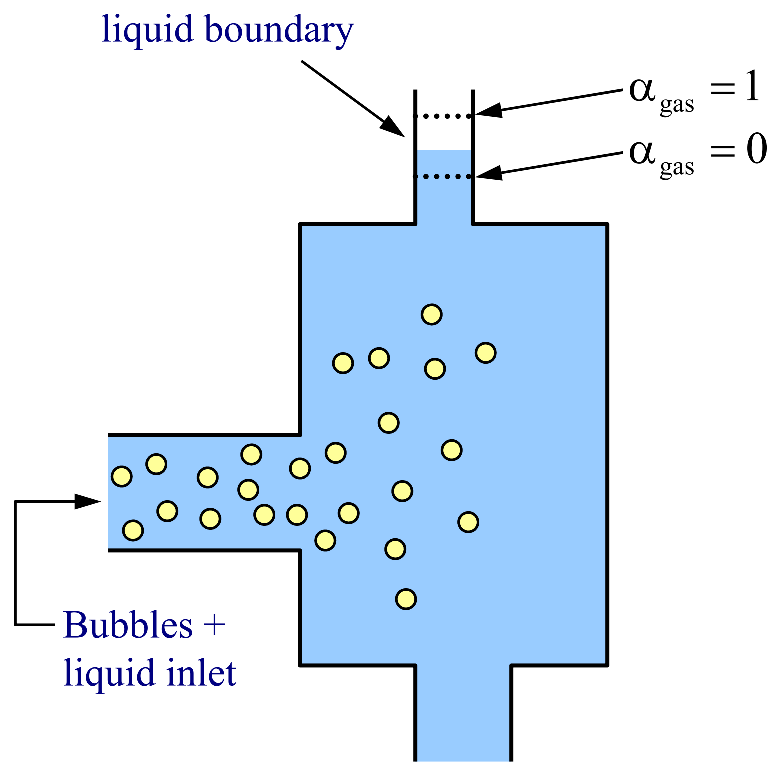


    - 3 equations and 3 unknowns – U, α_1 and α_2
    - Turbulence BC rules are the same as for single phase.

Eulerian Model Inputs

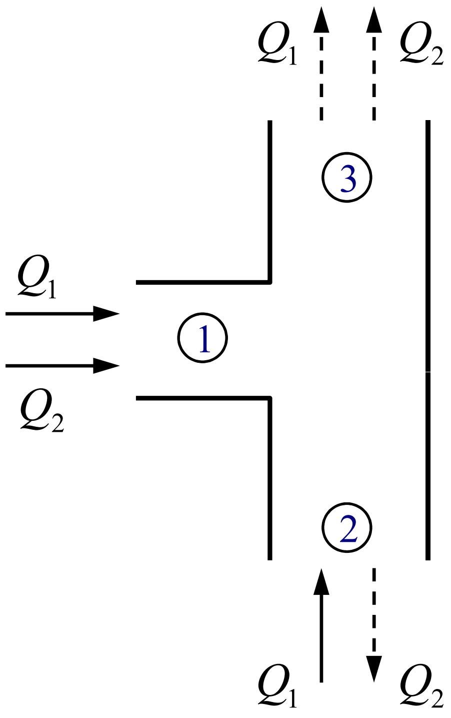
- Boundary conditions
  - Pressure inlet requires specification of VOF for all secondary phases
  - Make sure sum of all inlet VOFs do not exceed 1
  - Pressure outlet requires specification of VOF for all secondary phases when the Backflow occurs
    - Backflow VOF specification is very important when modeling dispersed flows with uknown free surface level
    - Specification of Backflow VOF may depend of location of boundary
    - In gas-liquid separator specification of backflow VOF at upper boundary may depend on its location. The safe practice is to move sufficiently high or low so it is well known whether upper domain boundary is under or above the liquid level

Gas-liquid separator

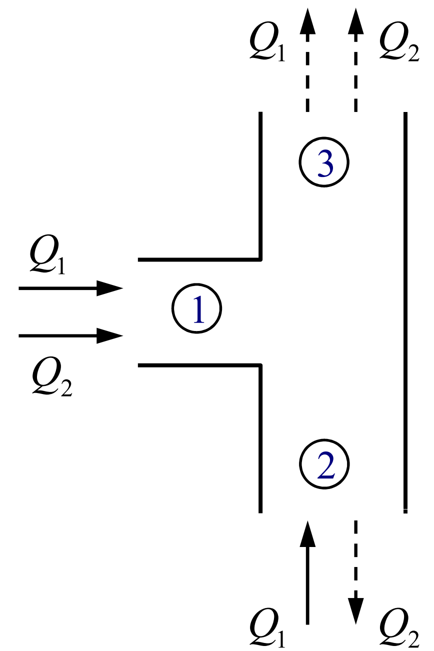


Eulerian Model Inputs

- Boundary conditions
  - Outflow boundary conditions are not allowed.
  - Periodic BCs (translational) are not allowed.
  - Rotational Periodic BCs are allowed only if periodic planes are parallel to the gravity vector.
  - We must prescribe separate boundary conditions for each phase including the mixture phase.
    - Situation when one surface is subject of one BC for one phase and another BC for another phase
    - Consider the following examples:
      - Inlet 1 – Flow rates for primary phase 1 and secondary phase 2 are prescribed.
      - Inlet 2 – Inlet for phase 1 but outlet for phase 2 (e.g. particles falling downward)
      - Inlet 3 – outlet for both phases



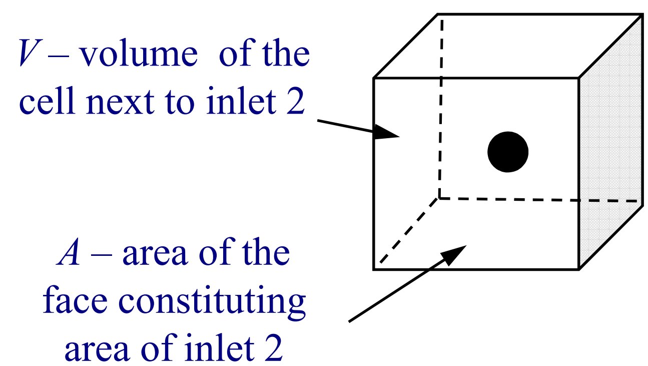
Eulerian Model Inputs



- Separate boundary conditions for phases
  - Solution – implement source term for continuity equation for phase 2 in cells next to inlet 2 using UDF
  - The purpose of the source is remove mass of phase 2 as soon at it appears in the cell
  - For time dependent model, the source term may have the following shape:

$$\dot{m}_2 = -\frac{\alpha_2 \rho_2}{\delta t}$$

which means that all mass of phase 2 will be removed from this cell during time step *dt*

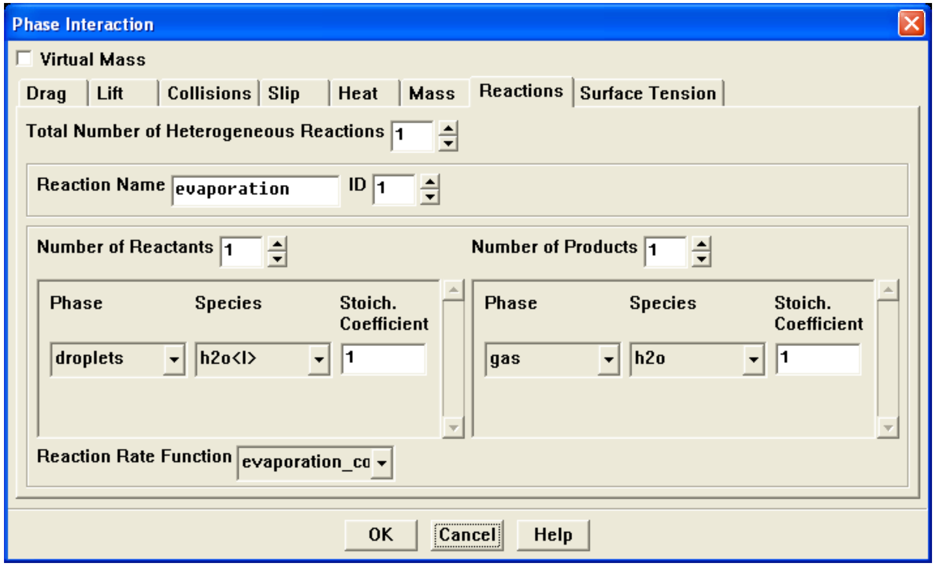


Eulerian Model Examples

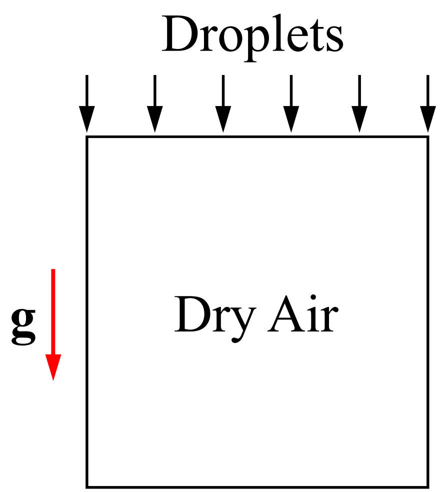
- Example – water droplets freely falling in dry air accompanied by evaporation
  - Water – single species mixture phase
  - Atmosphere – mixture of two species (air+vapor)
  - Evaporation from droplets into air is modeled as heterogeneous reaction
  - Reaction rate is given by

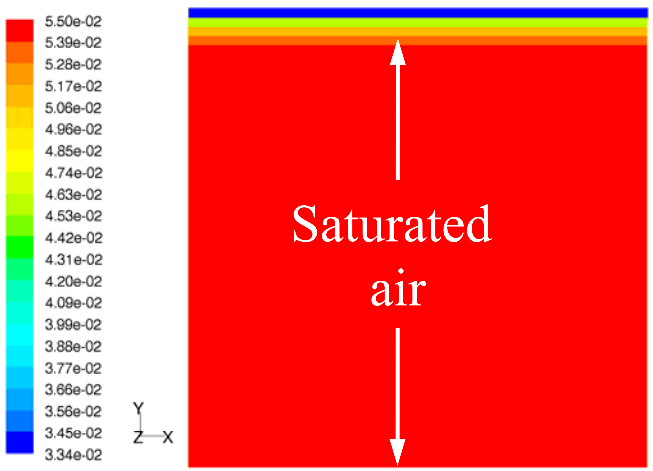
$$R = k_c \left[C_{\mathrm{H_2O,\,sat}}\left(T_{\mathrm{drop}}\right) - C_{\mathrm{H_2O,\,gas}}\left(T_{\mathrm{gas}}\right) \right] \frac{6\,\alpha_{\mathrm{drop}}}{d_{\mathrm{drop}}}$$

Molar concentration   Area density



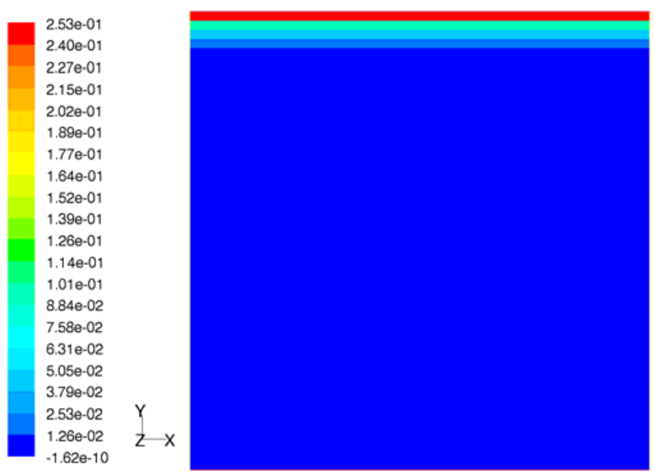
Eulerian Model Examples





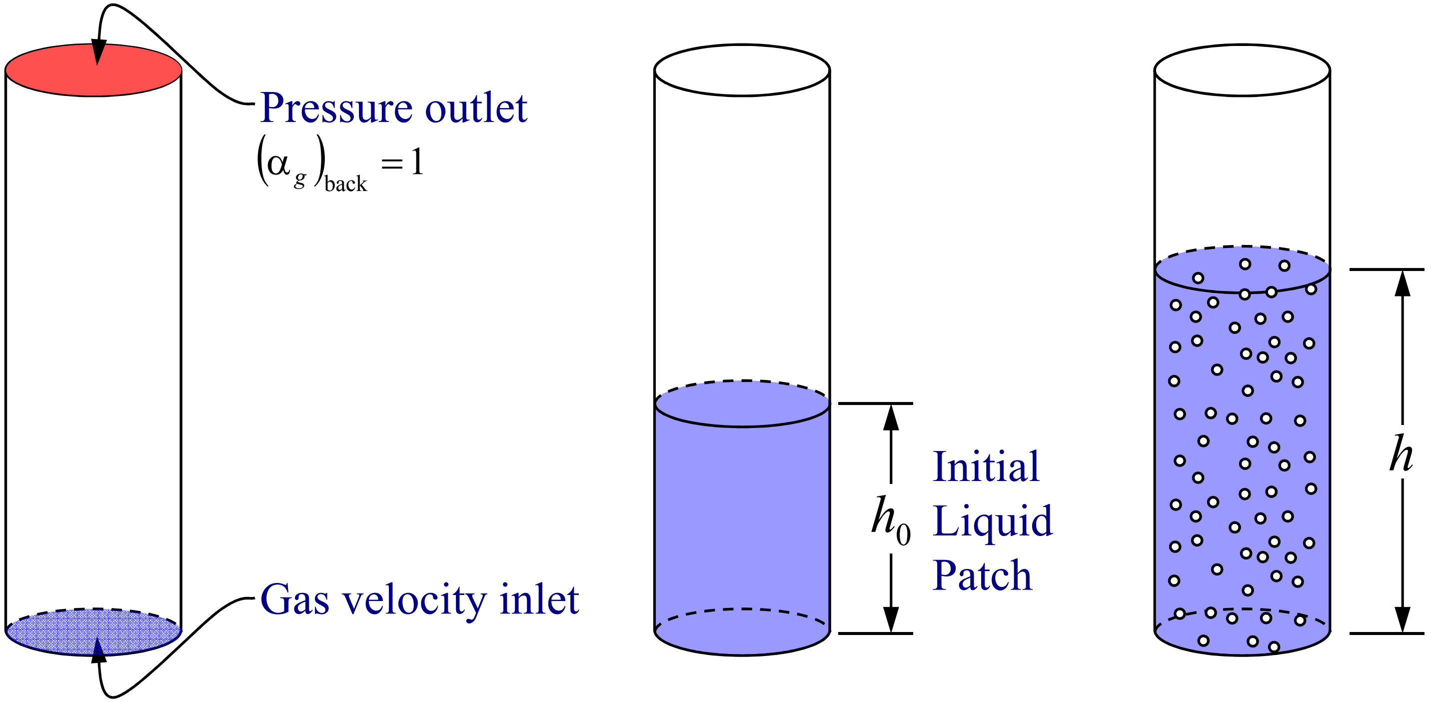
Vapor mole fraction

Evaporation rate



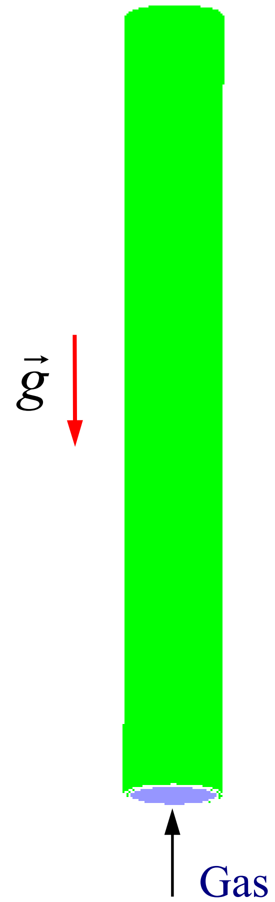
Eulerian Model Examples

- Bubble columns – liquid is sparged by bubbles emitted at the bottom
- Original liquid level rises from h_0 to h due to the displacement and drag of rising bubbles.
- Averaged bubble VOF determines the liquid holdup, $1 - (h_0 / h)$

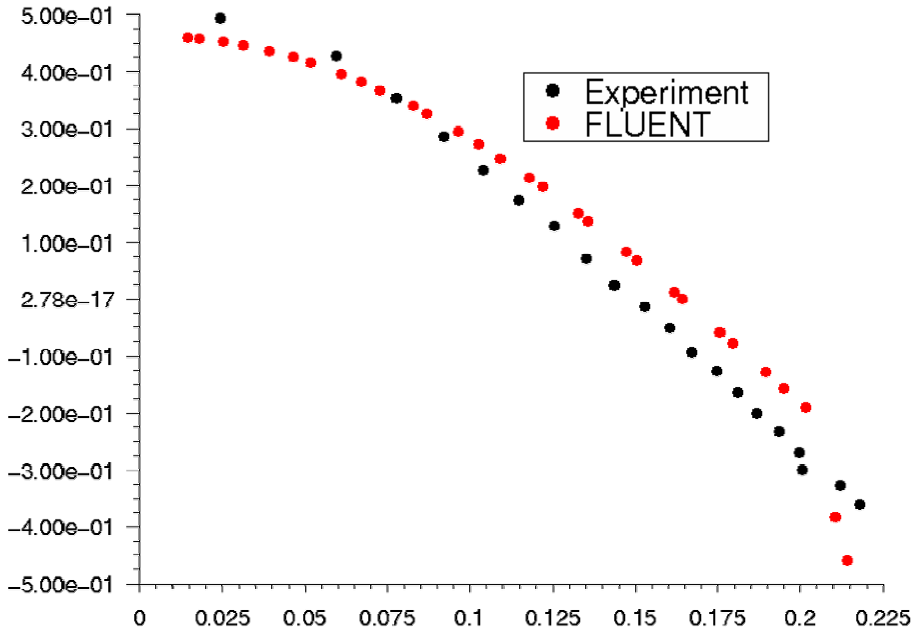


Eulerian Model Examples

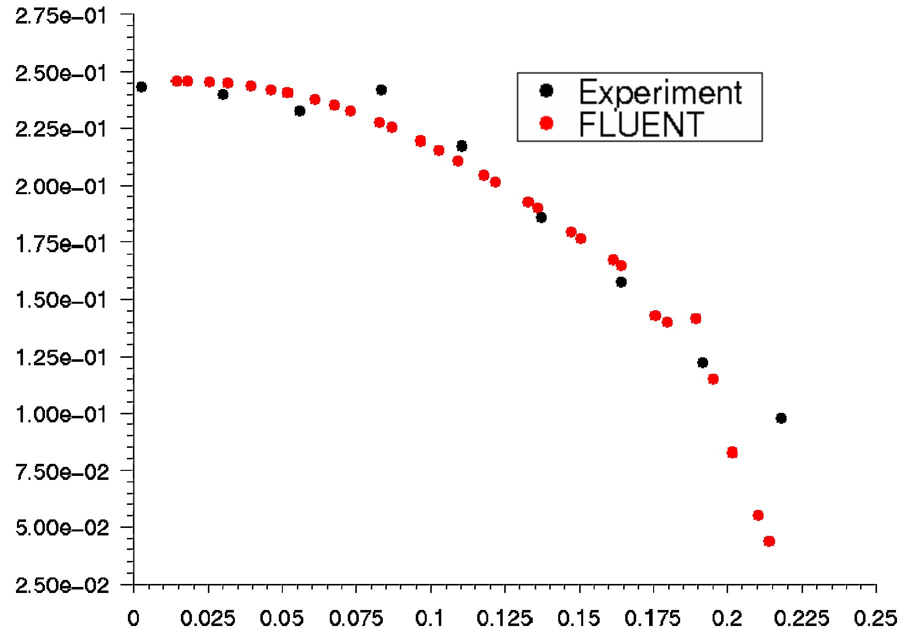
- 3D cylindrical bubble column, 44 cm diameter, 116 cm initial height of liquid (free board)
- Churn turbulent conditions, bubble size of 5 mm
- Unstructured grid, 37,800 cells
- Superficial gas velocity 10 cm/s
- RNG k–ε turbulence model
- QUICK scheme is used for all variables



Eulerian Model Examples

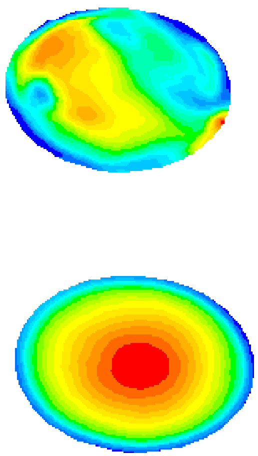


Comparison of time averaged axial liquid velocity

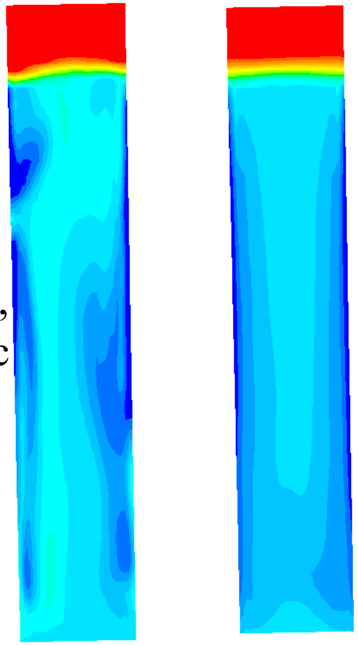


Comparison of time averaged air volume fraction

Eulerian Model Examples



Gas Volume Fraction
(Lateral Distribution)



Gas Volume Fraction
(Axial Distribution)

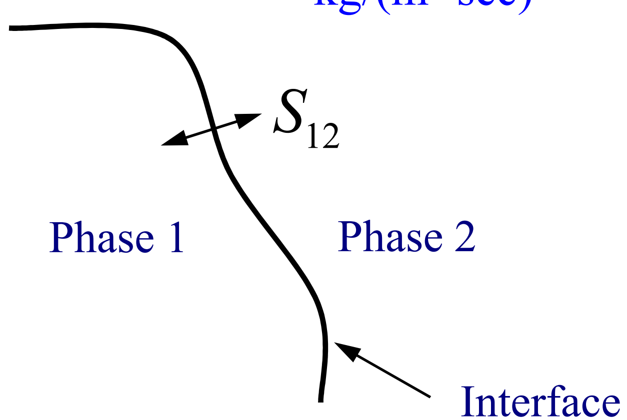
Eulerian Model Examples

- Interfacial mass transfer rate per unit of volume – source terms in phase mass conservation equation

$$S_{12} = \dot{\mathbf{m}}_i \cdot \mathbf{A}_i \quad \left[\frac{\text{kg}}{\text{m}^3 \cdot \text{sec}} \right]$$

Mass flux vector, kg/(m^2 sec)

Interfacial area density, 1/m



Eulerian Model Examples

- Interfacial mass flux is caused by mechanical/thermal non-equilibrium at the interface
- From ideal gas theory, deviation from equilibrium at gas side of interface will cause mass flux:

$$\|\dot{\mathbf{m}}\| = \left(\frac{2\sigma}{2-\sigma}\right)\left(\frac{M}{2\pi R}\right)^{1/2}\left(\frac{P_i}{T_i^{1/2}} - \frac{P_v}{T_v^{1/2}}\right)$$

- If gas is a mixture of vapor and air and temperature change is negligible, the above equation will reduce to evaporation rate:

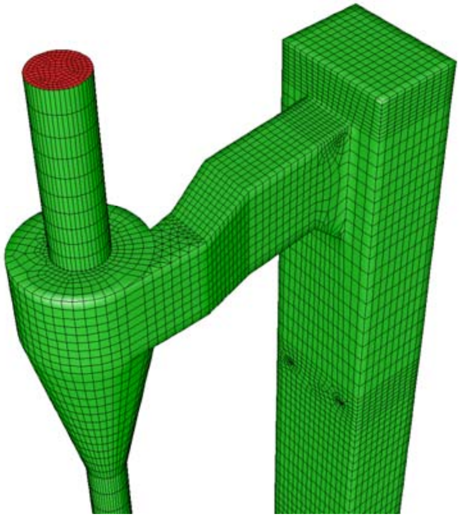
$$\|\dot{\mathbf{m}}\| = \left(\frac{2\sigma}{2-\sigma}\right)\left(\frac{M}{2\pi R T_g}\right)^{1/2}\left(P_{\mathrm{sat}} - P_{\mathrm{v}}\right)$$

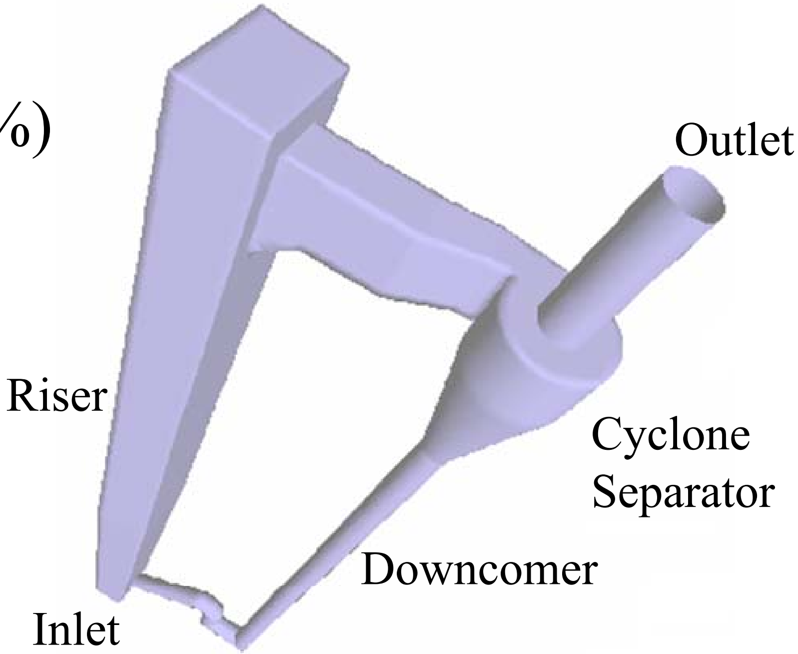
$$P_v = X_v P_{\mathrm{tot}} = \text{Vapor partial pressure}$$
$$P_g = X_g P_{\mathrm{tot}} = \text{Gas partial pressure}$$
$$P_{\mathrm{tot}} = P_v + P_g = \text{Dalton's law}$$

R. W. Schrage, “A Theoretical Study of Interphase Mass Transfer,” Columbia University Press, New York (1953).

Eulerian Model Examples

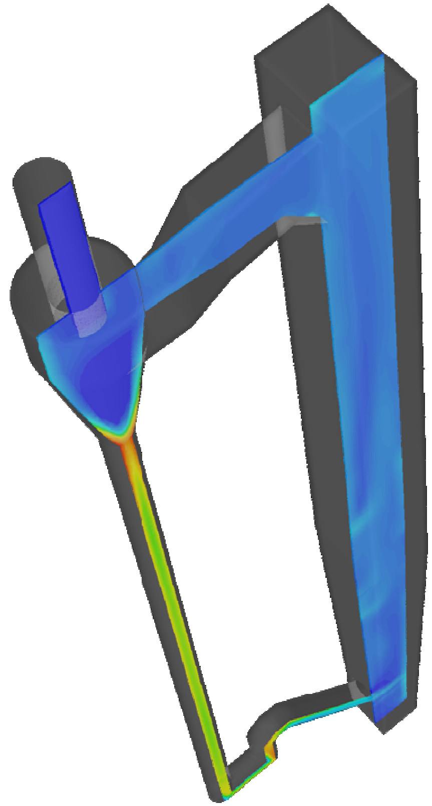
- A fully 3D circulating fluidized bed is modeled.
- 74,000 cell hybrid mesh
- Gas/Solids dilute flow (average solids volume fraction around 7%)

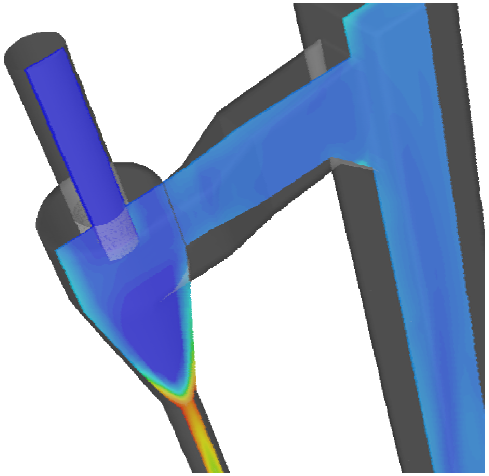


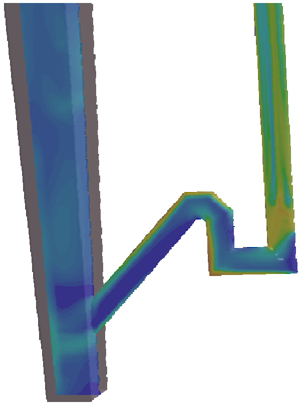


Eulerian Model Examples

- Solid volume fraction contours

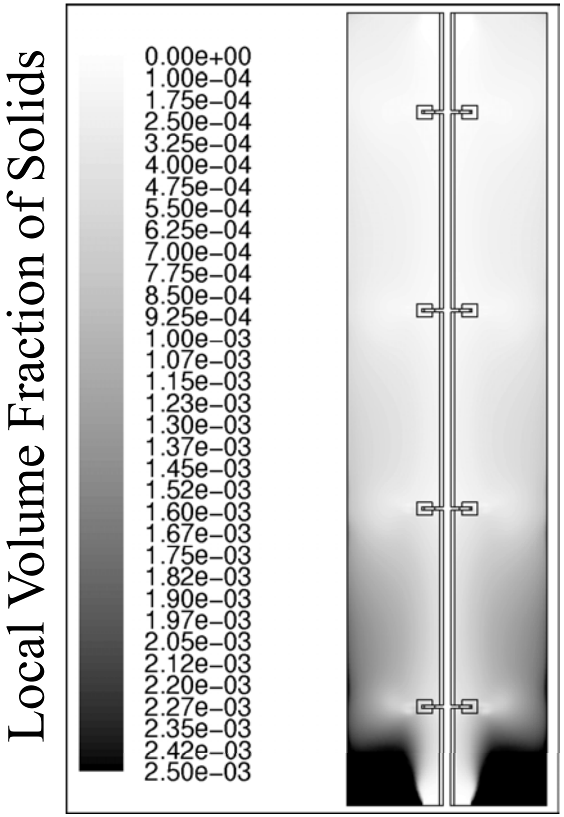


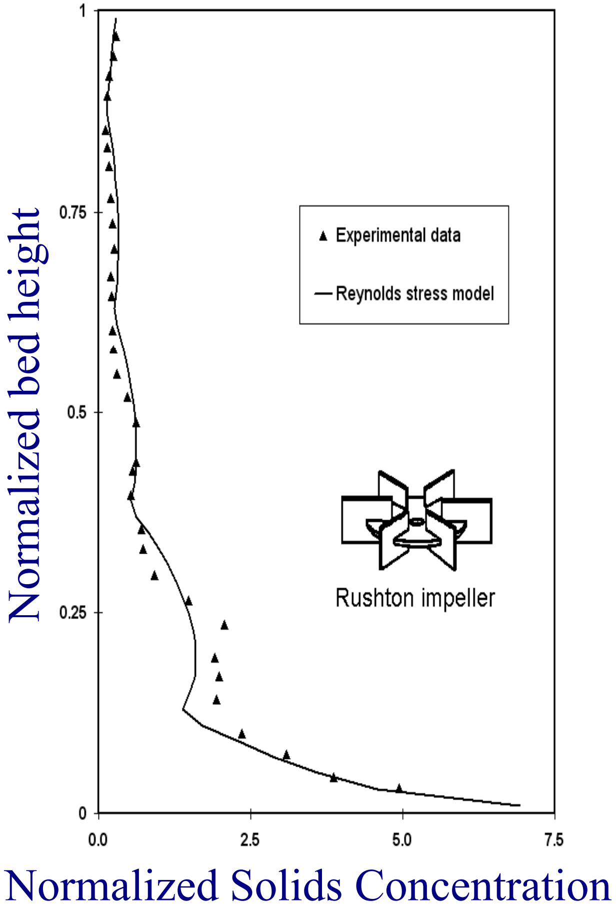




Eulerian Model Examples

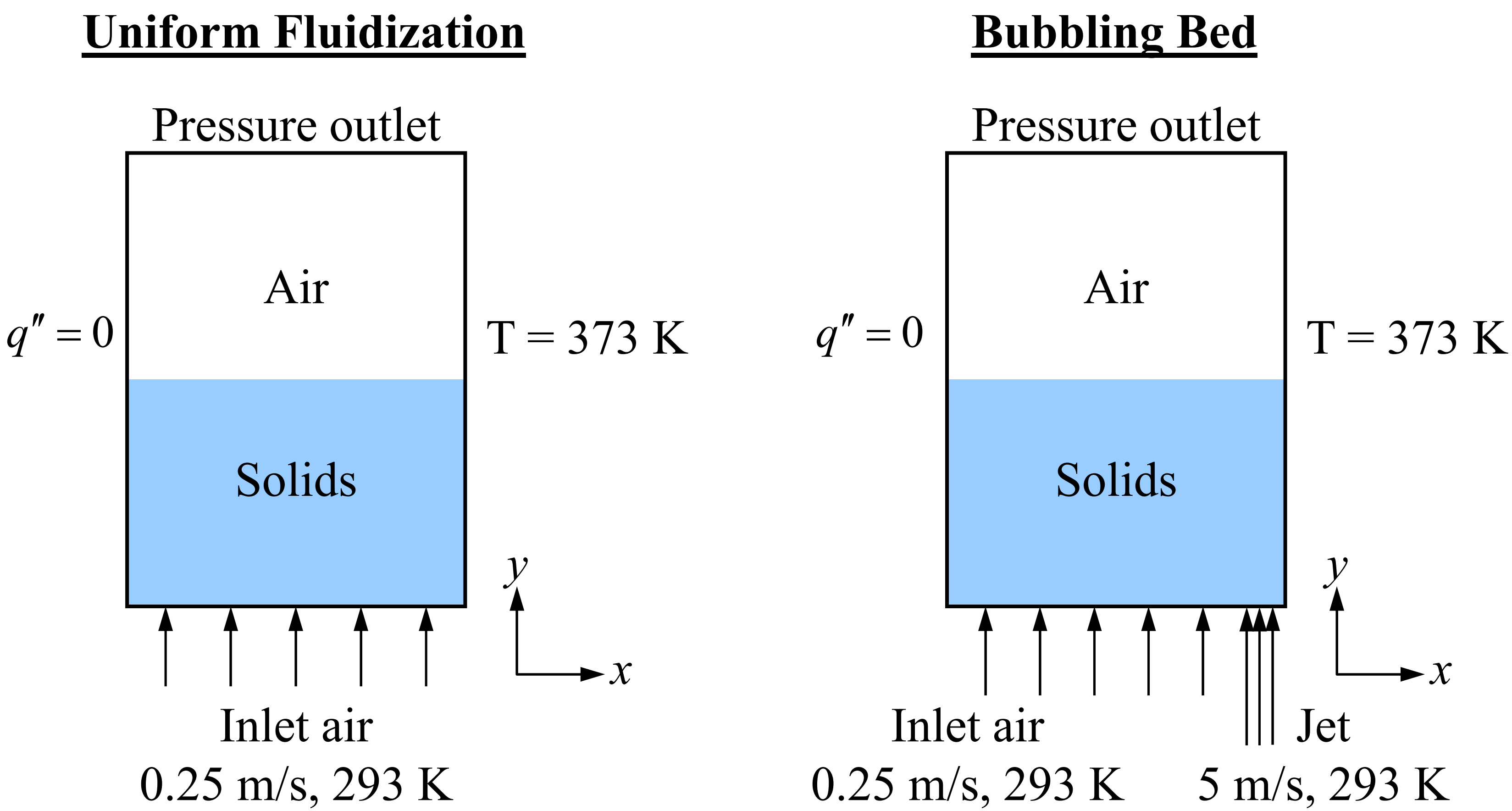
- Turbulence enhancement example water and glass beads in a stirred closed vessel.





Eulerian Model Examples

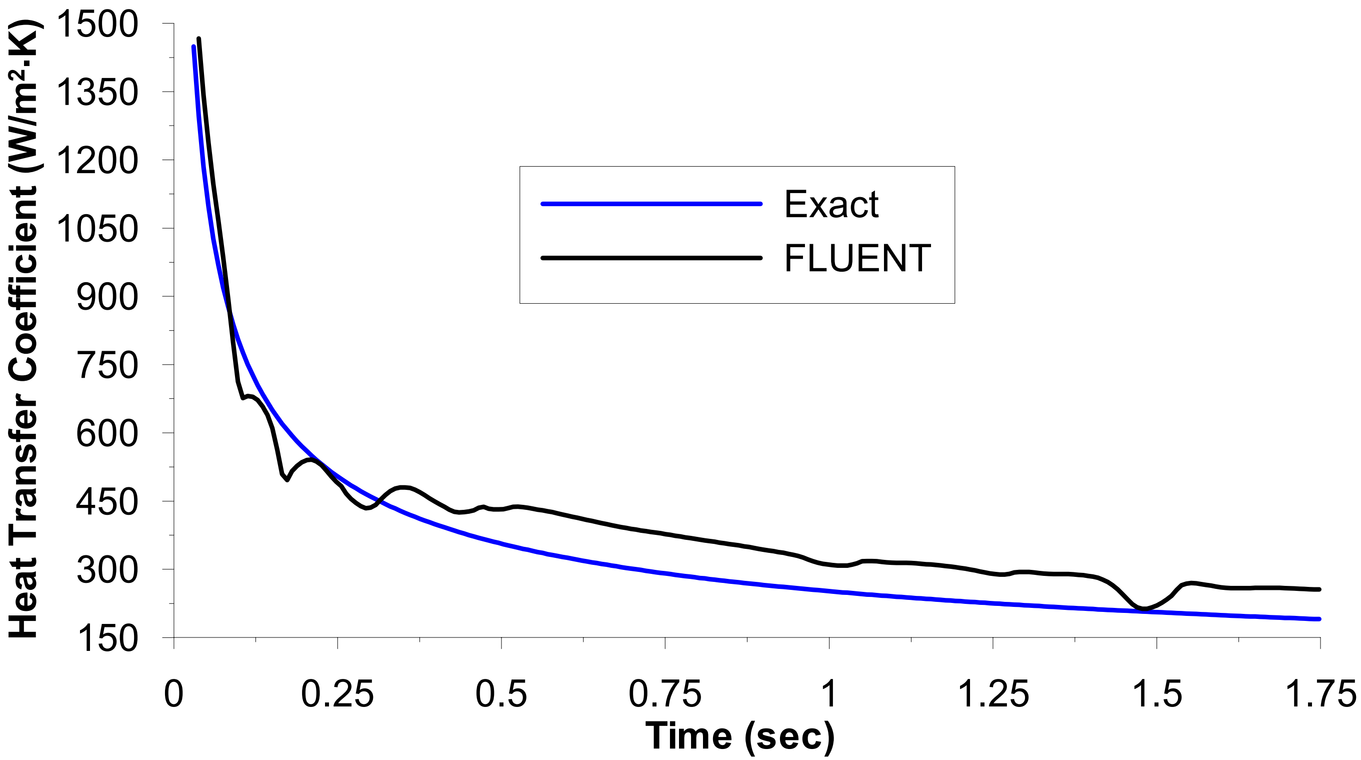
- Heat transfer to fluidized bed



Eulerian Model Examples

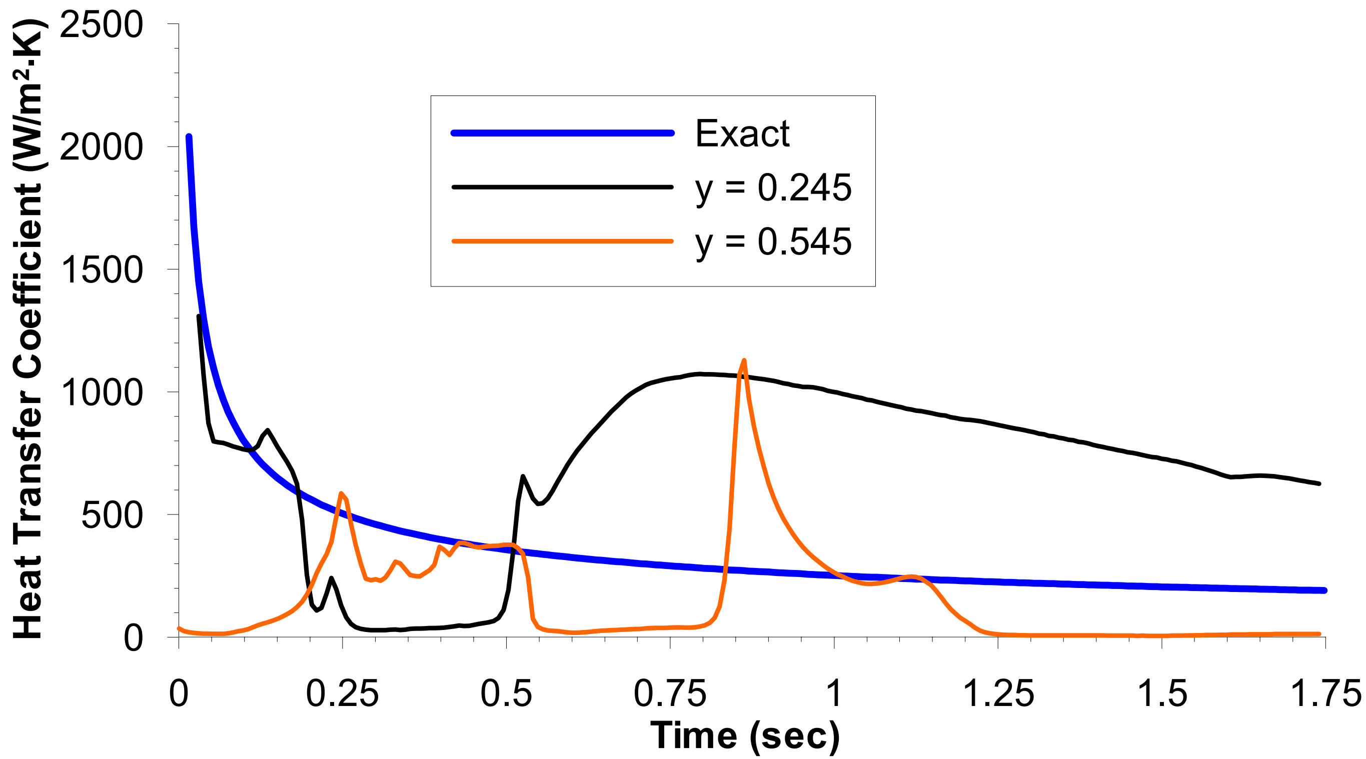
- Exact result based on penetration theory: $h = \dfrac{252}{\sqrt{t}}$

Uniform Fluidization



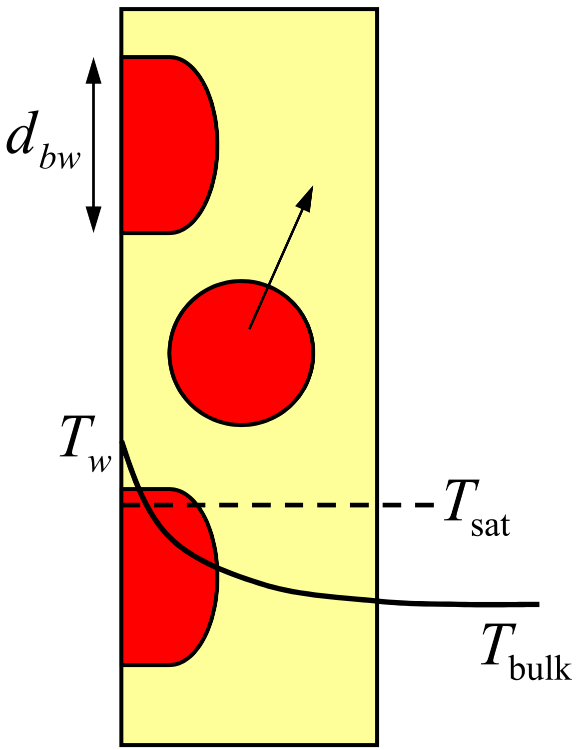
Eulerian Model Examples

- Bubbling bed - heat transfer coefficient history on different elevations



Eulerian Model Examples

- Wall heat flux partitioning (RPI model)

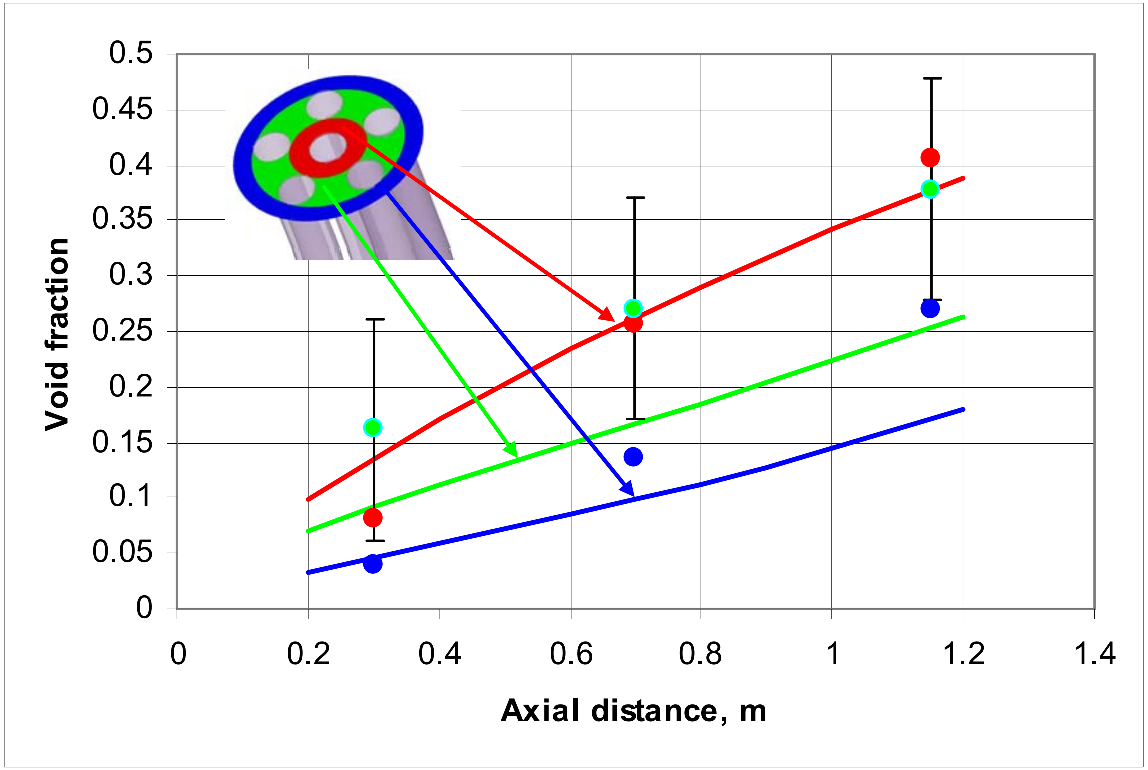


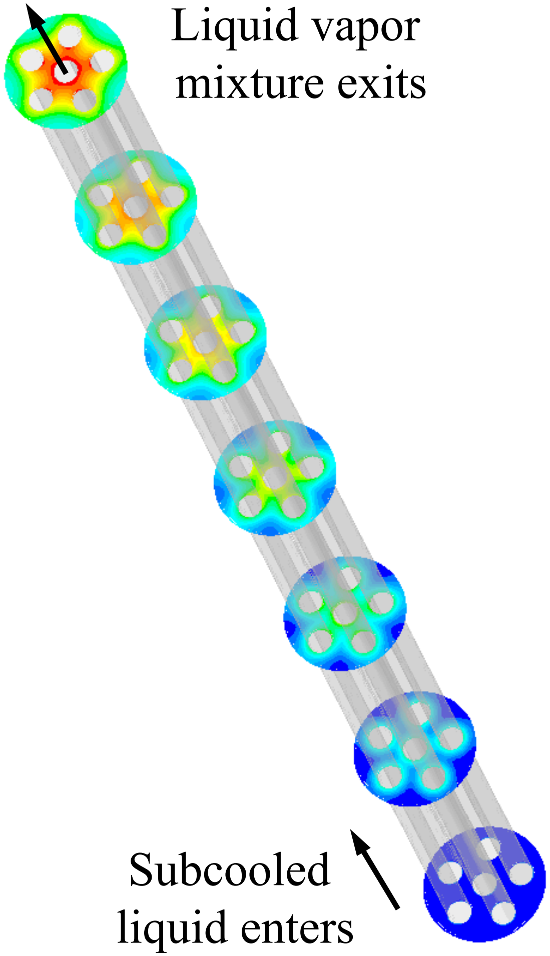
$$q''_{w,\text{ liquid}} = \overset{\text{Evaporation}}{q''_E} + \overset{\text{Convection}}{q''_C} + \overset{\text{Quenching}}{q''_Q}$$

$q''_E = \frac{\pi}{6} d^3_{bw} f n \rho_v h_{lv}$	Heat spent on evaporation (generation of vapor phase)
q''_C	Usual single phase log law
q''_Q	Transient refilling at bubble departure

Eulerian Model Examples

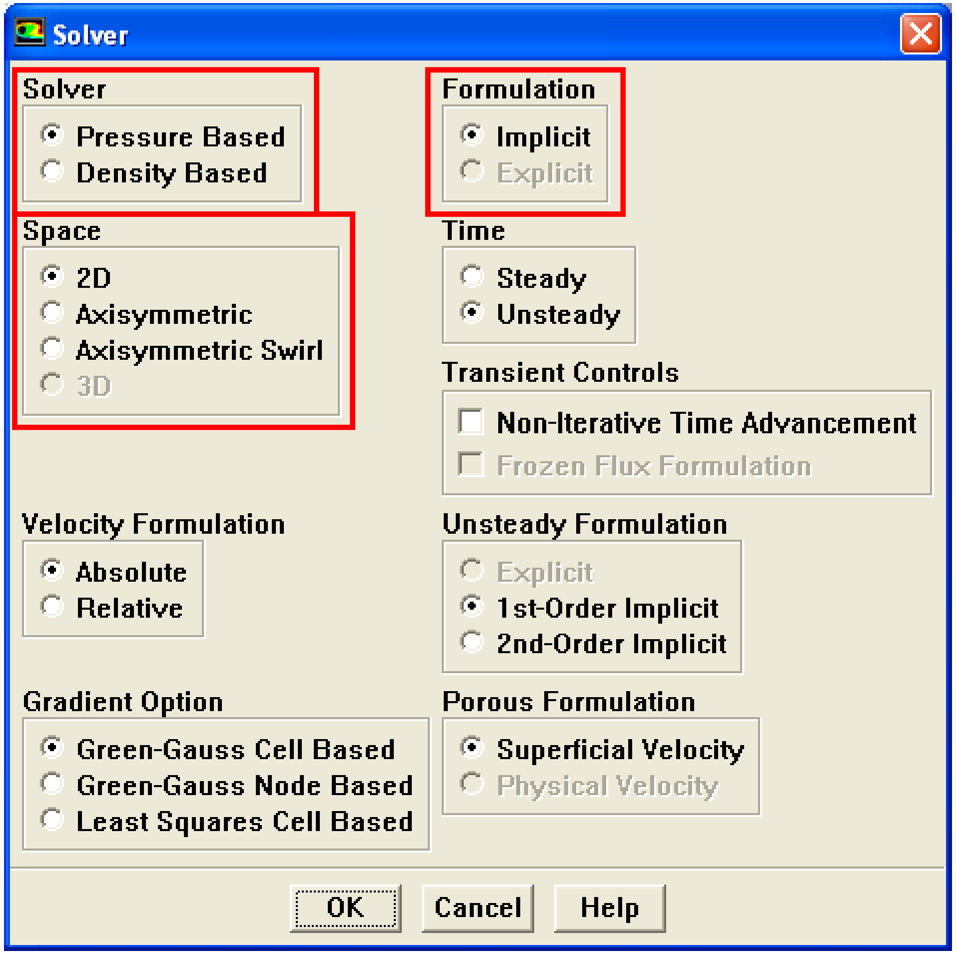
- Subcooled boiling in reactor channel with heated rods





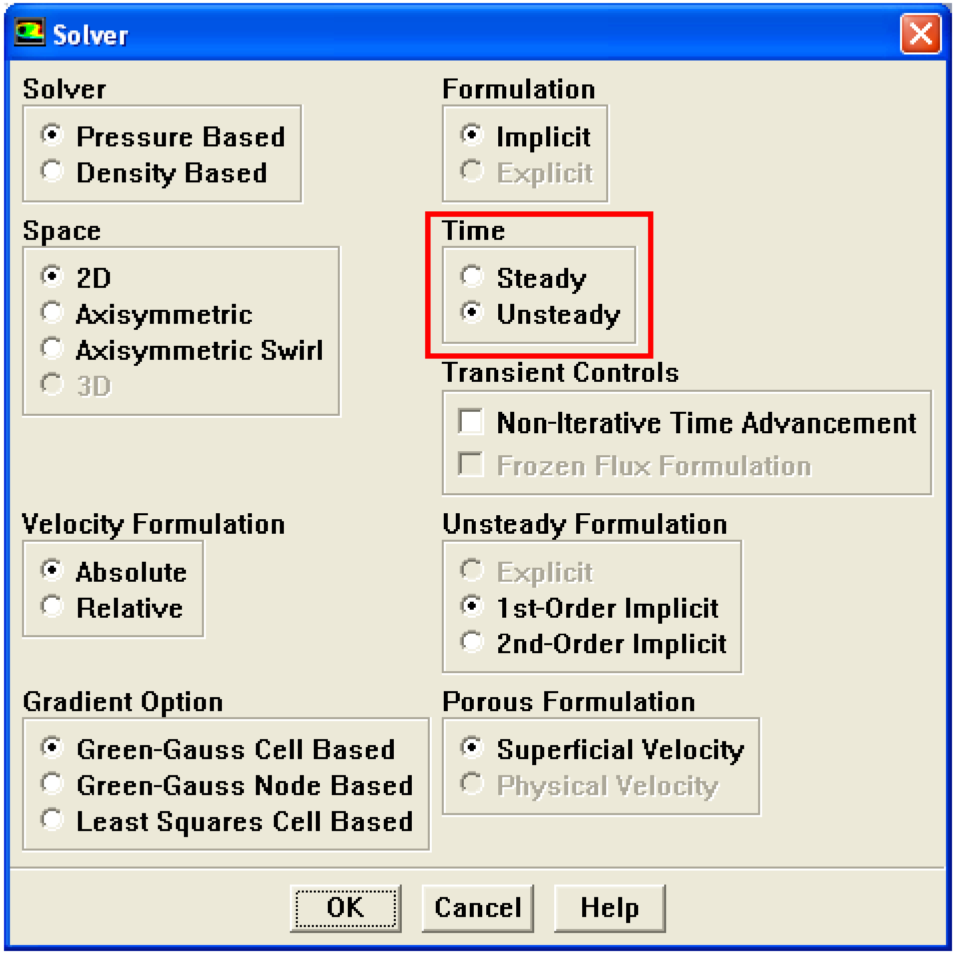
Solution Strategies and Common Mistakes

- Euler model is based on segregated solver so pressure based solver should be used.
- Formulation – only implicit is available (good stability)
- Space – if you use axisymmetric solver, make sure that gravity is parallel to the axis. Otherwise, your model is not axisymmetric!



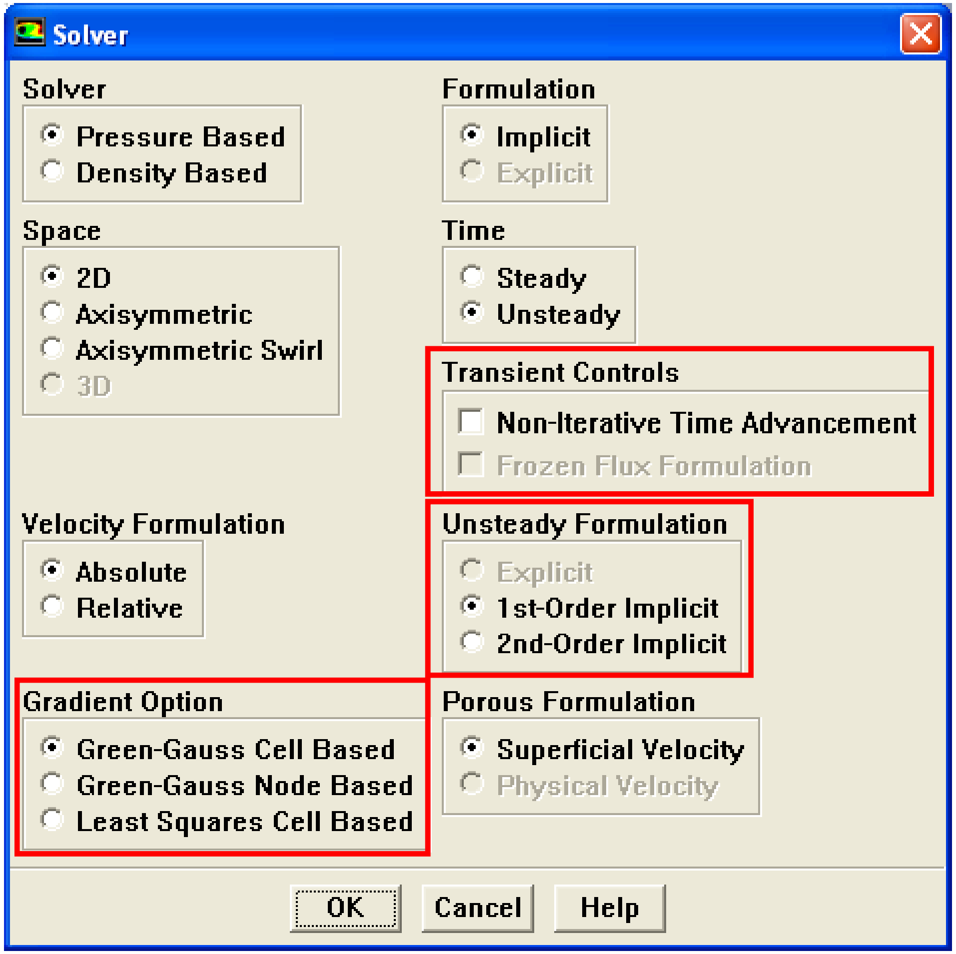
Solution Strategies and Common Mistakes

- Steady vs Unsteady – Euler model equations are connected to each other through non-linear exchange terms
- It is often difficult to converge even steady state problems using the steady solver.
- Workaround is to solve steady problems using the unsteady solver.
  - No need to resolve the transient so you can use large time steps and small Max Iterations per Time Step.
  - Monitor relevant quantities at every time step – flattening of monitors indicates convergence to steady state



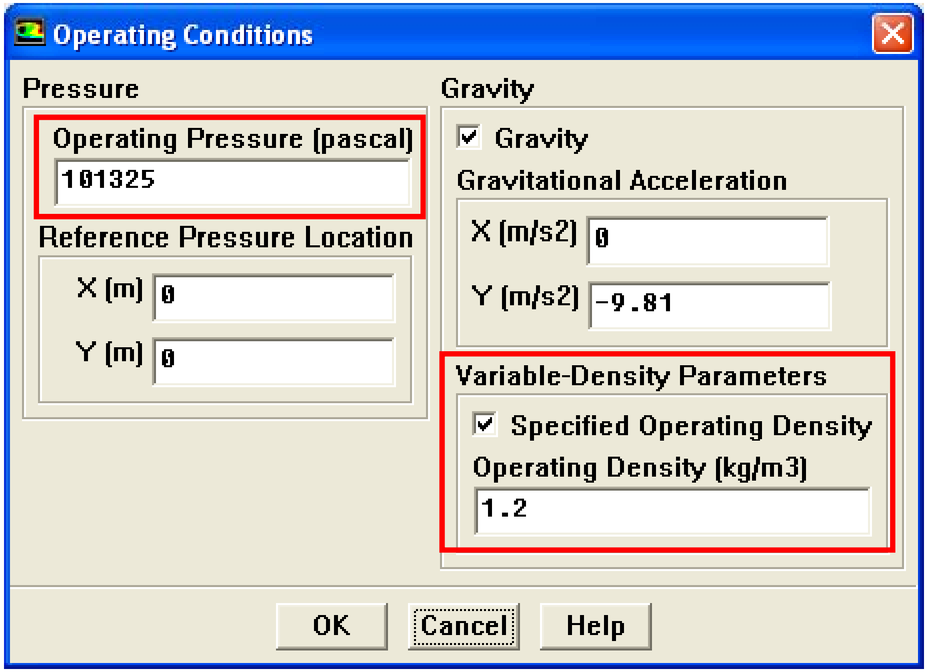
Solution Strategies and Common Mistakes

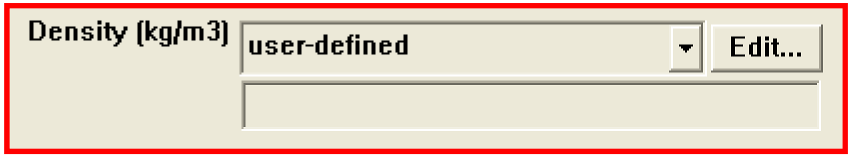
- Non-Iterative time advancement is not compatible with Eulerian.
- Gradient option – Eulerian model is tested mainly with Cell Based. Use node based for all tet meshes. Use Least Squares for polyhedral meshes.
- Unsteady formulation – 2nd order is not compatible with Eulerian.

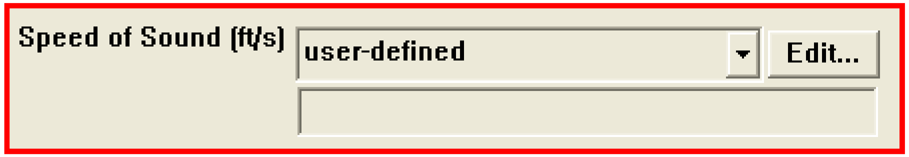


Solution Strategies and Common Mistakes

- Operating pressure is not important for incompressible phases
- Variable density – choose smallest density of all phases
- Phases can have constant, incompressible-ideal-gas, ideal-gas and UDF specified density
- If you use UDF – you must prescribe both density and speed of sound



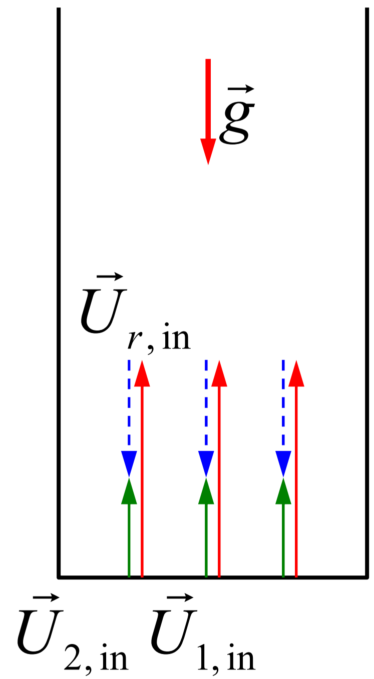




Solution Strategies and Common Mistakes

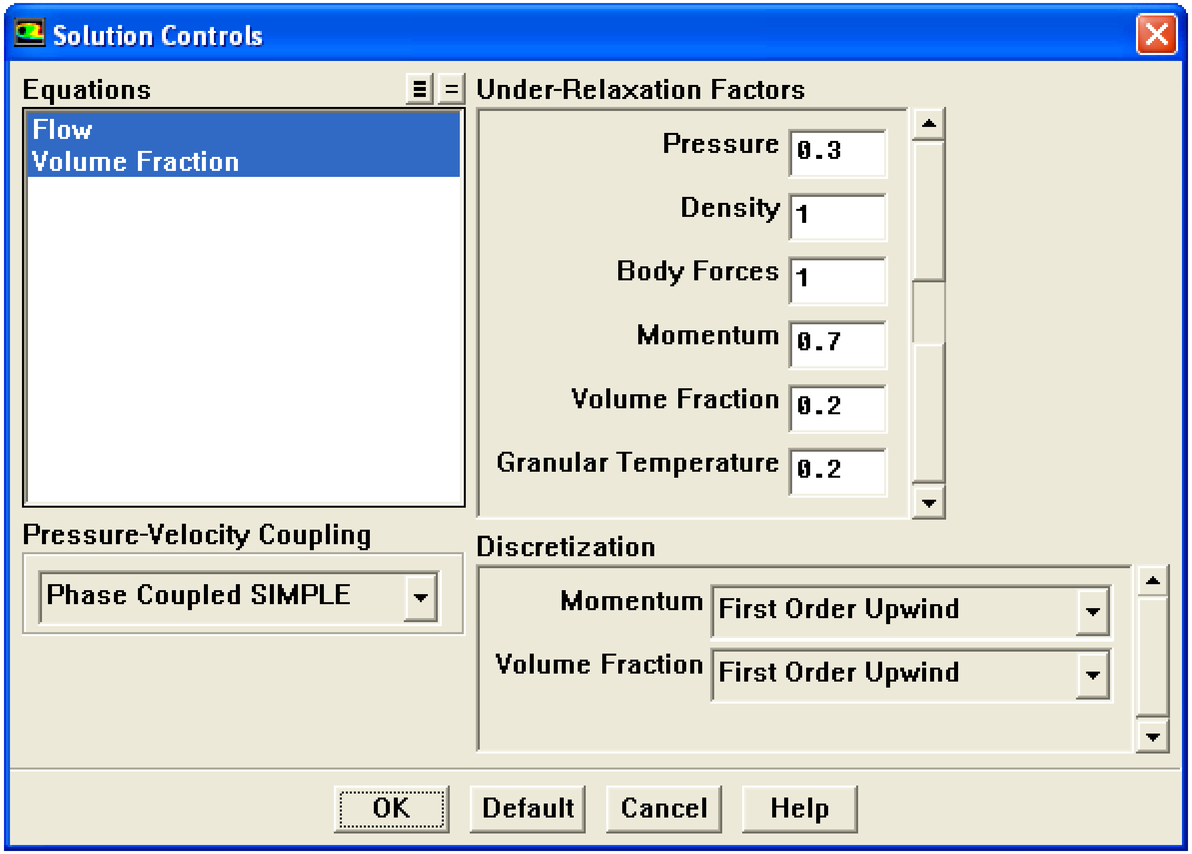
- Inlet, outlet and initial conditions with granular model – make sure that volume fraction of secondary granular phase never exceeds the packing limit
- Realizable BCs – if relative velocity vector is opposite to inlet direction, primary phase inlet velocity must be larger than relative velocity
  - Consider, for example, upward flow of gas loaded with solid particles. Boundary condition is realizable only when $U_{1,in} > U_{r,in}$
  - Relative velocity can be estimated from drag relation to relative velocity:

$$U_r = \left(\frac{4\, g \left| \rho_2 - \rho_1 \right| d}{3\, C_d} \right)^{1/2}$$



Solution Strategies and Common Mistakes

- Solution controls
  - Conservative solution control settings are shown.
  - If convergence is slow, try reducing URFs for volume fraction and turbulence.



Summary

- Euler model is a powerful tool to model dispersed flows.
- Accuracy is determined by accuracy of interfacial terms.
- The interfacial terms are usually nonlinear so convergence is often difficult.
  - Try solving the case using the unsteady solver.
  - Be careful with your choice of particle diameter.
- Boundary conditions
  - Remember that VOF and physical velocity are meaningless.
  - Rather, their product (superficial velocity) is related to volume flow rate (i.e. reality).
  - Backflow volume fraction of phases is important.