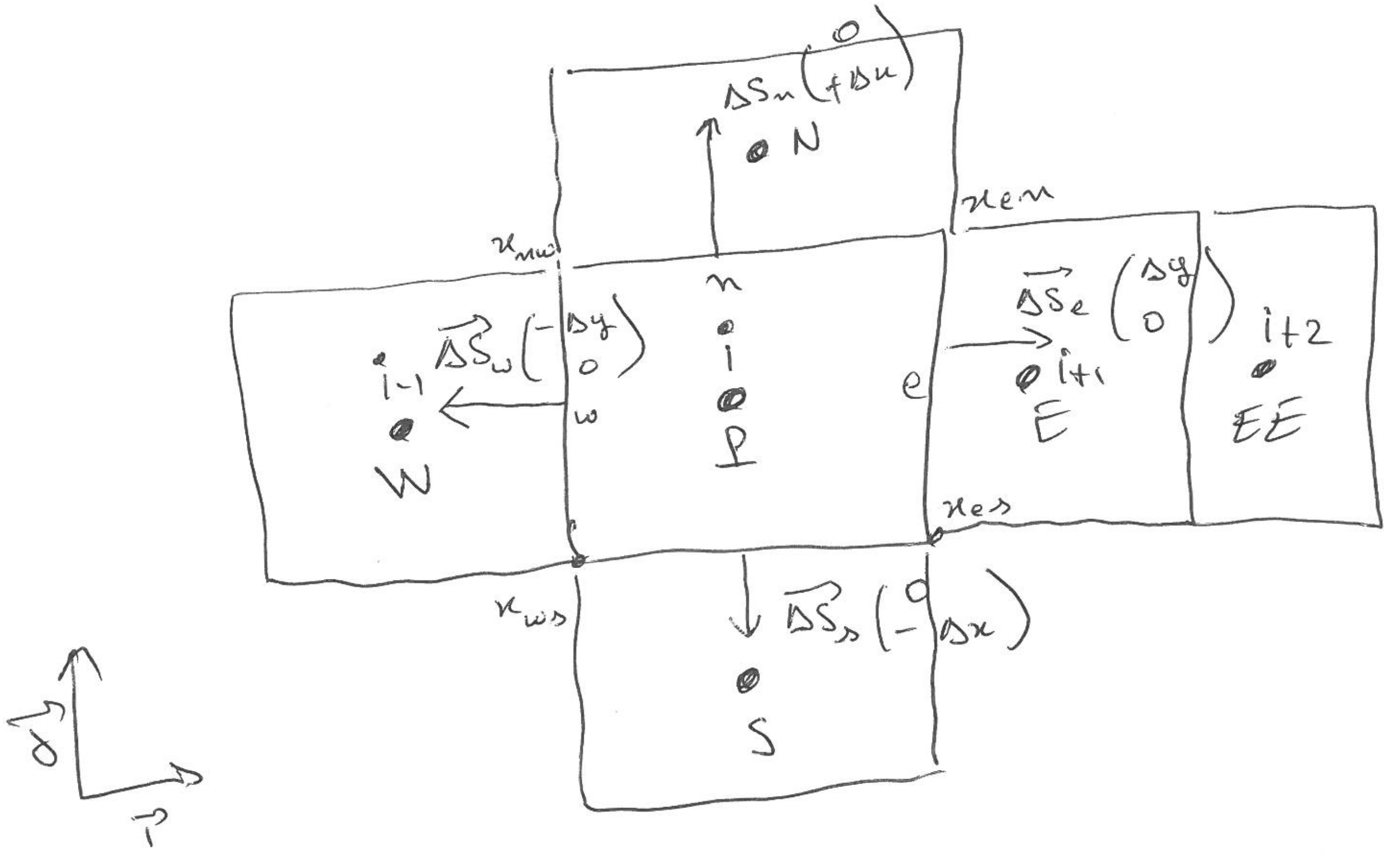
Exo A.6 — Gradient approximation & Spurious oscillations



1) cf Dessin. $\displaystyle\sum_f \overrightarrow{\Delta S_f} = 0.$

2) $\displaystyle V_P = \frac{1}{2} \sum_{faces} \vec{x}_f \cdot \overrightarrow{\Delta S_f}$

$$= \frac{1}{2}\left(\vec{x}_{es} \cdot \overrightarrow{\Delta S_e} + \vec{x}_{en} \cdot \overrightarrow{\Delta S_n} + \vec{x}_{nw} \cdot \overrightarrow{\Delta S_w} + x_{ws} \cdot \overrightarrow{\Delta S_s}\right)$$

avec $\vec{x}_{ws}\begin{pmatrix}0\\0\end{pmatrix}$ $\vec{x}_{es}\begin{pmatrix}\Delta x\\0\end{pmatrix}$ $\vec{x}_{ne}\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}$ $\vec{x}_{nw}\begin{pmatrix}0\\ \Delta y\end{pmatrix}$

$V_P = \frac{1}{2}\left(\Delta x \Delta y + \Delta y \Delta x\right) = \Delta x \Delta y.$ ok!

3)

$$\overline{\nabla\Phi}_P = \frac{1}{V_P}\sum_f \Phi_f \overrightarrow{\Delta S}_f$$

$$= \frac{1}{\Delta x \Delta y}\left[(\Phi_e - \Phi_w)\Delta y \vec{j} + (\Phi_n - \Phi_s)\Delta x \vec{i}\right]$$

puis $\Phi_e = \frac{1}{2}\Phi_E + \Phi_P$, etc....

$$\overline{\nabla\Phi}_P = \begin{pmatrix} \delta_x^o \Phi_P \\ \delta_y^o \Phi_P \end{pmatrix} \quad \text{avec } \delta_x^o \phi_P = \frac{\phi_E - \phi_W}{2\Delta x}$$

4)

$$\left(\frac{\partial\phi}{\partial x}\right)_e = \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)_P + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)_E$$

$$= \frac{1}{2}\left[\frac{\phi_E - \phi_W}{2\Delta x} + \frac{\phi_{EE} - \phi_P}{2\Delta x}\right]$$

$$= \frac{1}{2}\left[\frac{\phi_{i+2} + \phi_{i+1} - (\phi_i + \phi_{i-1})}{2\Delta x}\right]$$

si $\phi_i = c + (-1)^i$

$$\left(\frac{\partial\phi}{\partial x}\right)_e = \frac{1}{4}(-1)^i\left[(-1)^2 + (-1)^1 - \left((-1)^0 + (-1)^{-1}\right)\right] = 0.$$

Minimum correct approach: $N = \overrightarrow{\Delta S}_e$, $\vec{T} = \vec{0}$

$$\left(\frac{\partial\phi}{\partial x}\right)_e = \frac{\phi_E - \phi_P}{\Delta x} \quad \text{formule habituelle !}$$