Mixture Model in FLUENT

Advanced Multiphase Modeling Course

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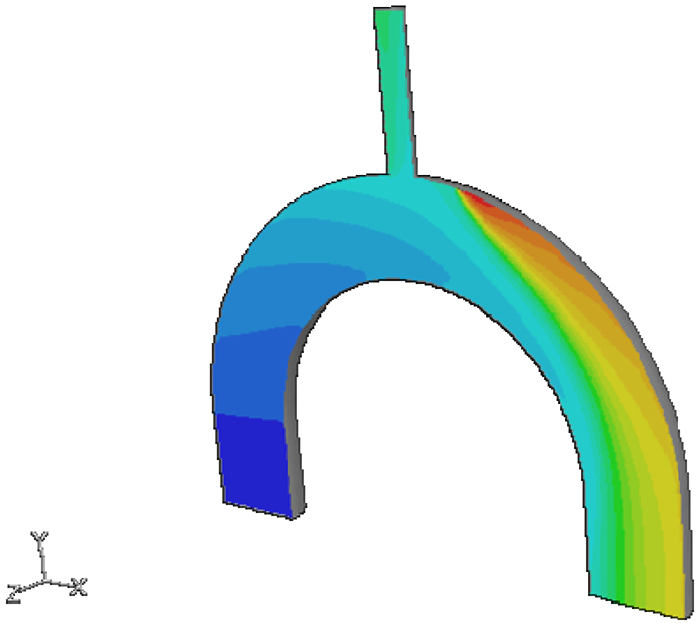


Overview

- The mixture model is a simplification of the Eulerian multiphase model.
  - Solves one set of momentum equations for the mass averaged velocity and tracks volume fraction of each fluid throughout domain.
  - Derives a constitutive relation for the relative velocities based on local equilibrium over the length scales.
  - For turbulent flows, a single set of turbulence transport equations is solved.
  - For nonisothermal flows, a single energy equation is solved based on mixture properties.

Equations

- Eulerian model solves momentum equation for each phase:

$$\frac{\partial\left(\alpha_q \rho_q \vec{v}_q\right)}{\partial t} + \nabla \cdot \left(\alpha_q \rho_q \vec{v}_q \vec{v}_q\right) = -\alpha_q \nabla p + \nabla \cdot \bar{\bar{\tau}}_q + \alpha_q \rho_q \vec{g}_q$$

$$+ \sum_{p=1}^{n} \left(\vec{R}_{pq} + \dot{m}_{pq} \vec{v}_{pq}\right) + \alpha_q \rho_q \left(\vec{F}_q + \vec{F}_{q,\mathrm{lift}} + \vec{F}_{q,\mathrm{vm}}\right)$$

- Summing up momentum equations for all phases and noticing that sum of all interfacial forces is zero we arrive to mixture momentum equation:

$$\frac{\partial\left(\rho \vec{u}_m\right)}{\partial t} + \nabla \cdot \left(\rho_m \vec{u}_m \vec{u}_m\right) = -\nabla p + \nabla\left[\mu_{\mathrm{eff}}\left(\nabla \vec{u}_m + \nabla \vec{u}_m{}^T\right)\right] + \rho_m \vec{g} + \vec{F} + \nabla \sum_{k=1}^{n} \alpha_k \rho_k \vec{u}_k^r \vec{u}_k^r$$

- Mixture velocity, density, and viscosity are defined as

$$\vec{u}_m = \frac{1}{\rho_m} \sum_{q=1}^{n} \alpha_q \rho_q \vec{u}_q \qquad \rho_m = \sum_{q=1}^{n} \alpha_q \rho_q \qquad \mu_m = \sum_{k=1}^{n} \alpha_k \mu_k$$

Equations

- Summing all phase continuity equations

$$\frac{\partial(\alpha_k \rho_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{u}_m) = -\nabla \cdot (\alpha_k \rho_k u_k^r)$$

one can write mixture mass conservation equation

$$\frac{\partial \rho_m}{\partial t} + \nabla(\rho_m \vec{u}_m) = 0$$

- In equations above, the drift velocity is introduced which is related to slip velocity as

$$\vec{u}_p^r \equiv \vec{u}_p - \vec{u}_m = \vec{u}_{pq} - \sum_{k=1}^{n} \frac{\alpha_k \rho_k}{\rho_m} \vec{u}_{qk}$$

Equations

- Now having a known mixture velocity (from the mixture momentum equation), we need to **reconstruct** the velocity of each phase.
- The main assumption of the mixture model is that fluid particles follow the mixture path closely. Then, the slip (relative) velocity between the phases is

$$\vec{u}_{pq} = \frac{\vec{a}\,\tau_p}{f_{\text{drag}}} \frac{\rho_p - \rho_m}{\rho_p} \qquad \tau_p = \text{Particle relaxation time}$$

where the particle acceleration is $\vec{a} = \vec{g} - \left(\vec{u}_m \cdot \nabla \vec{u}_m + \frac{\partial \vec{u}_m}{\partial t} \right)$

- In turbulent flows, the relative velocity should contain a diffusion term in the momentum equation for the disperse phase. FLUENT adds this dispersion to the relative velocity as follows:

$$\vec{u}_{pq} = \frac{\vec{a}\tau_p}{f_{\text{drag}}} \frac{(\rho_p - \rho_m)}{\rho_p} - \frac{\nu_m}{\alpha_p \sigma_D} \nabla \alpha_q$$

Equations

- The Energy equation for the mixture energy is written as

$$\frac{\partial(\rho_m E_m)}{\partial t} + \nabla \cdot [\vec{v}_m(\rho_m E_m + p)] = \nabla \cdot (k_m \nabla T) + S_E$$

- Where mixture energy and mixture thermal conductivity are defined as

$$E_m = \frac{1}{\rho_m}\sum_{q=1}^{n} \rho_q E_q \qquad k_m = \sum_{q=1}^{n} \alpha_q k_q$$

Restrictions

- The mixture model is applicable only to low particle relaxation times.

$$\tau_p = \frac{\rho_p d_p^2}{18\mu_f} < \frac{D}{U}$$

- One continuous phase and *N* dispersed phases.
- One velocity field can be used to describe all phases if the following conditions are satisfied
  - No countercurrent flow
  - No sedimentation

Cavitation

- The mixture cavitation model (first implemented in FLUENT 6.2) which accounts for
  - *N*-phase flows with multiphase species transport.
  - Effects of slip velocities between the liquid and gas phases.
  - Thermal effects and compressibility of both liquid and gas phases.

Cavitation

- Vapor mass conservation

$$\frac{\partial(\rho f)}{\partial t} + \nabla \cdot (\rho \vec{u}_v f) = \nabla \cdot (\gamma \nabla f) + R_e - R_c$$

- The phase change rate expressions assume that maximum vapor bubble size is determined by surface tension σ and fluctuating velocity scale

$$V_{ch} = \sqrt{k}$$

$$p \le p_{\text{sat}},\ C_e = 0.02$$

$$R_e = C_e \frac{V_{ch}}{\sigma} \rho_l \rho_v \left[\frac{2(p_{\text{sat}} - p)}{3\rho_l} \right]^{1/2} (1 - f)$$

$$p > p_{\text{sat}},\ C_c = 0.01$$

$$R_c = C_c \frac{V_{ch}}{\sigma} \rho_l \rho_v \left[\frac{2(p - p_{\text{sat}})}{3\rho_l} \right]^{1/2} f$$

Cavitation

- Phase slip velocity
  - As in the FLUENT general mixture model, velocity slip between phases is allowed using an algebraic slip formulation.
  - Basic assumption – a local equilibrium between phases should be reached over short spatial length.
  - The slip velocity computed from Manninen et al. or user-defined formulation.
  - Drift flux term appears in the vapor transport equation.
- Variable liquid and vapor density
  - Both liquid and vapor density are allowed to vary (in reality, mainly with pressure /temperature).
  - This can be accomplished either through prescribed functions and/or physical laws, or user defined functions (UDFs)

Cavitation

- Turbulence-induced pressure fluctuations
  - The cavitation model accounts for turbulence-induced pressure fluctuations by simply raising the phase-change threshold pressure from p_{sat} to

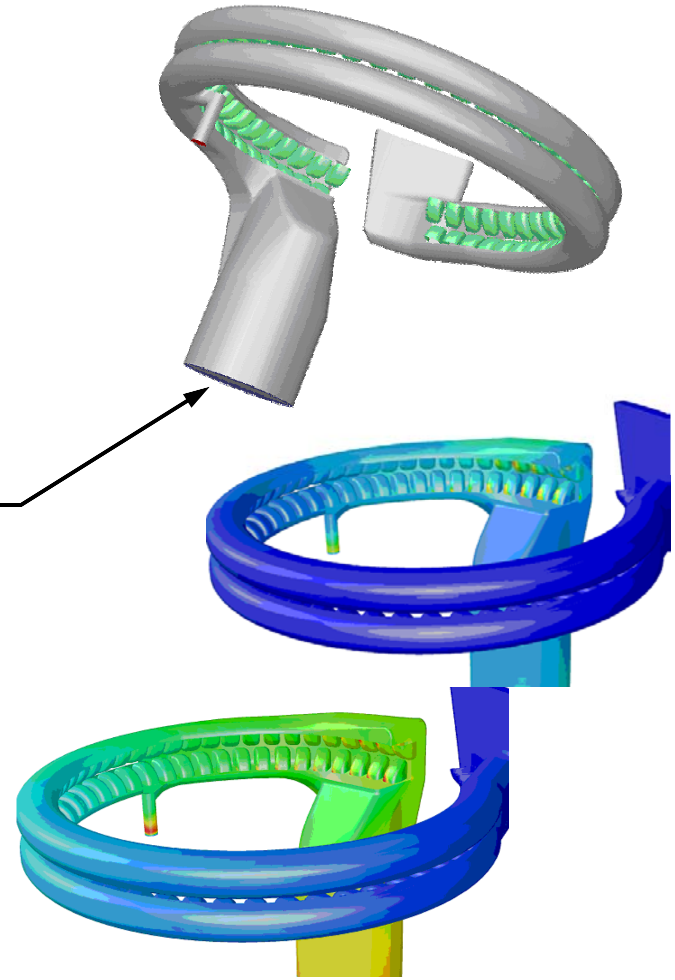
$$p_v = p_{\text{sat}} + \frac{0.39\rho k}{2}$$

- Effects of non-condensable gases
  - The working fluid is assumed to be a mixture of the liquid phase and the gaseous phase, with the gaseous phase comprising of the liquid vapor and the non-condensable gases.
  - The density of the mixture, ρ, is calculated as

$$\rho = \alpha_v \rho_v + \alpha_g \rho_g + (1 - \alpha_v - \alpha_g)\rho_l$$

Cavitating Fuel Pump

- Cavitation in an automotive fuel pump is studied using FLUENT.
- The pump is immersed in the fuel tank
- Fuel (or a mixture of fuel and vapor) is ingested through the inlet, below
- A rotor transports the fuel around to the outlet, above the pump
- Contours of vapor on the pump surfaces for moderate (top) and substantial (bottom) vapor ingestion.



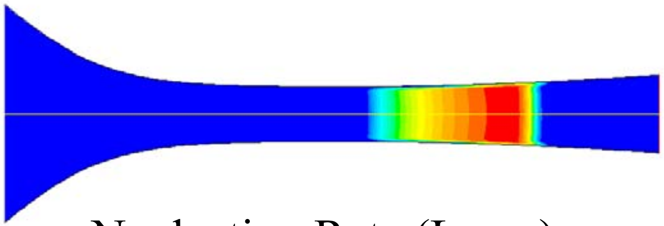
Wet Steam Model

- To model wet steam, two additional transport equations are solved for – mass fraction of the condensed liquid phase and number density of the droplets per unit volume.
  - Mass generation rate is given by the sum of mass increase due to nucleation (the formation of critically sized droplets) and also due to growth/demise of these droplets.
  - Nucleation rate is described by the steady-state classical homogeneous nucleation theory and corrected for non-isothermal effects.
- Restrictions on the wet steam model
  - The wet steam model is available for the coupled solvers only.
  - Pressure inlet, mass-flow inlet, and pressure outlet are the only inflow and outflow boundary conditions available.

Wet Steam Model

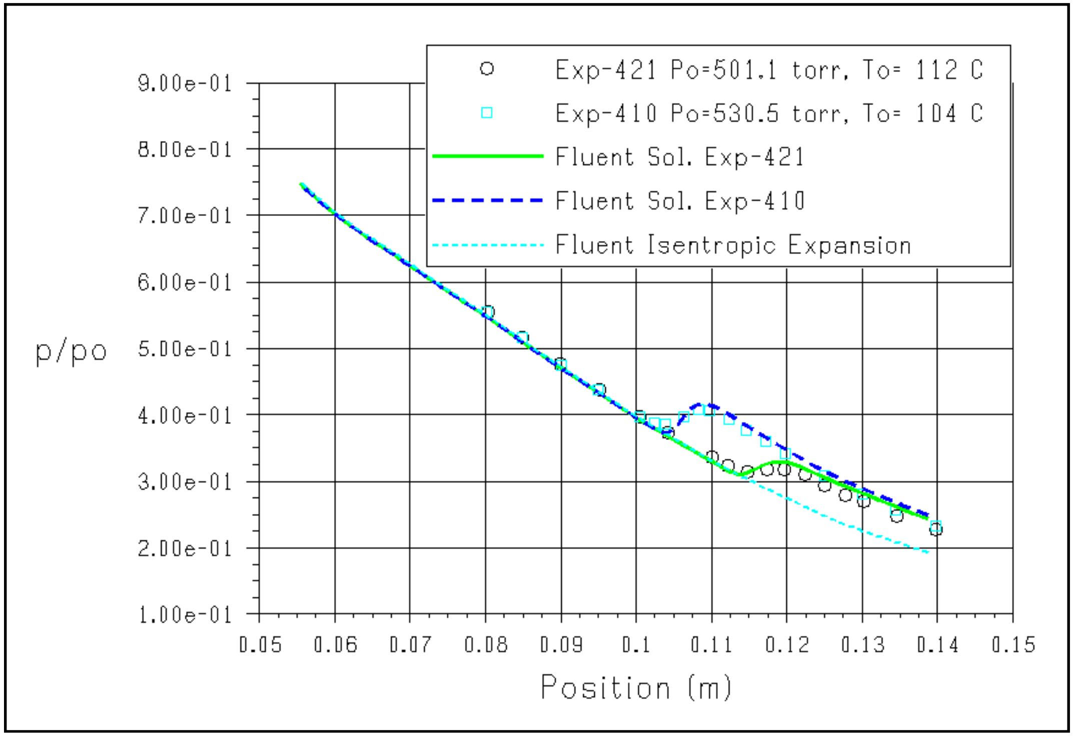
- During the rapid expansion of steam, a condensation process will take place shortly after the state path crosses the vapor-saturation line.
- Phase change model, which involves the formation of liquid-droplets in a homogeneous non-equilibrium condensation process, is based on the classical non-isothermal nucleation theory.
- Assumptions
    - The velocity slip between the droplets and gaseous-phase is negligible.
    - The interactions between droplets are neglected.
    - Since droplet sizes are typically very small it is assumed that the volume of the condensed liquid phase is negligible.

Wet Steam Example – deLaval Nozzle



Nucleation Rate (Log_{10})

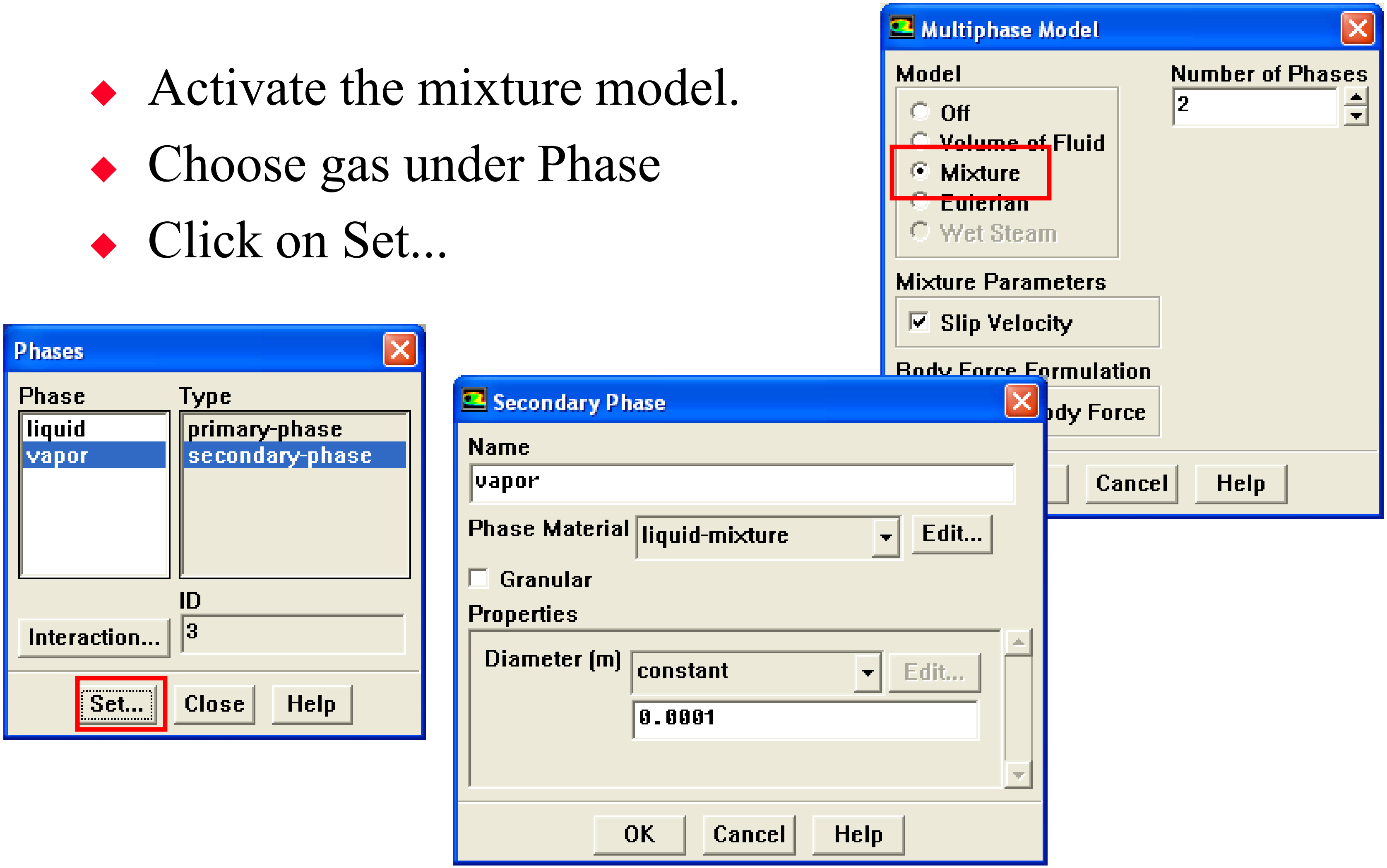
Liquid Mass Fraction



Pressure Profile Comparison

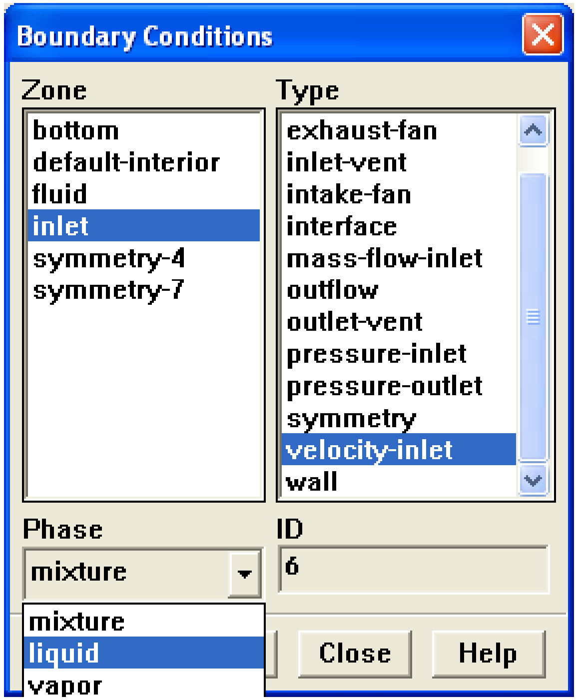
Interface

- Activate the mixture model.
- Choose gas under Phase
- Click on Set...



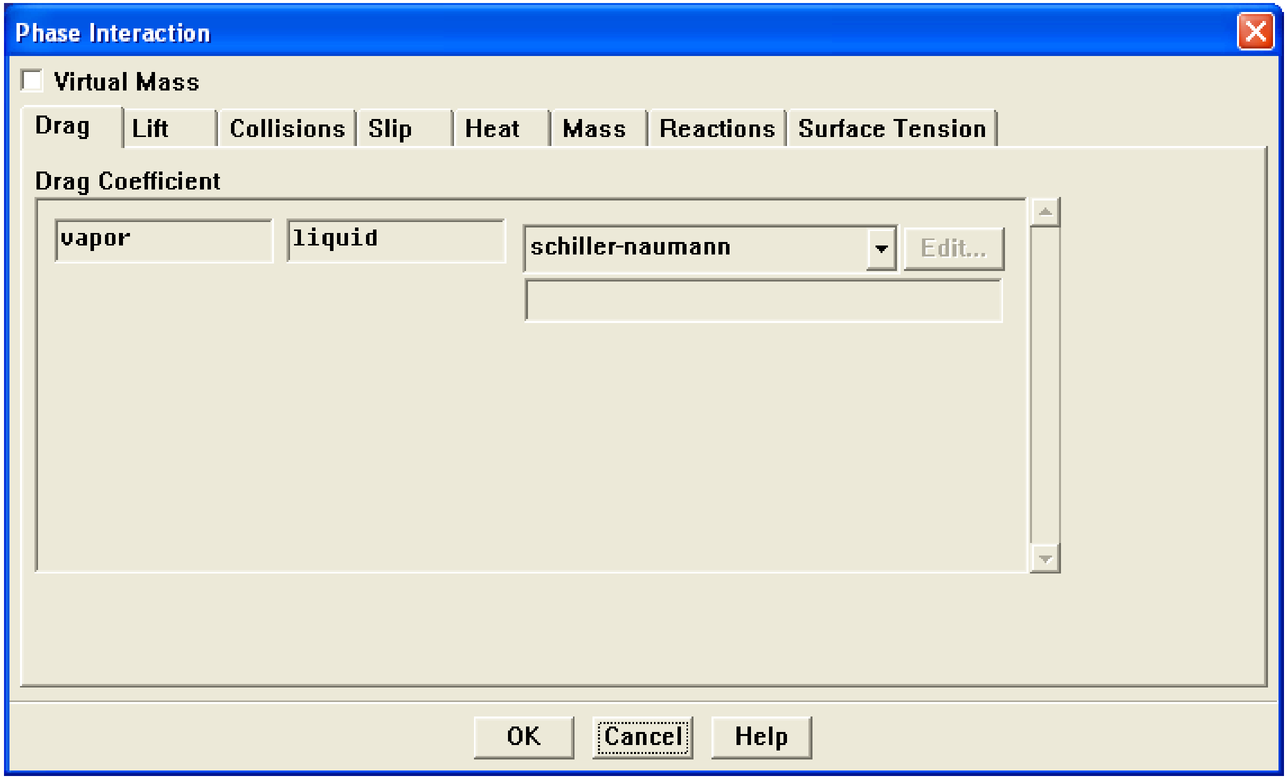
Interface

- Boundary Conditions for mixture and individual phases

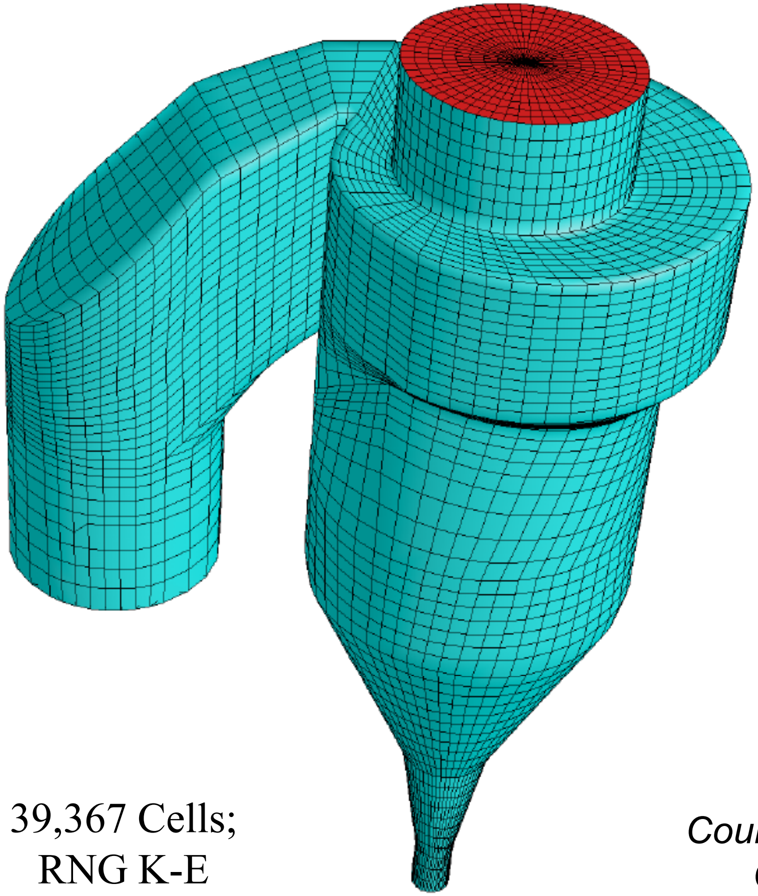


Interface

- Define…Phases…Interaction

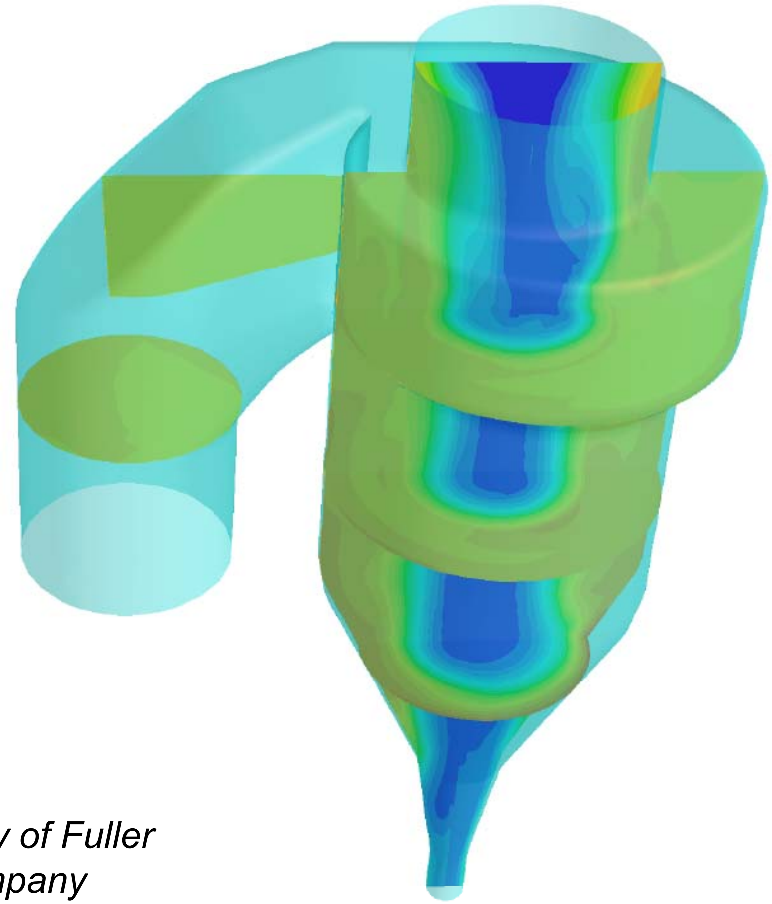


Application of Mixture Model to Cyclones



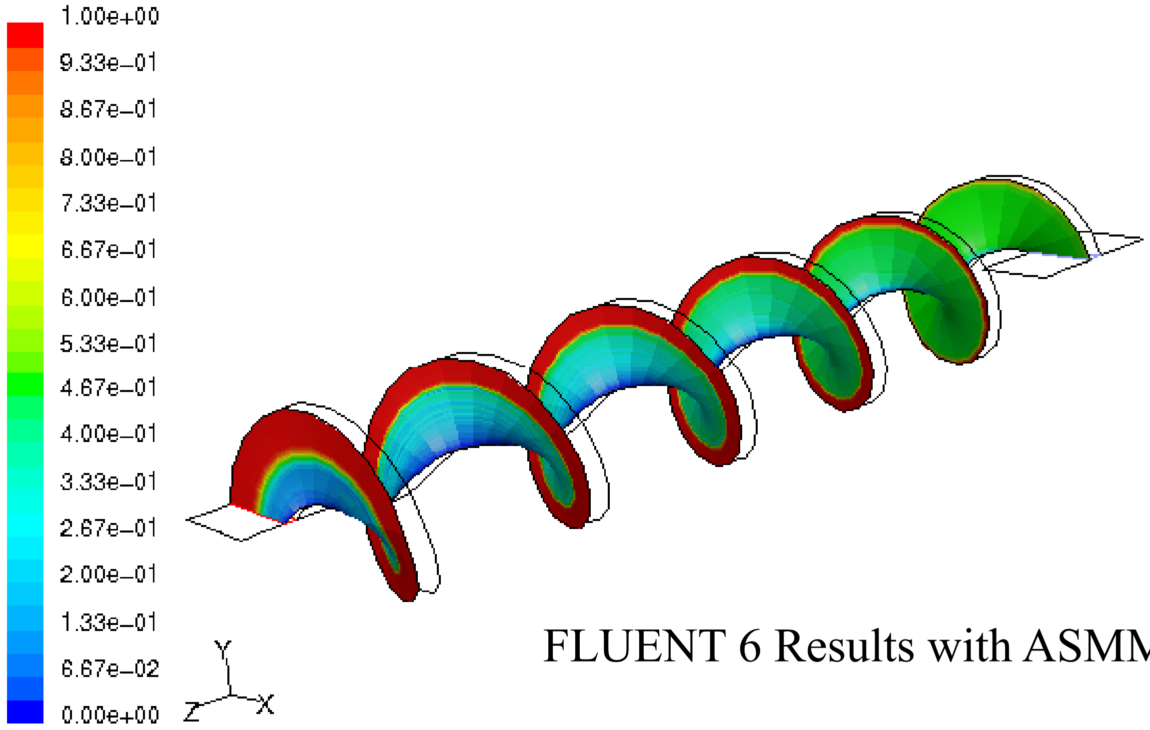
39,367 Cells; RNG K-E Turbulence Model Used

Courtesy of Fuller Company



Solids Volume Fraction Contours (5 micron particles)

Oil-Water Separation

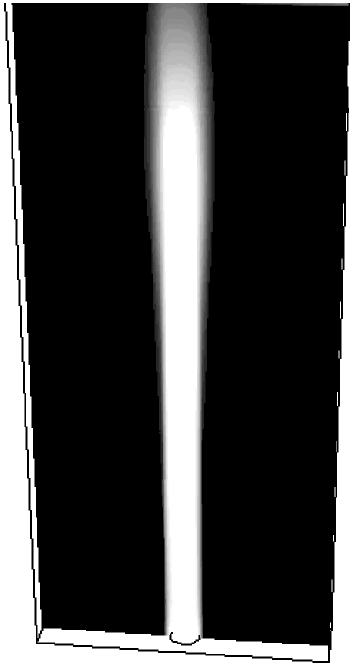


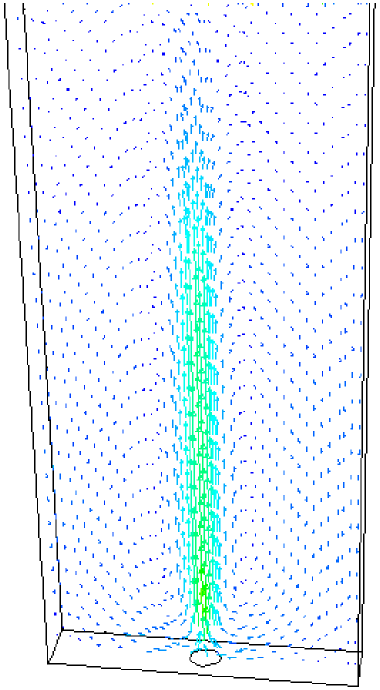
FLUENT 6 Results with ASMM

Courtesy of Arco Exploration and Production Technology

Churn Turbulent Bubble Column

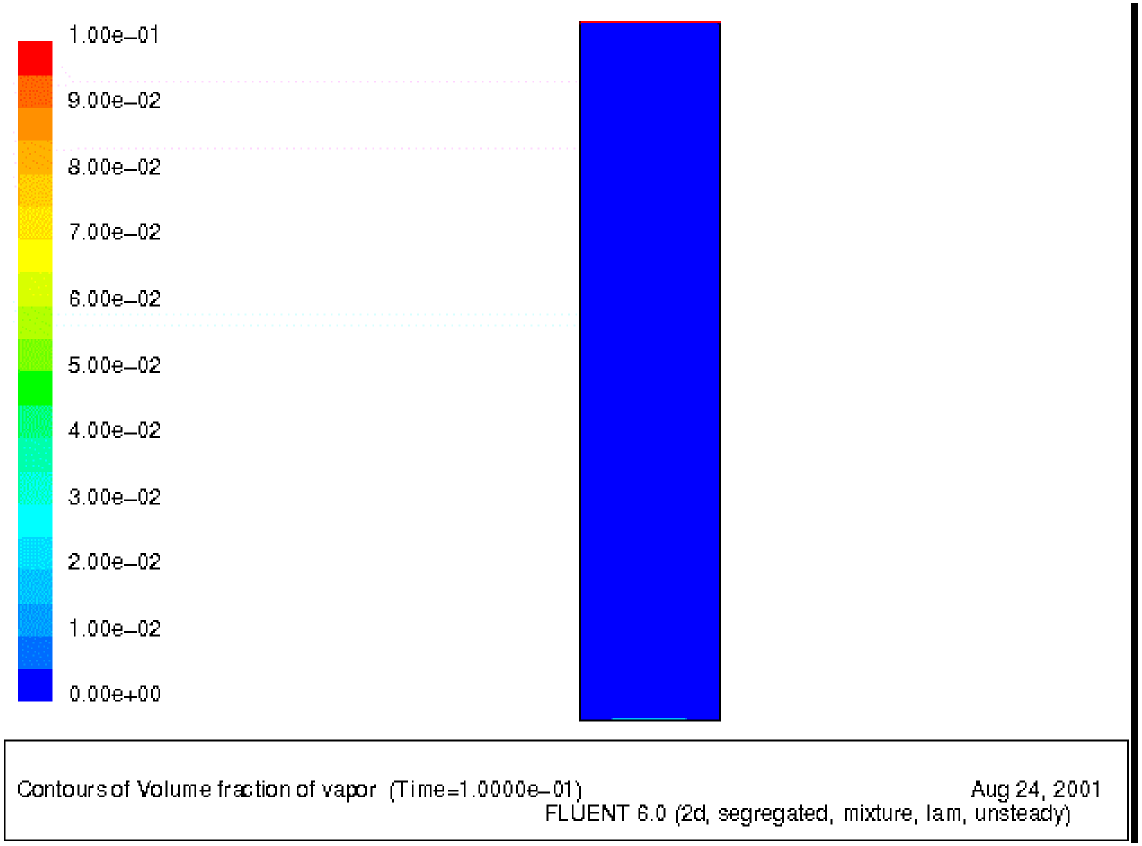
- Churn turbulent bubble column – dynamic simulation





Heat and Mass Transfer

- Boiling example



Solution Strategy

- Is model really applicable?
  - Check particle relaxation time
  - Restrictions of one velocity field for both phases
- Patch volume fraction of dispersed phase.
- Activate Implicit Body Forces.
- Increase underrelaxation for volume fraction 0.2 to 0.5
- Use low underrelaxation for slip velocity
- Solve problem transient.

Summary

- Simple approach
- Easy to use
- Applicable to many multiphase flow problems
- Consider restrictions