Géologie Structurale

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Ce qui nous attend aujourd’hui

1- Déformation

2- Contrainte

3 - Élasticité linéraire

(4 – Détermination de ces paramètres)

Previously… Strain (les déformations)

- La deformation est une somme entre une **translation**, une **rotation** et une **transformation**
- Nous l'avons définie comme $\varepsilon = \frac{\Delta L}{L}$
- La deformation est une grandeur tensorielle : $\varepsilon_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$
- Il est défini de telle sorte qu'il ne prend en compte que la transformation

Strain - déformation

$$\bar{\bar{\varepsilon}} = \begin{Bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{Bmatrix}$$

Strain - déformation

Allongement relatif

$$\bar{\bar{\varepsilon}} = \begin{Bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{Bmatrix}$$

Strain - déformation

Variation angulaire

$$\bar{\bar{\varepsilon}} = \left\{ \begin{matrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{matrix} \right\}$$

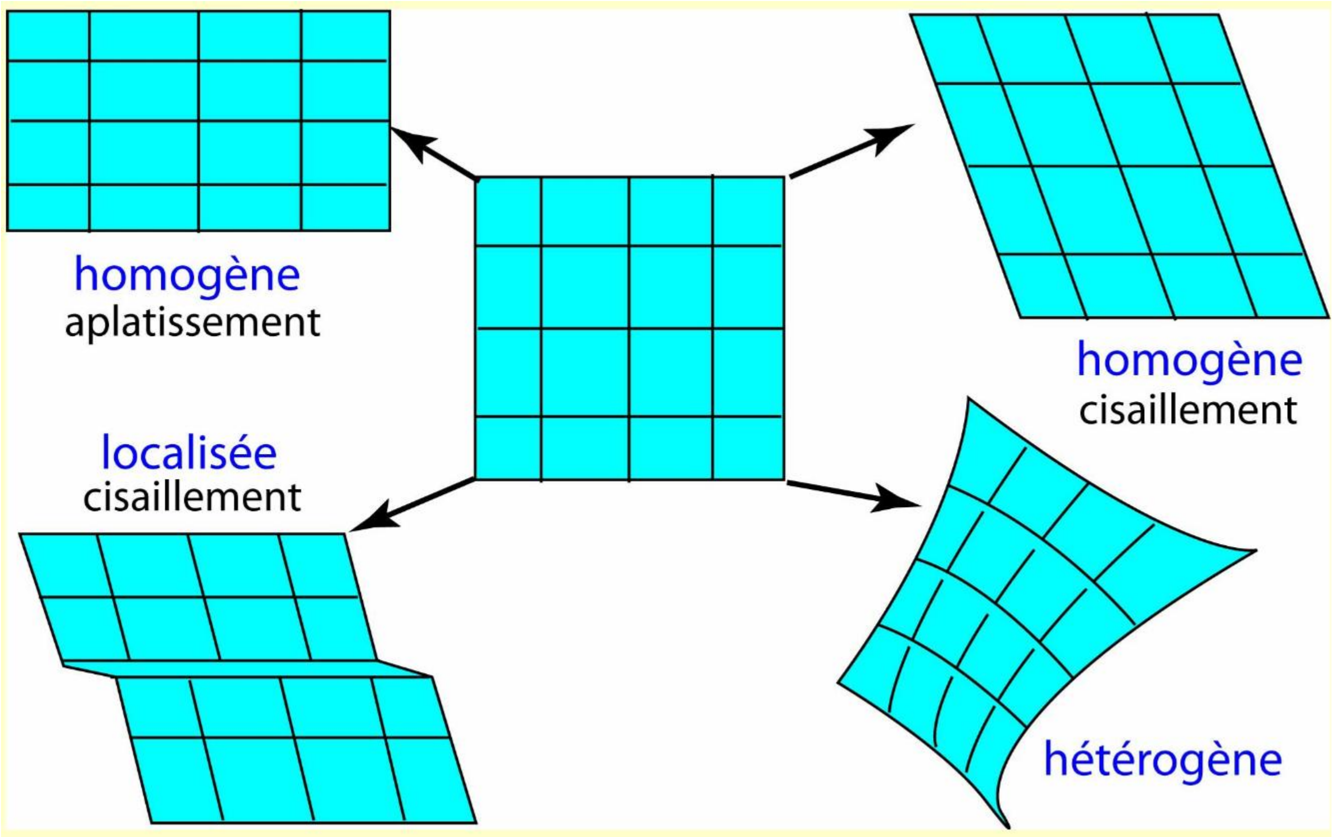
Glossaire

- Déformation cassante ≠ deformation non cassante
- Déformation fragile ≠ deformation ductile

- Déformation élastique : reversible
- Déformation inélastique : irreversible
- Mécanismes de deformation plastiques => mécanismes autres que la fissuration

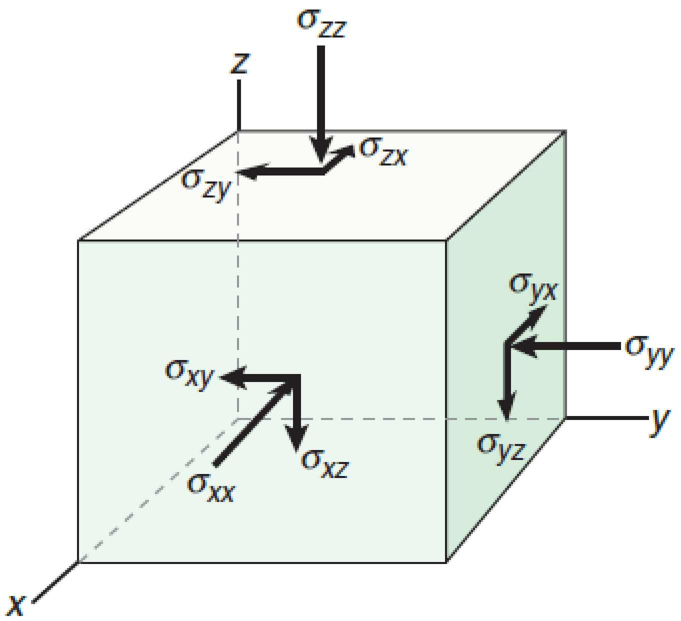
Glossaire

- Déformation homogène
- Déformation hétérogène
- Déformation localisée



Previously… Stress (la contrainte)

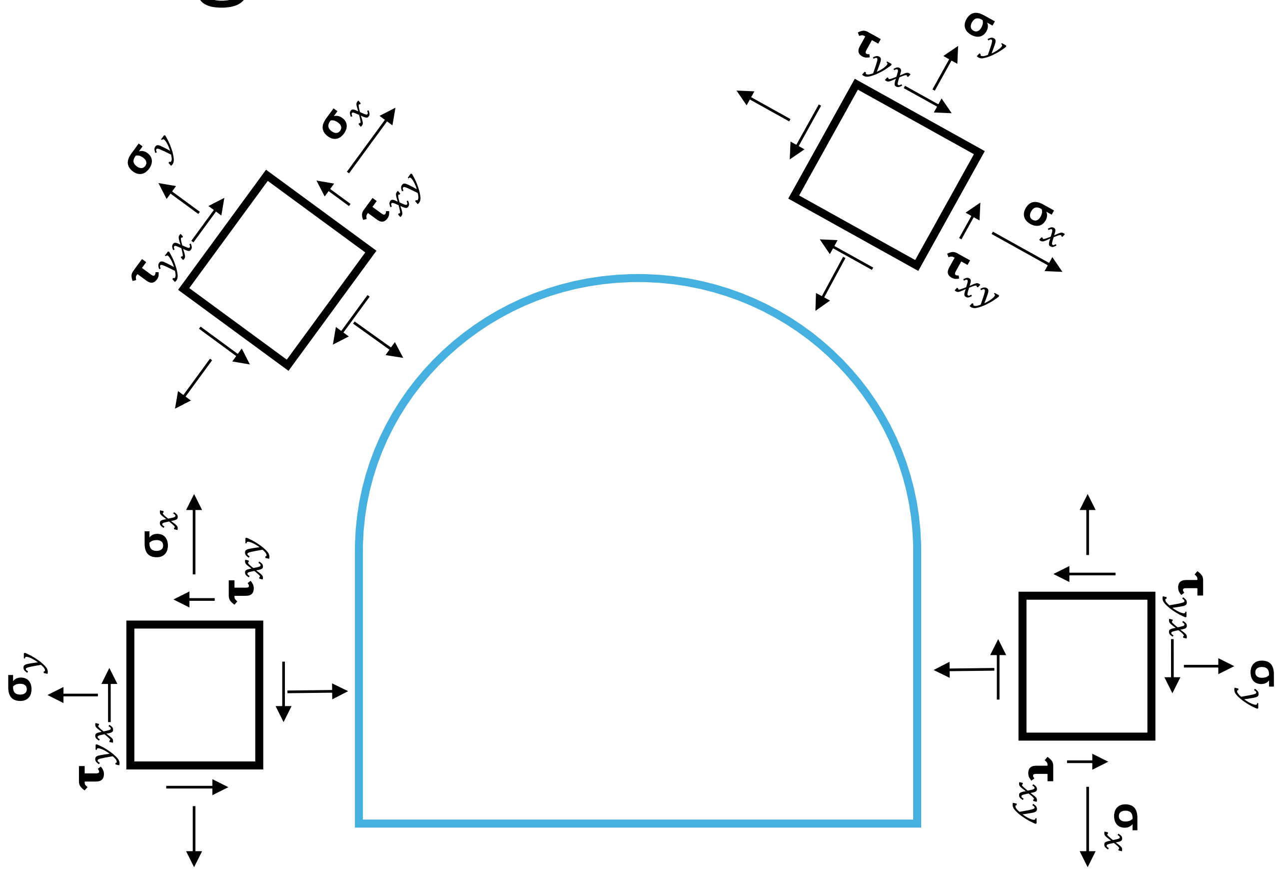
- Nous avons défini le tenseur des contraintes :



$$\bar{\bar{\sigma}} = \begin{pmatrix} \boldsymbol{\sigma}_{xx} & \boldsymbol{\sigma}_{xy} & \boldsymbol{\sigma}_{xz} \\ \boldsymbol{\sigma}_{yx} & \boldsymbol{\sigma}_{yy} & \boldsymbol{\sigma}_{yz} \\ \boldsymbol{\sigma}_{zx} & \boldsymbol{\sigma}_{zy} & \boldsymbol{\sigma}_{zz} \end{pmatrix}$$

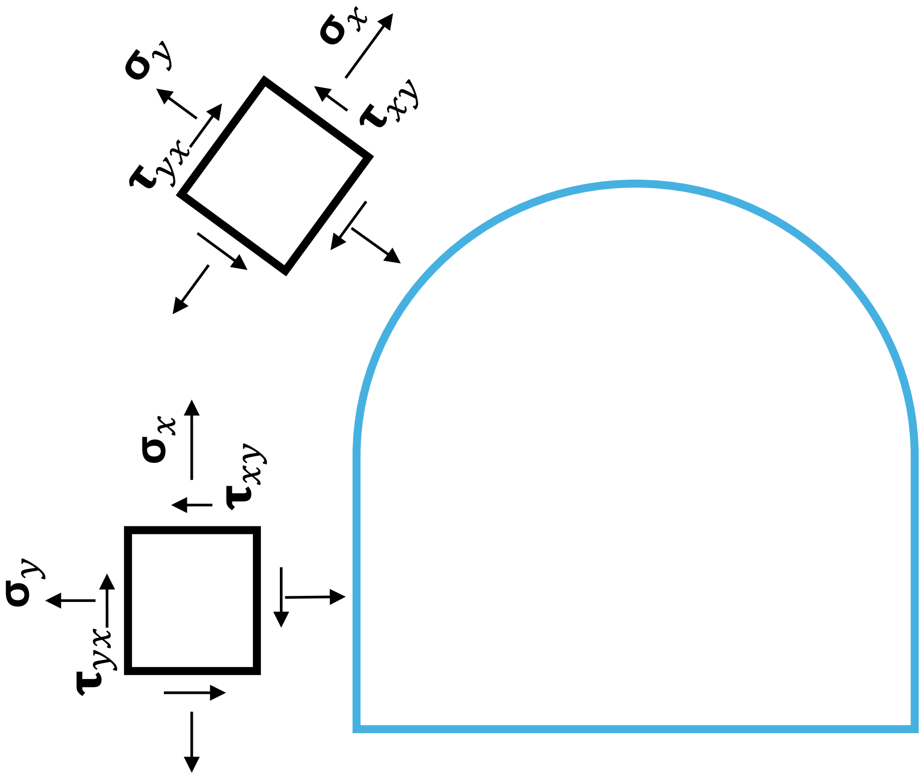
Lorsque l’on représente un élément de volume, on ne représente pas directement le tenseur des contraintes. On représente une projection du tenseur par rapport à chaque face (selon la normale entrante ou sortanteen fonction de la convention de signe utilisée)

Quel est l'état de contrainte sur la paroi de mon ouvrage?



Stress (la contrainte)

Expression des contraintes normales et tangentielles (problème plan):



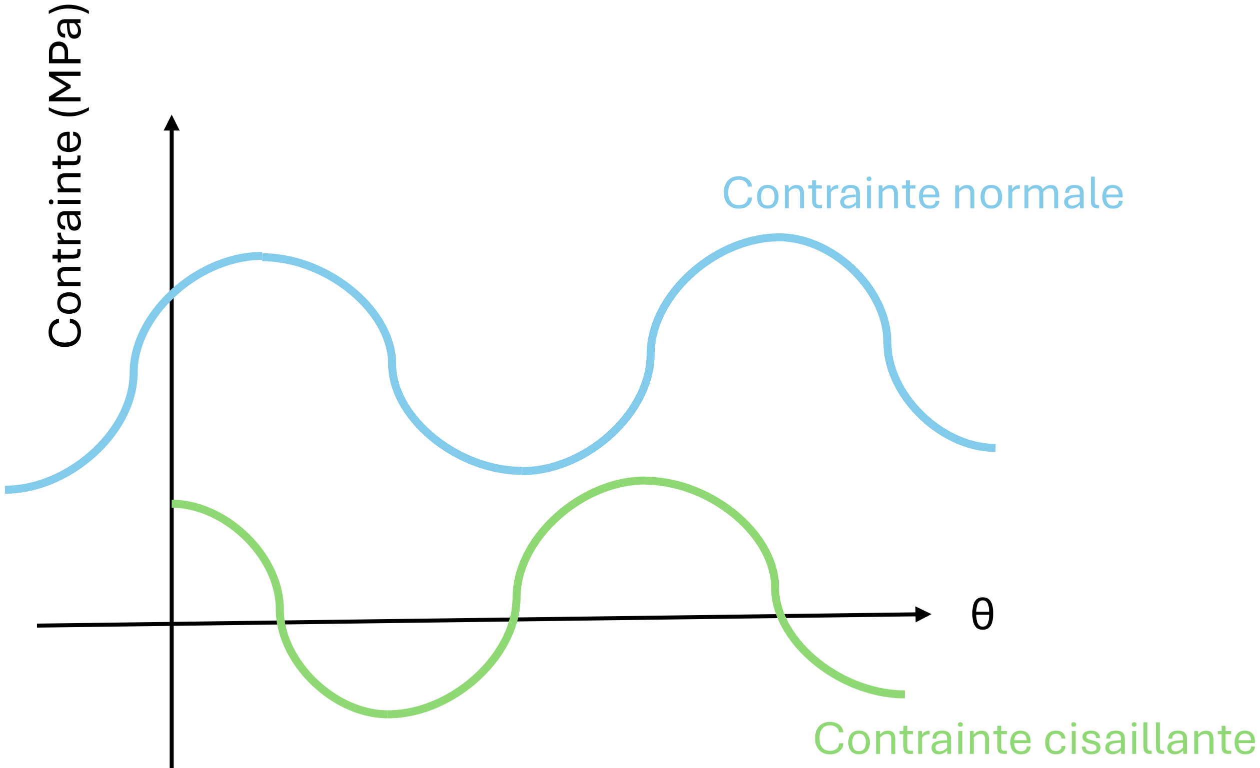
Composante σ_n selon n :

$$\sigma_n = \vec{n} \cdot \bar{\bar{\sigma}}\, \vec{n} = \cos(\theta)^2 \sigma_x + 2\cos(\theta)\sin(\theta)\tau_{xy} + \sin(\theta)^2 \sigma_y = \boxed{\frac{\sigma_x + \sigma_y}{2} + \cos(2\theta)\frac{\sigma_x - \sigma_y}{2} + \sin(2\theta)\tau_{xy}}$$

Composante $\boldsymbol{\tau}_n$ selon n :

$$\tau_n = \sin(\theta)\cos(\theta)(\sigma_y - \sigma_x) - \sin(\theta)^2 \tau_{xy} + \cos(\theta)^2 \tau_{xy} = \boxed{\sin(2\theta)\frac{\sigma_y - \sigma_x}{2} + \cos(2\theta)\tau_{xy}}$$

Stress (la contrainte)



Points remarquables :

- Lorsque σ_n est max ou min, τ s'annule
- On appelle ces valeurs les contraintes principales

Composante σ_n selon n :

$$\sigma_n = \vec{n} \cdot \bar{\bar{\sigma}}\,\vec{n} = \cos(\theta)^2\sigma_x + 2\cos(\theta)\sin(\theta)\tau_{xy} + \sin(\theta)^2\sigma_y = \boxed{\frac{\sigma_x + \sigma_y}{2} + \cos(2\theta)\frac{\sigma_x - \sigma_y}{2} + \sin(2\theta)\tau_{xy}}$$

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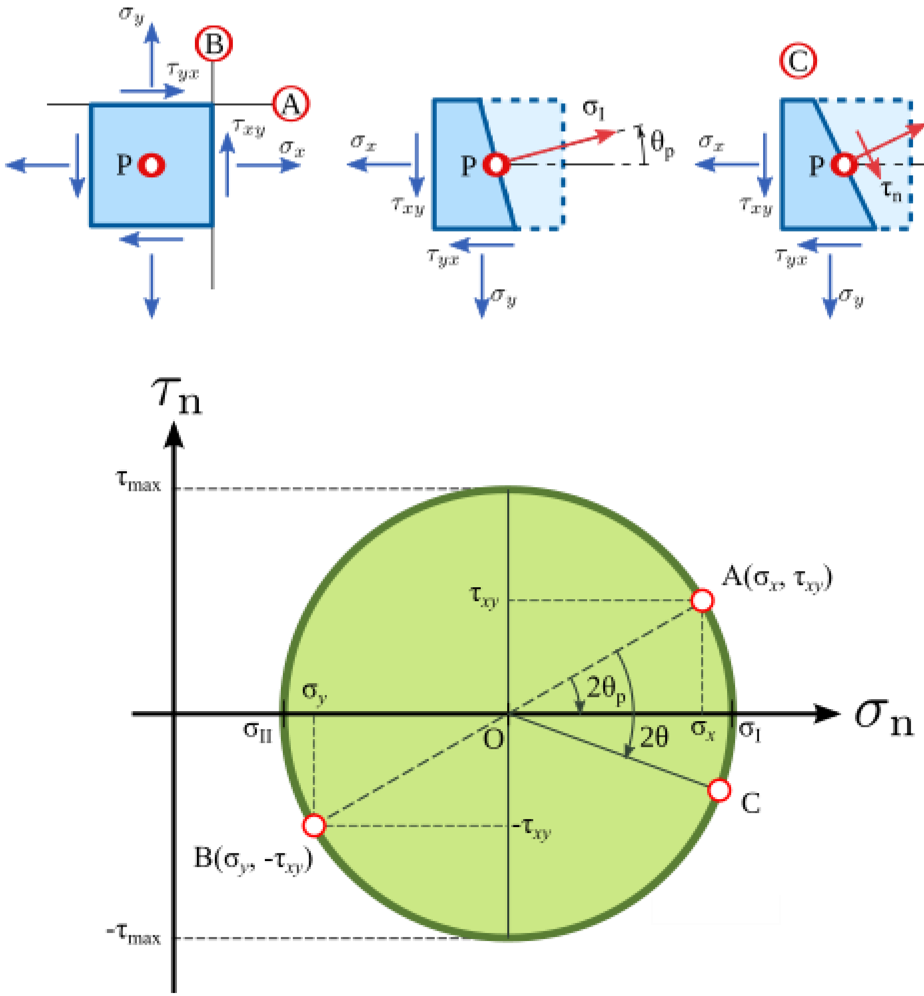
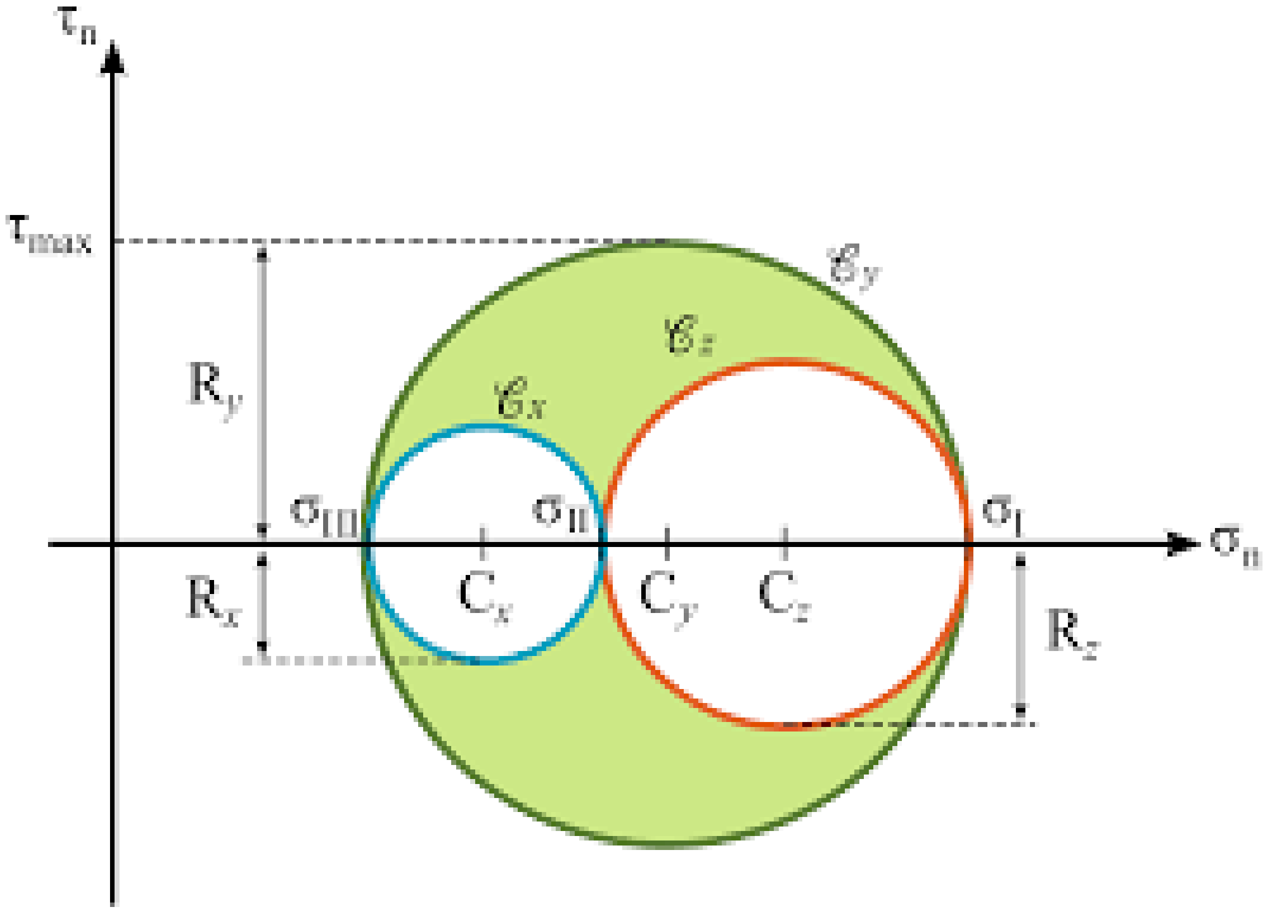
Cercle de Mohr (contraintes planes)



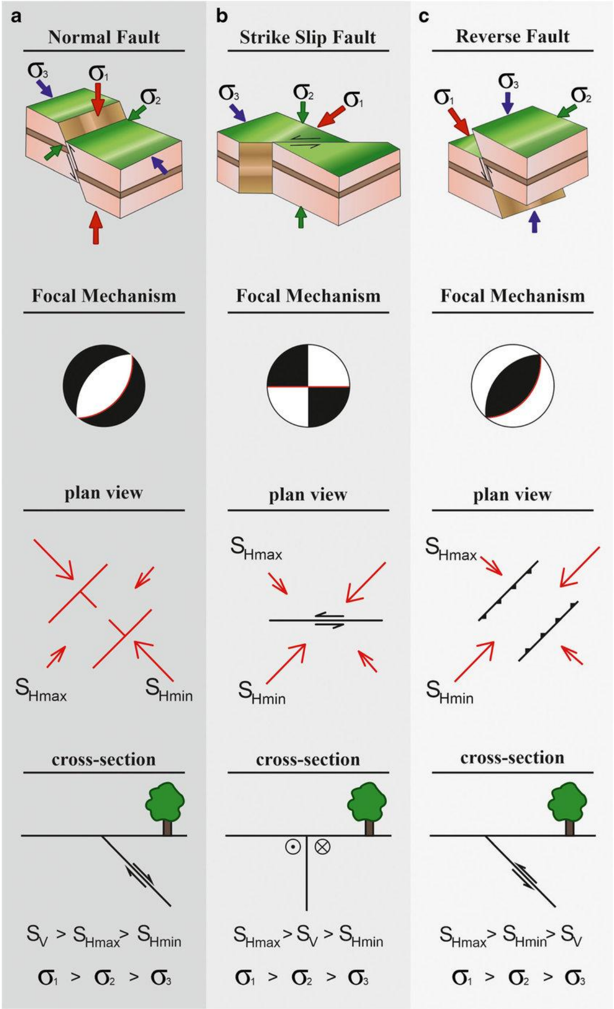
On combine les deux expressions:

$$\boxed{(\sigma_n - \frac{\boldsymbol{\sigma}_x + \boldsymbol{\sigma}_y}{2})^2 + \boldsymbol{\tau}_n{}^2 = (\frac{\boldsymbol{\sigma}_x + \boldsymbol{\sigma}_y}{2})^2 + \boldsymbol{\tau}_{xy}{}^2}$$

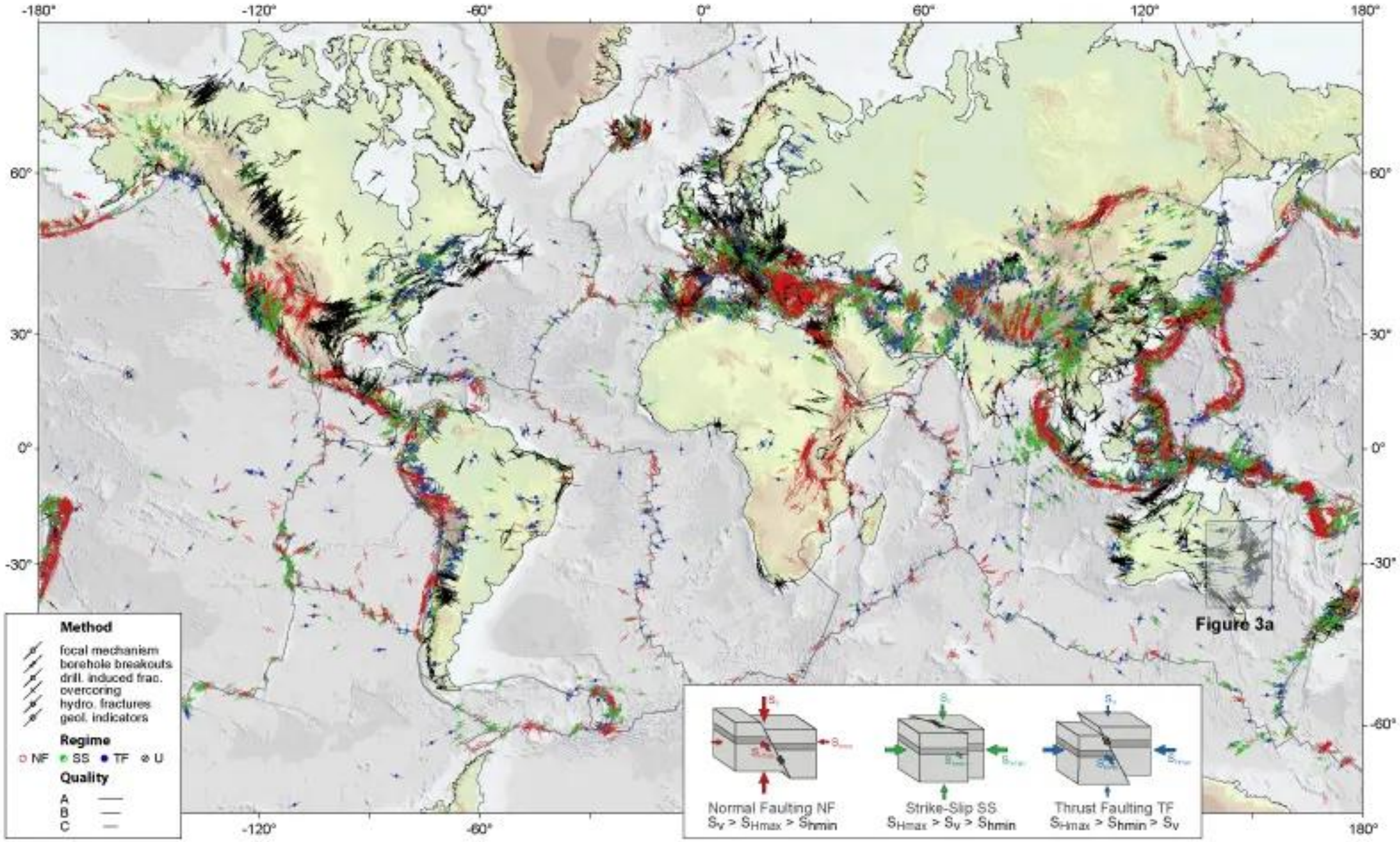
Tri-Cercle de Mohr



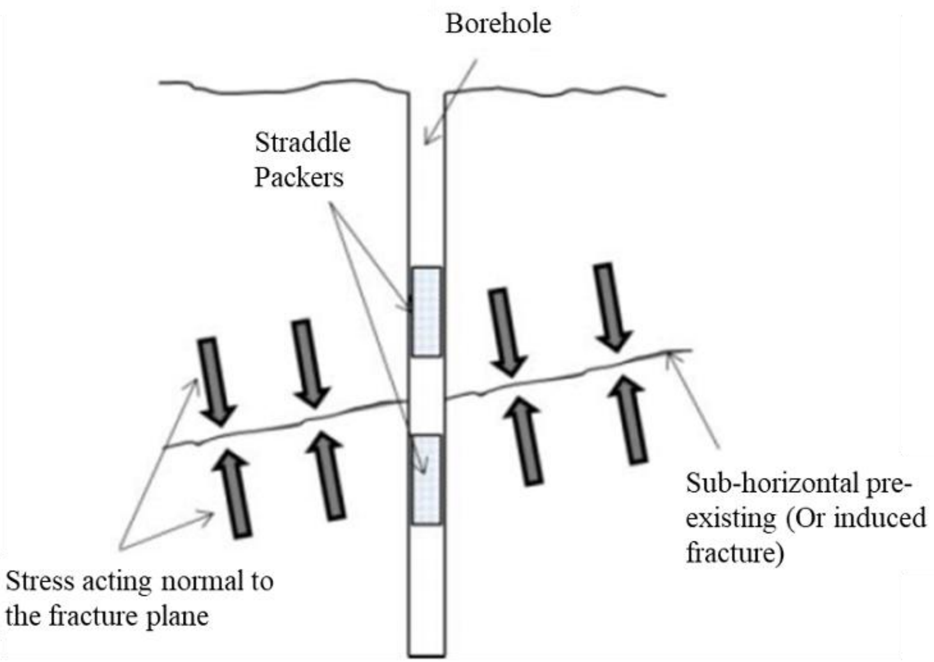
Contraintes principales

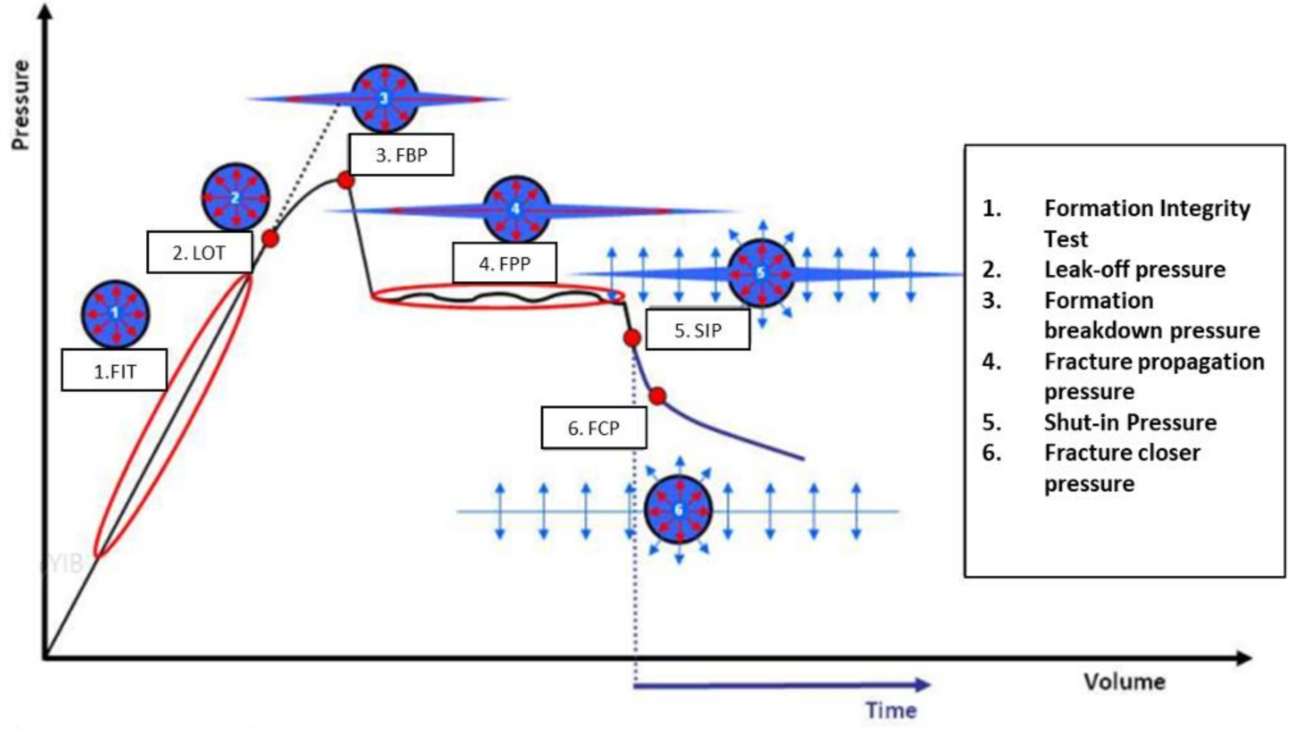


World Stress map : https://www.world-stress-map.org/



Estimer les contraintes in-situ





Estimer les contraintes in-situ (2)

Overcoring method

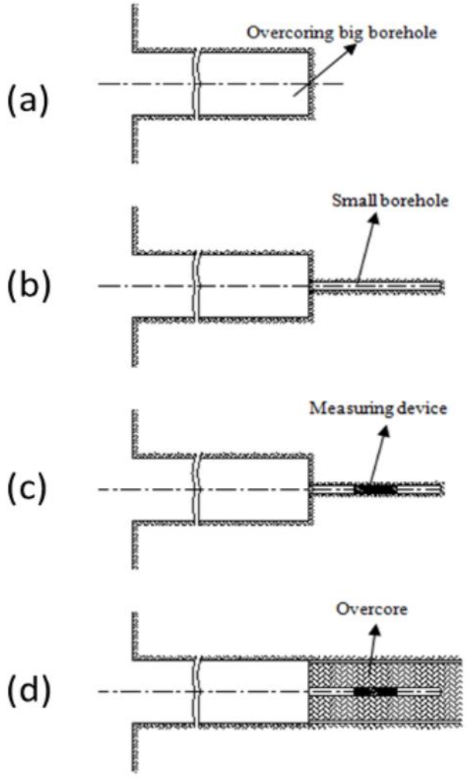


Figure 14 The sequence of overcoring method: (a) Drilling a large diameter hole. (b) Drilling a smaller pilot hole. (c) Placing the measuring device in the smaller hole. (d) Drilling the large diameter hole is resumed and the measuring device is overcored. (from Guo et al.[11]).

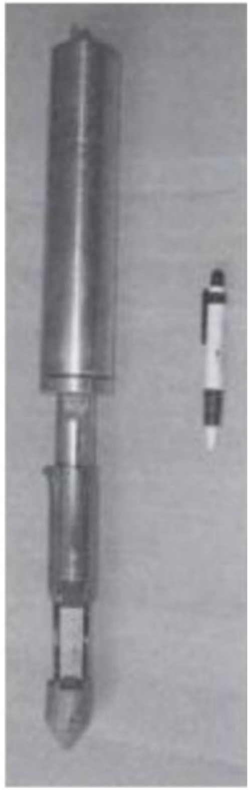
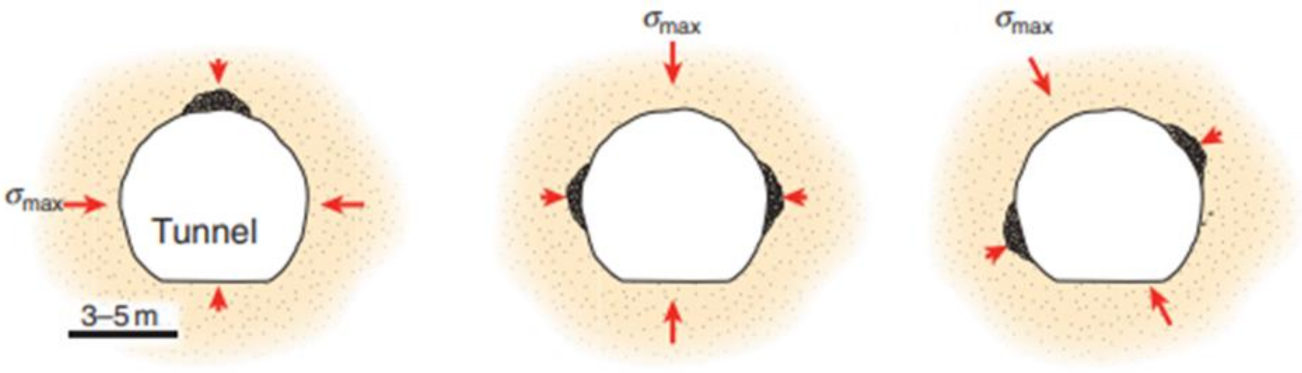
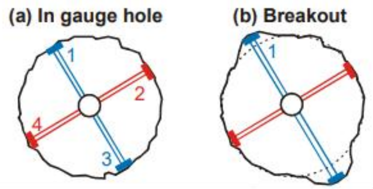


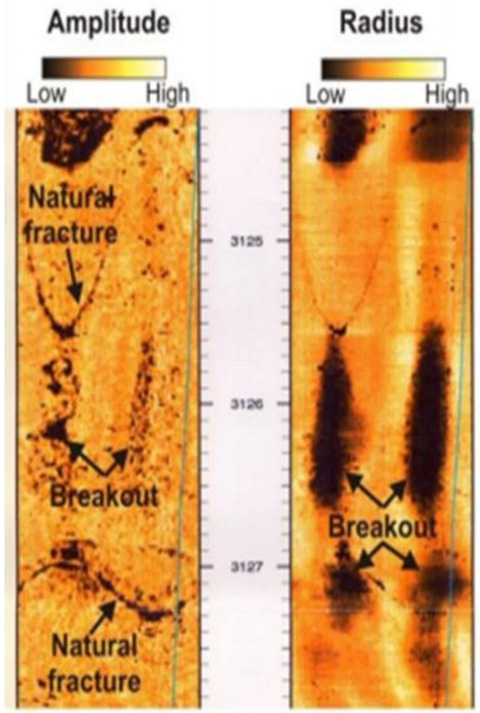
Figure 15 Overcoring tool (from Hudson[12]).

Estimer les contraintes in-situ (3)

Borehole breakouts

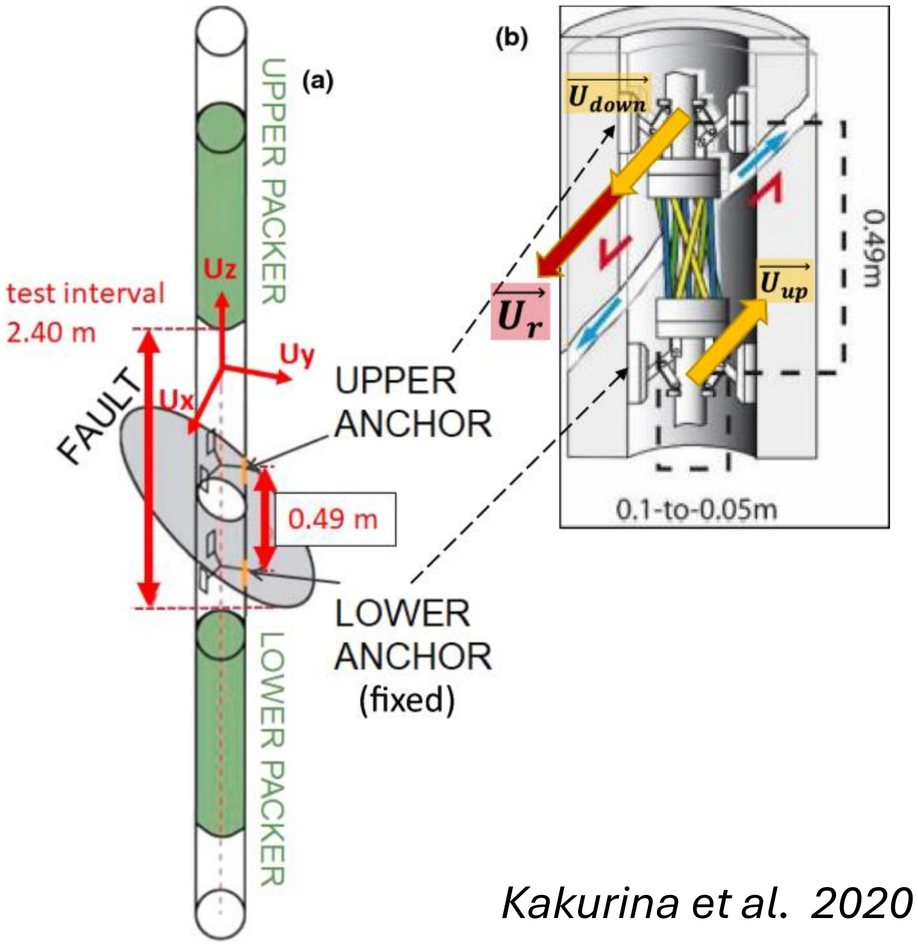






Heidbach et al. 2016

Estimer les contraintes in-situ (4)



Kakurina et al. 2020

Elasticité linéaire

Dans le cas isotrope élastique, on peut relier le tenseur des contraintes au tenseur des deformations par la relation suivante :

$$\sigma = C\varepsilon$$

Où :

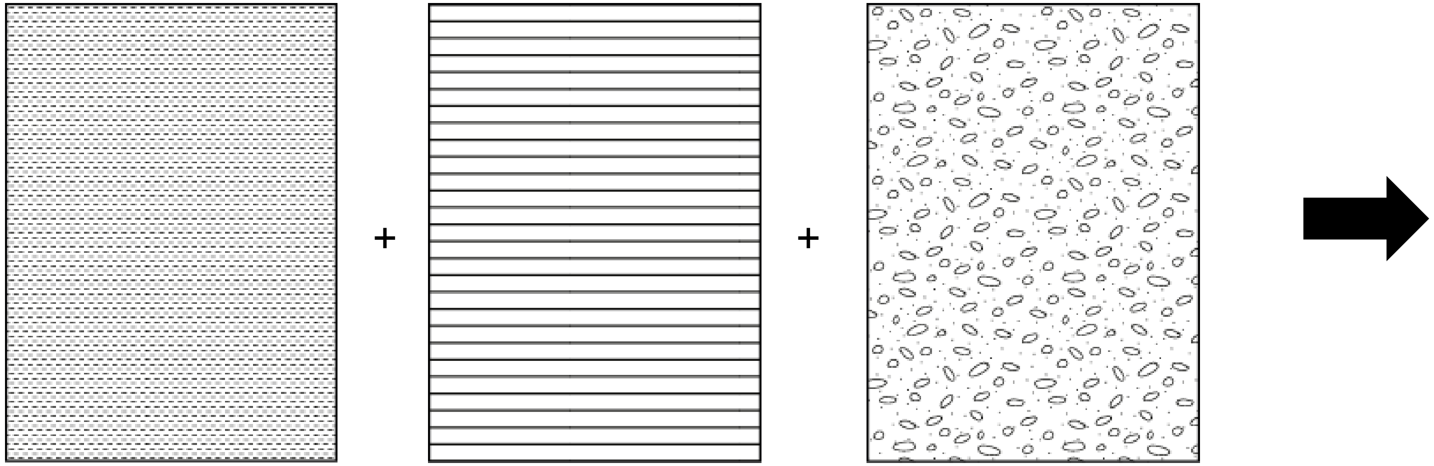
$$c = \frac{E}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{pmatrix}$$

Isotrope : ne dépend pas de la direction
Élastique : deformation réversible

What is anisotropy?

« Lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les effets produits.» (Pierre Curie, 1894)

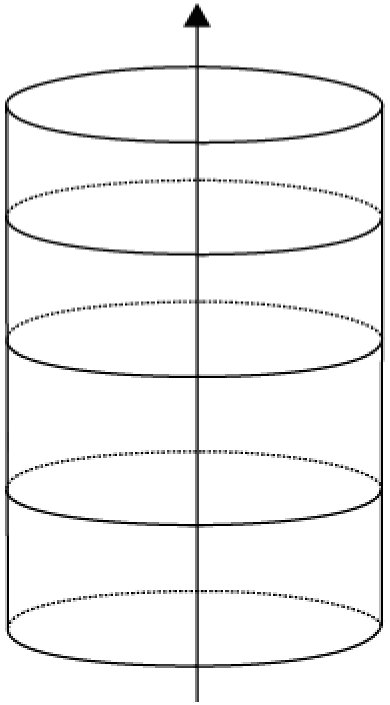
Physical properties depend on direction



Mineral shape

Bedding

Pores and cracks alignment



Transversly isotropic material

Horne, 2013; Bandyopadhyay, 2009b

Rhéologie

DEFORMATION OF ROCKS IN THE LABORATORY

David Griggs

Introduction--In the search for understanding of the causes of deformation of the Earth's crust, it is particularly important to know two physical properties of the rocks constituting the crust--properties which are analogous to strength and viscosity as measured in engineering testing laboratories. Experiments on the deformation of rocks show that neither of these is a simple or easily defined property. In the following, "strength" will be used rather loosely as the stress which causes rupture under specified conditions. "Equivalent viscosity" will be used to describe the flow of rocks under constant stress. It is a term which is derived in the same manner as ordinary viscosity of liquids, but different in that it varies as a function of the applied shear-stress.

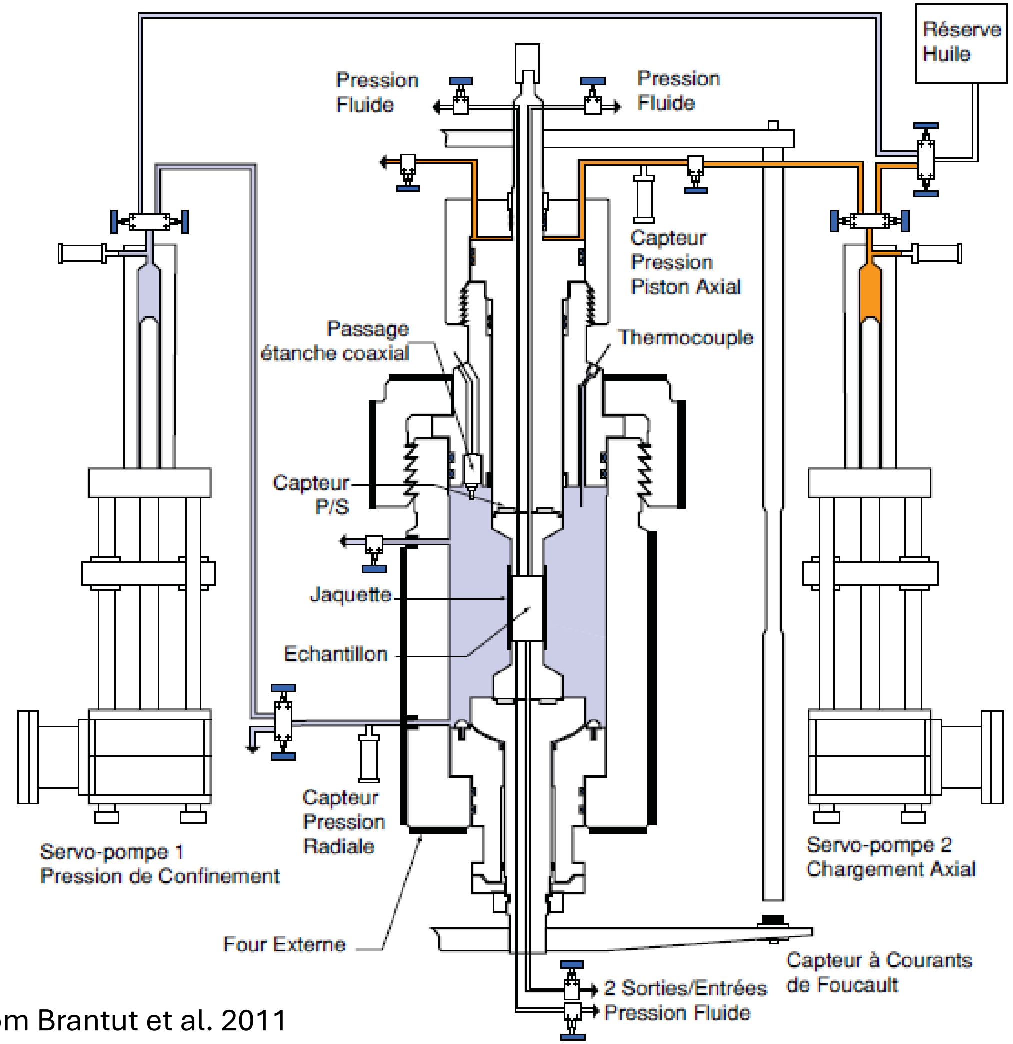
Strength and equivalent viscosity vary as functions of all five of the environmental variables in the Earth's crust, and an experimental approach to the determination of these two properties must determine first the effect of varying each variable separately with all the rest held constant, and then the effect of simultaneous variation of combinations of the variables. These variables are:

(1) Confining (hydrostatic) pressure
(2) Temperature
(3) Time
(4) Solutions
(5) Shear-stress

Griggs, 1940

Experimental setup

From 0-100MPa of confining pressure
(less than 10km depth)



From Brantut et al. 2011

Comportement mécanique des roches